Math 10860, spring 2020

First midterm exam, Monday March 2

NAME:

Instructions

- The exam goes from 11.35pm to 12.35pm.
- There are 4 questions, plus a bonus question. Present your answers in the space provided. Use the back of each page if necessary; if you do, clearly indicate this.
- Present your answers clearly and neatly. Remember that the exam is a chance for you both to show me what you know, and a chance to show me that you can *clearly tell me* what you know.
- **Justify all your assertions**, even if a question does not explicitly say this. Partial credit can be given, but only if your answers are supported.
- Calculators are not allowed, nor should they be needed.
- No notes, books or any other external resources are allowed.
- Remember the Academic Code of Honor Pledge:

> "As a member of the Notre Dame community, I acknowledge that it is my responsibility to learn and abide by principles of intellectual honesty and academic integrity, and therefore I will not participate in or tolerate academic dishonesty."

MAY THE ODDS BE EVER IN YOUR FAVOR!

Question	score	out of
1		12
2		9
3		9
4		10
5 (bonus)		2
Total		40

1. (4+4+4 points)

(a) State the first part of the Fundamental Theorem of Calculus (FTOC1) (with all necessary hypotheses).

(b) Use FTOC1 to prove that if f is a continuous function defined on an interval I then there are functions g defined on I for which $g' = f$, and that for any such g and any $a < b$ in I

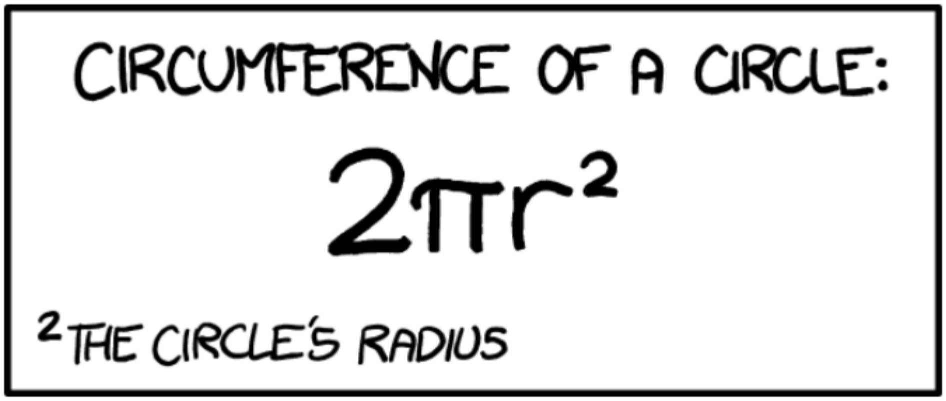
$$\int_a^b f = g(b) - g(a).$$

(c) State the second part of the Fundamental Theorem of Calculus (with all necessary hypotheses).

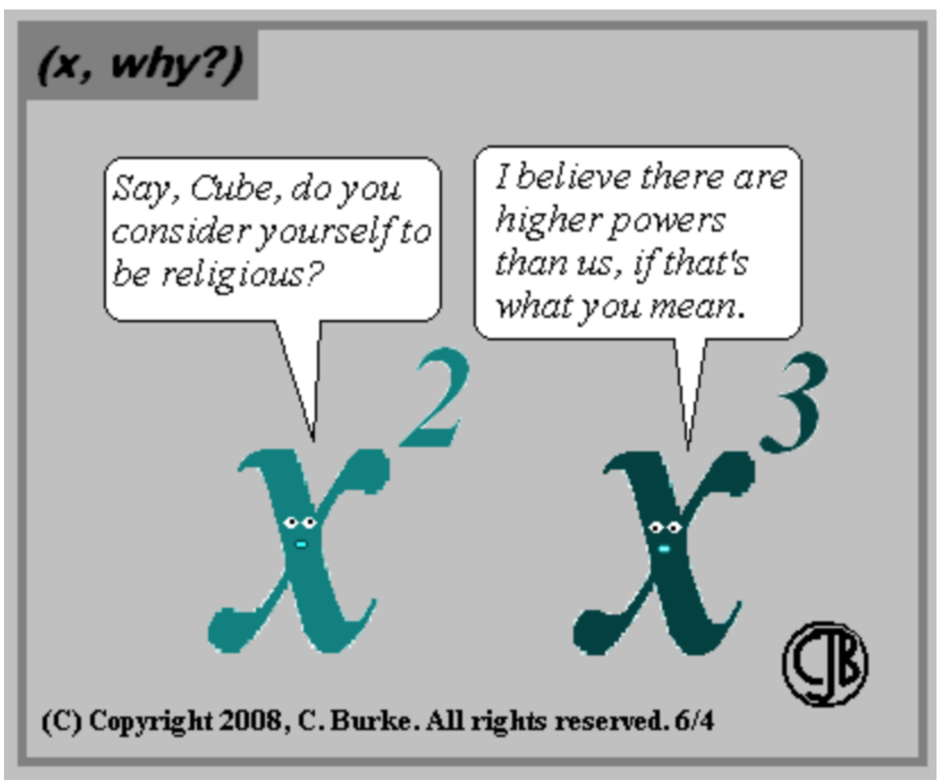
2. (4+5 points)

(a) A function $f : (a, b) \to \mathbb{R}$ is bounded on every closed interval contained in (a, b), but is unbounded on (a, b), and in fact $\lim_{x \to a^+} f(x) = \lim_{x \to b^-} f(x) = \infty$. What is the correct way to interpret the improper integral $\int_a^b f$? (Your answer should address when the improper integral exists).

(b) Find all real numbers r for which $\int_0^\infty \frac{dx}{(1+x)^r}$ exists (with justification; you can state without proof any (correct) properties of any functions that you use in your justification).



xkcd by Randall Munroe



(x, why?) by Christopher Burke

3. (4+5 points)

   (a) Give the definition of the function cos on the domain $[0, 2\pi]$.

   (b) State the domain and range of arccos (the function also known as $\cos^{-1}$) and compute its derivative. You may assume any facts you know about sin, cos. You should state any facts you use about derivatives of inverse functions. Your final answer should not involve any trigonometric functions.

4. (5+5 points) (**NB**: these two parts are not intended to be related)

(a) Find, with proof, all functions $f : \mathbb{R} \to \mathbb{R}$ that satisfy $f' = -f$. (**Note**: $f' = -f$, not $f' = f$.)

(b) Using what you know about functions satisfying $f'' + f = 0$ (or otherwise, but in this case not for full credit) prove that sin is an odd function.

5. (Bonus question, 2 points) What is $\sin(\pi/8)$?