Math 10850, Honors Calculus 1

Things to think about for first tutorial

Thursday, August 29 2019

1. In class we built the truth-table of "p or (q and r)" (symbolically, $p \vee (q \wedge r)$), and found it to be:

p	q	r	q and r	p or (q and r)
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

   Construct the truth table of "(p or q) and (p or r)", and compare it with the table above (by which I mean, compare the two final columns; to make a comparison meaningful, be sure that the first three columns of the table you construct are identical to the first three columns above).

   You have just verified one of the laws of logical operators, the distributive law:

   "p or (q and r)" is the same[1] as "(p or q) and (p or r)".

   Convince yourself that this is a perfectly reasonable law!

2. The *exclusive or* operator, often written "p xor q", is true exactly when either p is true, or q is true, *both not both are true.*

   (a) Construct the truth table of xor.

   (b) Find an expression that only uses $\neg$, $\wedge$ and $\vee$, that has the same truth table as "p xor q".

[1] What do I mean, "is the same"? Since this will come up frequently, I should clarify it here. If A is one statement built from the simpler statements p, q and r, using combinations of $\neg$, $\wedge$ and $\vee$, and B is another one, then A and B are *the same* (more correctly, *equivalent*) if: for each possible assignment of truth values to p, q and r, the truth value of A is the same as the truth value of B. Effectively this means that if you use a single truth table to figure out what A and B look like, then the column corresponding to A is the same as the column corresponding to B. Of course, this can be extended to pairs of statements built from any number of simpler statements. This is discussed in the class notes

3. (a) Confirm that

$$\neg(p \vee q) \quad \text{and} \quad (\neg p) \wedge (\neg q)$$

have the same truth tables. This is an example of one of *De Morgan's* laws.

(b) Confirm that

$$\neg(p \wedge q) \quad \text{and} \quad (\neg p) \vee (\neg q)$$

have the same truth tables. This is an example of the other of De Morgan's laws.

In general, DeMorgan's laws say:

> The negation of "all these things are simultaneously true" is "(at least) one of these things is false"

and

> The negation of "(at least) one of these things is true" is "all these things are false".

More formally, for all numbers n, and all collections of propositions $p_1, p_2, \ldots, p_n$,

$$\neg(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \text{ is the same as } (\neg p_1) \vee (\neg p_2) \vee \cdots \vee (\neg p_n)$$

and

$$\neg(p_1 \vee p_2 \vee \cdots \vee p_n) \text{ is the same as } (\neg p_1) \wedge (\neg p_2) \wedge \cdots \wedge (\neg p_n).$$

You should convince yourself that this is reasonable; until we reach proof by induction, we really have no way of *proving* that it is correct (it's really infinitely many different statements, since there are infinitely many choices for n).

4. Some books are *autological* — they refer to themselves. For example, the Guinness Book of Records is mentioned in the Guinness Book of Records, as the best-selling copyrighted book of all time; and the Bible is frequently mentioned in the Bible. Other books are *non-autological* — they don't refer to themselves. For example, at no point in "Lady in the Lake" by Laura Lippman, does the heroine pick up or discuss "Lady in the Lake" by Laura Lippman!

My wife, who is a librarian at Mishawaka-Penn-Harris Public Library, was recently asked by the director there to compile a list of all the non-autological books in the library's collection. She did this, and produced a book from the list, inspiringly called

> "A list of all the non-autological books in the MPHPL collection", by K. Cashman.

The library has decided to buy this book, and include it in their collection. When she revises her book to take into account this new addition to the library's collection, should she include her book in the revised list, or not?

(This is the famous "Library Paradox", a variant of Bertrand Russell's "Barber Paradox".)

5. Given a statement p, there are a four different statements that you can build from p using $\neg$, $\wedge$ and $\vee$:

   - p itself — true when p is true, false when p is false
   - $\neg p$ — false when p is true, true when p is false
   - $p \vee (\neg p)$ — true when p is true, true when p is false
   - $p \wedge (\neg p)$ — false when p is true, false when p is false.

   (Check the truth tables if you have any doubts). Any other statement that you build will have the same truth table as one of these four. (For example, $p \vee (\neg(p \wedge p))$ is the same as which of the four above?)

   (a) Suppose you are given *two* statements p and q. Figure out how many different statements you can build from p and q, using $\neg$, $\wedge$ and $\vee$. (Think about the last column of the truth table. How many possibilities are there for it? *And can all these possibilities be realized by some statement?*)

   (b) Generalize! Suppose you are given n statements $p_1, \ldots, p_n$. Figure out how many different statements you can build from $p_1, \ldots, p_n$, using $\neg$, $\wedge$ and $\vee$.