

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**Math 10560, Practice Exam 3**  
**April 17, 2007**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
.....					
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
.....					
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
.....					
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)

<b>Please do NOT write in this box.</b>	
Multiple Choice	_____
9.	_____
10.	_____
11.	_____
12.	_____
Total	_____

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

Multiple Choice

1.(7 pts.) Calculate

$$\lim_{n \rightarrow \infty} \frac{(\ln n)^2}{n}.$$

- (a) 1                                      (b)  $\infty$                                       (c) does not exist  
(d)  $e^2$                                       (e) 0

2.(7 pts.) Find  $\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}}$ .

- (a)  $\frac{5}{12}$                       (b)  $\frac{20}{3}$                       (c)  $\frac{5}{3}$                       (d)  $\frac{4}{15}$                       (e)  $\frac{5}{4}$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

3.(7 pts.) The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

- (a) diverges because  $\lim_{n \rightarrow \infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0$ .
- (b) diverges because the terms alternate.
- (c) does not converge absolutely but does converge conditionally.
- (d) diverges even though  $\lim_{n \rightarrow \infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0$ .
- (e) converges absolutely.

4.(7 pts.) Use Comparison Tests to determine which **one** of the following series is divergent.

(a)  $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}} + 1}$

(b)  $\sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{1}{2}\right)^n$

(c)  $\sum_{n=1}^{\infty} 7 \left(\frac{5}{6}\right)^n$

(d)  $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^3 + 100}$

(e)  $\sum_{n=1}^{\infty} \frac{1}{n^2 + 8}$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

5.(7 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for  $\frac{x^2}{1+x}$ ?

(a)  $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n+2}$

(b)  $\sum_{n=0}^{\infty} (-1)^n x^{n+2}$

(c)  $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$

(d)  $\sum_{n=0}^{\infty} x^{2n+2}$

(e)  $\sum_{n=0}^{\infty} (-1)^n x^{2n}$

6.(7 pts.) Find the degree 3 MacLaurin polynomial (Taylor polynomial centered at 0) for the function

$$\frac{e^x}{1-x^2}$$

(a)  $1 + x + \frac{3x^2}{2} + \frac{7x^3}{6}$

(b)  $1 + x - \frac{5x^3}{3}$

(c)  $1 - \frac{x^2}{2} + \frac{x^3}{5}$

(d)  $1 + x + \frac{x^2}{6} + 0x^3$

(e)  $1 + x - \frac{x^3}{6}$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

7.(7 pts.) Which series below is a power series for  $\cos(\sqrt{x})$  ?

(a)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-\frac{1}{2}}}{(2n)!}$

(b)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$

(c)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2+1}$

(d)  $\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^n}{(2n)!}$

(e)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$

8.(7 pts.) Calculate

$$\lim_{x \rightarrow 0} \frac{\sin(x^3) - x^3}{x^9}.$$

**Hint:** Without MacLaurin series this may be a long problem.

(a)  $\frac{9}{7}$

(b) 0

(c)  $-\frac{1}{6}$

(d)  $\infty$

(e)  $\frac{7}{9}$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

Partial Credit

You must show your work on the partial credit problems to receive credit!

9.(11 pts.) Does the series

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{2n}}$$

converge or diverge? Show your reasoning and state clearly any theorems or tests you are using.

**Remark:** The correct answer with no justification is worth 2 points.

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

10.(11 pts.) Use the Integral Test to discuss whether the series  $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$  converges.

**Remark:** Be sure to check that the Integral Test can be applied. The correct answer with no justification is worth 2 points.

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**11.**(11 pts.) Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x - 3)^n$$

**Remark:** The correct answer with no justification is worth 2 points.



Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**12.**(11 pts.)

(a) Show that

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2}$$

provided that  $|x| < 1$ .

(b) Find

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}.$$

**(Hint:** First use term-by-term integration on the series in part (a).)

Name: \_\_\_\_\_

Instructor: ANSWERS

**Math 10560, Practice Exam 3**  
**April 17, 2007**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(a)	(b)	(c)	(d)	(●)
2.	(a)	(●)	(c)	(d)	(e)
.....					
3.	(a)	(b)	(●)	(d)	(e)
4.	(a)	(b)	(c)	(●)	(e)
.....					
5.	(a)	(●)	(c)	(d)	(e)
6.	(●)	(b)	(c)	(d)	(e)
.....					
7.	(a)	(b)	(c)	(d)	(●)
8.	(a)	(b)	(●)	(d)	(e)

<b>Please do NOT write in this box.</b>	
Multiple Choice	_____
9.	_____
10.	_____
11.	_____
12.	_____
Total	_____