

Math 10120 Finite Mathematics
Practice Final Exam 2
May 8, 2018

Name: SOLUTIONS

- Be sure that you have all 15 pages of the test.
- The exam lasts for 2 hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- | | |
|-------------------------|-------------------------|
| 1. (a) (b) (c) (d) (e) | 17. (a) (b) (c) (d) (e) |
| 2. (a) (b) (c) (d) (e) | 18. (a) (b) (c) (d) (e) |
| | |
| 3. (a) (b) (c) (d) (e) | 19. (a) (b) (c) (d) (e) |
| 4. (a) (b) (c) (d) (e) | 20. (a) (b) (c) (d) (e) |
| | |
| 5. (a) (b) (c) (d) (e) | 21. (a) (b) (c) (d) (e) |
| 6. (a) (b) (c) (d) (e) | 22. (a) (b) (c) (d) (e) |
| | |
| 7. (a) (b) (c) (d) (e) | 23. (a) (b) (c) (d) (e) |
| 8. (a) (b) (c) (d) (e) | 24. (a) (b) (c) (d) (e) |
| | |
| 9. (a) (b) (c) (d) (e) | 25. (a) (b) (c) (d) (e) |
| 10. (a) (b) (c) (d) (e) | 26. (a) (b) (c) (d) (e) |
| | |
| 11. (a) (b) (c) (d) (e) | 27. (a) (b) (c) (d) (e) |
| 12. (a) (b) (c) (d) (e) | 28. (a) (b) (c) (d) (e) |
| | |
| 13. (a) (b) (c) (d) (e) | |
| 14. (a) (b) (c) (d) (e) | |
| | |
| 15. (a) (b) (c) (d) (e) | |
| 16. (a) (b) (c) (d) (e) | |

Name: _____

Multiple Choice

1. (5 pts.) Consider the following sets, which are both subsets of the universe set $\{1, 2, 3, 4, 5, 6, a, b, c, e, i, o, u\}$:

$$A = \{1, 2, 3, a, b, c\} \text{ and } B = \{a, e, i, o, u, 2, 4, 6\}$$

What is $A' \cap B$?

(a) \emptyset

(b) $\{1, 3, b, c\}$

~~(c)~~ $\{e, i, o, u, 4, 6\}$

(d) $\{2, a\}$

(e) $\{1, 2, 3, 4, 6, a, b, c, e, i, o, u\}$

Things not in A that are in B : 4, 6, e, i, o, u

2. (5 pts.) A group of tourists visits an amusement park. 25 tourists ride the roller coaster, 30 tourists ride the ferris wheel, 12 tourists ride both the roller coaster and the ferris wheel, and 4 tourists do not ride the roller coaster or the ferris wheel. How many tourists are in the group?

(a) 43

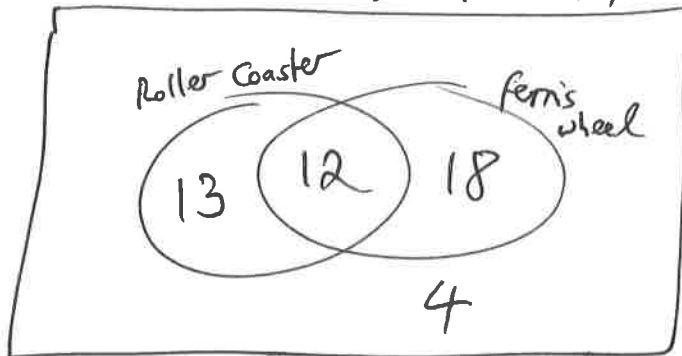
(b) 81

~~(c)~~ 47

(d) 77

(e) 59

$$n(U) = 13 + 12 + 18 + 4 = 47$$



Name: _____

3. (5 pts.) An ice cream parlor has 6 different toppings available to go on their ice cream cones. I want to order an ice cream cone with at least 2 toppings. How many different ways can I do this?

- (a) 42 (b) 30 (c) 15
(d) 64 ~~(e)~~ 57

$$\begin{aligned} & C(6, 2) + C(6, 3) + C(6, 4) + C(6, 5) + C(6, 6) \quad \leftarrow \text{order doesn't matter} \\ = & 15 + 20 + 15 + 6 + 1 \\ = & 57 \quad \left[\text{OR: } 2^6 - C(6, 0) - C(6, 1) \right] \end{aligned}$$

4. (5 pts.) 438 people bought tickets to a very expensive music festival, which was canceled due to poor management. The festival organizers only have enough money left to refund three people. How many ways can the organizers pick three people to give refunds to?

- (a) $438 \cdot 437 \cdot 436$ (b) $438 + 437 + 436$
(c) 438^3 ~~(d)~~ $C(438, 3)$
(e) $P(438, 3)$

Order doesn't matter: $C(438, 3)$

Name: _____

5. (5 pts.) Suppose you have baked twelve different cakes, and you want to give away six of them to your professors. You are going to give one cake to your chemistry professor, one cake to your history professor, one cake to your anthropology professor, and three cakes to your math professor. How many different ways can this be done?

(a) 665,280

(b) 1,404

(c) 924

~~(d)~~ 110,880

(e) 290,400

$$12 \times 11 \times 10 \times \binom{9}{3} = 110880$$

↑ ↑ ↑ ↙

chem hist anthro Math (order doesn't matter
in choice of 3 for math)

6. (5 pts.) A juice stand sells fruit juice blends and vegetable juice blends. To order a fruit juice blend, the customer picks 3 different kinds of fruit from 10 different choices available on the fruit menu. To order a vegetable juice blend, the customer picks 3 different kinds of vegetables from 15 different choices available on the vegetable menu. How many ways are there to order juice from the juice stand?

~~(a)~~ $C(10,3) + C(15,3)$

(b) $P(10,3) + P(15,3)$

(c) $C(10,3) \cdot C(15,3)$

(d) $P(10,3) \cdot P(15,3)$

(e) $C(25,3)$

$$C(10,3) + C(15,3)$$

↑

fruit or veggie

Name: _____

7. (5 pts.) How many 3-digit codes can be made if the first two digits are different numbers from 0 to 9, and the last digit is a lowercase letter from the 26-letter English alphabet?

(a) 2,600

~~(b)~~ 2,340

(c) 1,170

(d) 1,872

(e) 936

$$10 \times 9 \times 26 = 2340$$

8. (5 pts.) How many ways are there to arrange the letters in the word

FINAL

so that the two vowels (I and A) appear next to each other?

(a) 24

~~(b)~~ 48

(c) 120

(d) 12

(e) 6

4 possibilities for where the two vowels appear:

vvccc, cvvcc, ccvvc, cccvv

2 choices for order of vowels (IA or AI)

3! choices for order of consonants

So $4 \times 2 \times 3! = 48$ ways in total

Name: _____

9. (5 pts.) How many ways are there to split a group of 30 people into 3 teams each of size 10?

(a) $\frac{30!}{10!^3}$

(b) $\frac{30!}{10! \cdot 3!^{10}}$

(c) $\frac{30!}{3!^{10}}$

~~(d)~~ $\frac{30!}{3! \cdot 10!^3}$

(e) $C(30, 10) \cdot C(30, 3)$

Assuming the three teams aren't in any particular order:
$$\binom{30}{10} \binom{20}{10} \binom{10}{10} / 3! = \frac{30!}{20! \cdot 10!} \cdot \frac{20!}{10! \cdot 10!} \cdot \frac{10!}{10! \cdot 0!} \cdot \frac{1}{3!} = \frac{30!}{(10!)^3 3!}$$

first second third \uparrow divide out to eliminate order

10. (5 pts.) A jar contains 10 marbles of which 3 are red and the rest are black. If you reach in and grab 4 marbles, what is the probability that exactly 2 of them are black?

~~(a)~~ $\frac{3}{10}$

(b) $\frac{1}{10}$

(c) 0

(d) $\frac{4}{10}$

(e) $\frac{2}{10}$

Number of outcomes: $C(10, 4) = 210$

Number of successful outcomes: $C(7, 2) \times C(3, 2) = 63$

probability = $\frac{63}{210} = \frac{3}{10}$

Name: _____

11. (5 pts.) I have a coin which comes up heads with probability $\frac{3}{4}$. If I toss the coin six times, what is the probability that the first two tosses are the same, and of the last four tosses exactly two are tails? (Answers are rounded to four decimal places.)

(a) 0.1187

(b) 0.0132

~~(c)~~ 0.1318

(d) 0.0220

(e) 0.1875

$$P(\text{first two the same}) = \left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2$$

$$P(\text{exactly two heads among last four}) = C(4, 2) \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2$$

$$P(\text{both happening}) = \text{product (independence)} = 0.1318\dots$$

12. (5 pts.) Two fair six-sided dice are rolled and the the numbers on the uppermost faces of the dice are recorded. Consider the following events:

E : the sum of the numbers is even

F : the sum of the numbers is divisible by 3

G : the sum of the numbers is 5

H : the two numbers are the same

Which of the following statements is FALSE?

(a) E and G are mutually exclusive. $\rightarrow \checkmark$ can't happen simultaneously

(b) G and H are mutually exclusive. $\rightarrow \checkmark$ " " "

~~(c)~~ E and F are mutually exclusive. $\rightarrow \times$ numbers could be 5,1 \rightarrow sum even and divisible by 3

(d) F and G are mutually exclusive. $\rightarrow \checkmark$ can't happen simultaneously

(e) Every outcome in H is also in E . $\rightarrow \checkmark$ if both numbers the same, sum must be even.

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13. (5 pts.) A warehouse produces sheet metal using two machines, which run independently. The probability of machine 1 breaking down is 0.12 and the probability of both machines breaking down is 0.03. What is the probability of machine 2 breaking down?

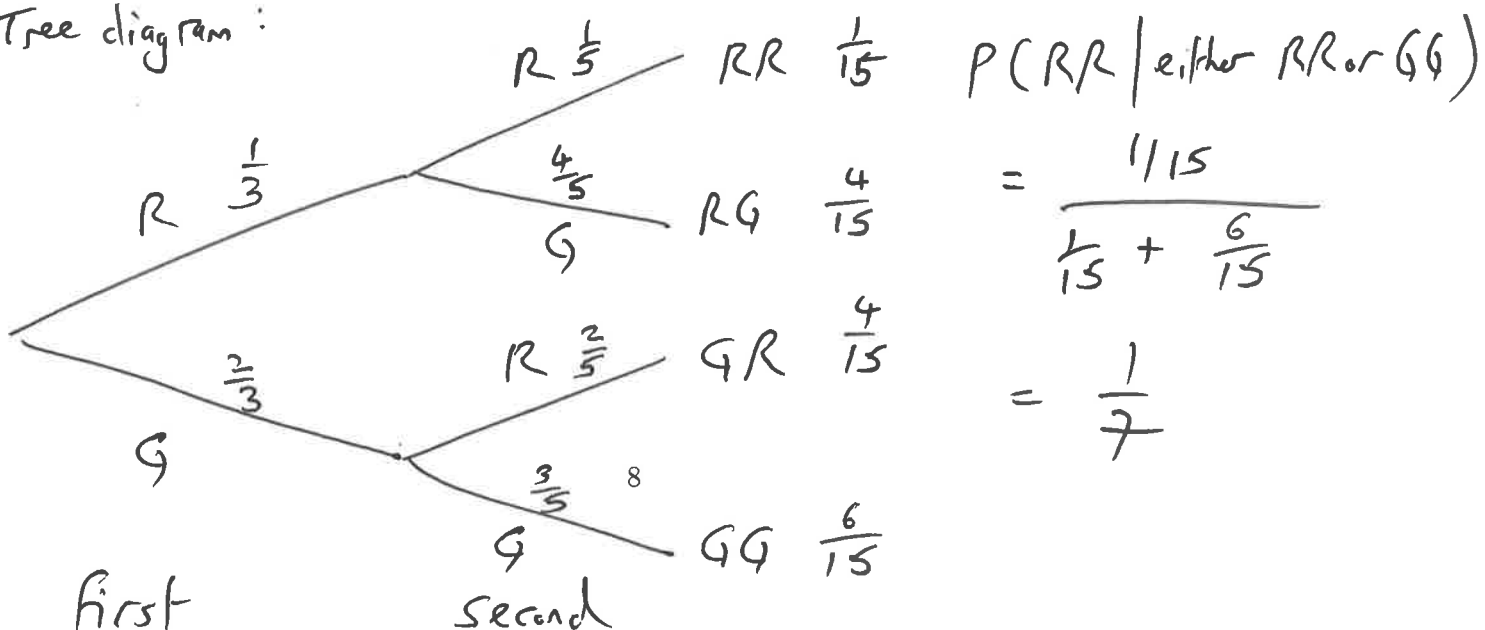
- (a) 0.15 (b) 0.0036 ~~(c) 0.25~~
(d) 0.85 (e) 0.09

By independence, $P(\text{both}) = P(1)P(2)$
So $0.03 = 0.12 \times P(2)$,
So $P(2) = \frac{0.03}{0.12} = 0.25$

14. (5 pts.) A jar contains 2 red marbles and 4 green marbles. While you are not looking, I draw two marbles in a row *without replacement*. Then I tell you that the two marbles I drew were the same color. Given this information, what is the probability that I drew two red marbles?

- (a) $\frac{1}{9}$ (b) $\frac{1}{5}$ (c) $\frac{1}{15}$
~~(d) $\frac{1}{7}$~~ (e) $\frac{1}{3}$

Tree diagram:



Name: _____

15. (5 pts.) If a fair coin is tossed three times, what is the probability that the first two tosses are heads **OR** the last two tosses are heads?

(a) $\frac{3}{8}$

(b) $\frac{1}{2}$

(c) $\frac{1}{16}$

(d) $\frac{5}{8}$

(e) $\frac{3}{16}$

$A = \{ \text{first two heads} \}, P(A) = \frac{1}{4}$

$B = \{ \text{last two heads} \}, P(B) = \frac{1}{4}$

$A \cap B = \{ HHH \}, P(A \cap B) = \frac{1}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{4} + \frac{1}{4} - \frac{1}{8} = \frac{3}{8}$

16. (5 pts.) The following frequency table shows the number of cups of coffee I drank each week, during each of the last 50 weeks (note that one frequency is missing from the table):

Number of cups drank	Frequency (number of weeks)
12	8
13	10
14	? $\rightarrow 12$ (so numbers add to 50)
15	12
16	8

What is the **relative** frequency of my drinking 14 cups of coffee a week?

(a) $\frac{38}{50}$

(b) 12

(c) $\frac{14}{50}$

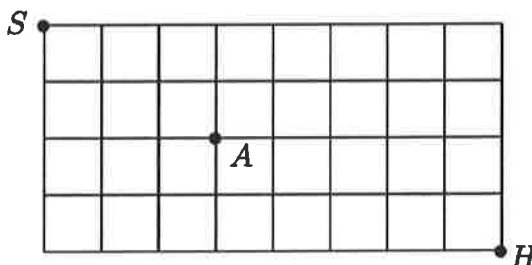
(d) $\frac{12}{50}$

(e) The answer cannot be determined from the information given.

$\frac{12}{50}$

Name: _____

17. (5 pts.) Rudy walks 12 blocks every day from school (marked S on the map) to his house (marked H), always stopping along the way at the amusement arcade (marked A). How many different routes can he choose, assuming that he only walks east or south?



- (a) 31 (b) 15 (c) 495 ~~(d) 210~~ (e) 105

$$S \text{ to } A : C(5, 3) = 10$$

$$A \text{ to } H : C(7, 5) = 21$$

$$S \text{ to } H \text{ via } A : 10 \times 21 = 210$$

18. (5 pts.) The table below shows how many singles, doubles, triples and home-runs Ichiro Suzuki hit during his last 200-hit season, and how many of those hits were against left-handed pitchers.

Type of hit	Number of hits	Number of hits against left-handed pitchers
Single	130	20
Double	40	8
Triple	12	4
Home run	18	6

What is the probability that a randomly chosen hit was against a left-handed pitcher, given that it was a triple?

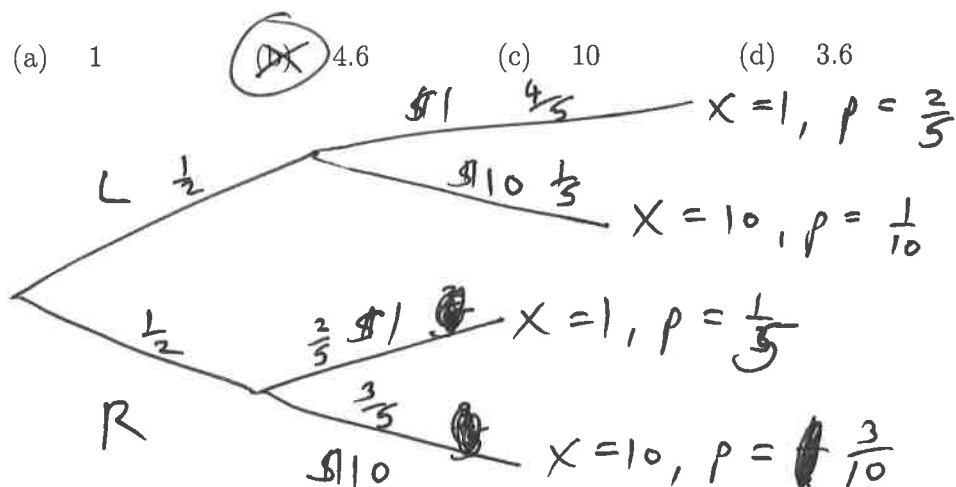
- (a) $\frac{3}{50}$ (b) $\frac{8}{119}$ ~~(c) $\frac{1}{3}$~~ (d) $\frac{2}{19}$ (e) $\frac{3}{4}$

$$P(\text{Left} | \text{triple}) = \frac{P(\text{left and triple})}{P(\text{triple})} = \frac{\frac{4}{200}}{\frac{12}{200}} = \frac{1}{3}$$

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19. (5 pts.) In my right hand I have a wallet with two \$1 bills and three \$10 bills. In my left hand I have a wallet with four \$1 bills and one \$10 bill. You choose a wallet at random, and choose a bill at random from the wallet. You get to keep the bill you selected. Let X be the amount of money that you win in this game. What is the expected value of X ?

- (a) 1 ~~(b) 4.6~~ (c) 10 (d) 3.6 (e) 5.5



value	prob
1	$\frac{3}{5}$
10	$\frac{2}{5}$

$$E(X) = \frac{3}{5} + 10\left(\frac{2}{5}\right) = \frac{23}{5} = 4.6$$

20. (5 pts.) A certain random variable X , whose mean is known to be 0, is given by the following distribution table:

k	$P(X = k)$
-2	0.2
-1	0.2
0	0.1
1	0.4
2	0.1

What is the variance of X ?

- (a) 1.2 (b) 0 (c) 1.44 ~~(d) 1.8~~ (e) 1.34

$$\begin{aligned} \text{Variance}(X) &= 0.2[-2-0]^2 + 0.2[-1-0]^2 \\ &\quad + 0.1[0-0]^2 + 0.4[1-0]^2 + 0.1[2-0]^2 \\ &= 0.8 + 0.2 + 0 + 0.4 + 0.4 = 1.8 \end{aligned}$$

Name: _____

21. (5 pts.) I roll a dice 12 times. How likely is it that I roll either two or three 6's? (Round your answer to three decimal places.)

- (a) .677 (b) 0.197 (c) 0.875
 0.493 (e) 0.296

Binomial experiment, $n = 12$, $p = \frac{1}{6}$

$$P(X=2 \text{ or } 3) = C(12, 2) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{10} \\ + C(12, 3) \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^9 \\ = 0.493\dots$$

22. (5 pts.) Of the 8,000 undergraduates on campus, 1000 are from the midwest. I select 120 students from campus at random, with replacement. Let X be the number of times that I select a student from the midwest. What is the expected value and standard deviation of X ?

IGNORE

- Expected value 15, standard deviation 13.125
(b) Expected value 60, standard deviation 5.48
(c) Expected value 1000, standard deviation 29.58
 Expected value 15, standard deviation 3.62
(e) Expected value 125, standard deviation 10.46

Binomial, $n = 120$

$$p = \frac{1000}{8000} = \frac{1}{8}$$

$$E(X) = \frac{120}{8} = 15$$

$$\text{Var}(X) = 120 \left(\frac{1}{8}\right) \left(\frac{7}{8}\right) = 13.125$$

$$\text{Standard deviation: } \sqrt{13.125} \approx 3.62\dots$$

Name: _____

23. (5 pts.) John, Mary and Ellis took the MCAT exam on three different years. Each year, the scores were normally distributed. The table below shows the overall test mean and standard deviation, and the individual test taker's score, in each year.

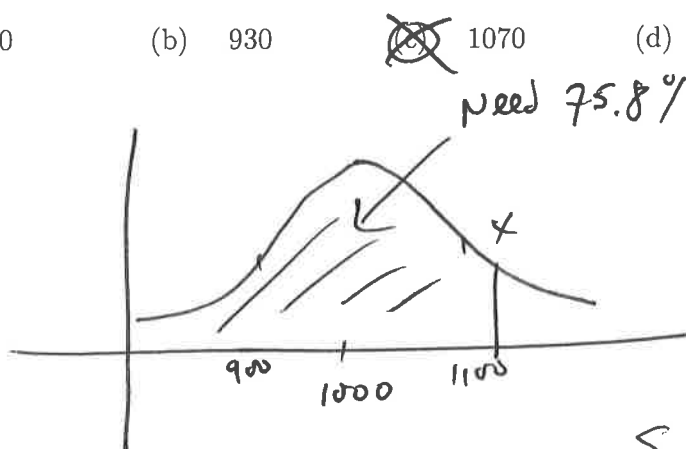
Year	test taker	mean	standard deviation	test taker's score	z score
2014	John	110	10	125	1.5
2015	Mary	100	8	114	1.75
2016	Ellis	112	10	125	1.3

Which of the following statements is FALSE?

- (a) All three performed above average **TRUE**
- (b) Based on z-scores, Mary did the best of the three on the test **TRUE**
- (c) Ellis scored better than at least 90% of the test takers in 2016 **TRUE**
- (d) Based on z-scores, John did better than Ellis **TRUE**
- (e) Fewer than 5% of test takers in 2014 did better than John **FALSE, need z-score of 1.64 for this**

24. (5 pts.) Cantaloupes at Martins supermarket have weights that are normally distributed with mean 1000 grams, standard deviation 100 grammes. Of which number x is it true to say "75.8% of the Cantaloupes at Martins have weight at most x grams"?

- (a) 70
- (b) 930
- (c) 1070
- (d) 1110
- (e) 965



From table,
 $P(0 \leq z \leq 0.7) = 25.8\%$,
 $\therefore P(z \leq 0.7) = 75.8\%$

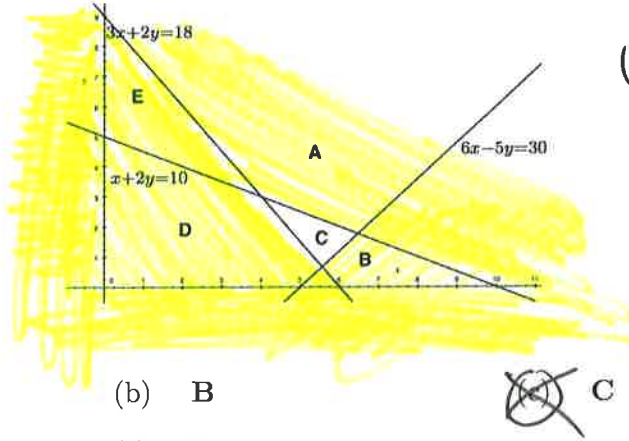
So need x to have z-score of 0.7,

$$x = 1000 + (0.7)100 = 1070$$

Name: _____

25. (5 pts.) Which of A, B, C, D, E is the correct feasible set for the following collection of inequalities?

$$3x + 2y \geq 18, \quad x + 2y \leq 10, \quad 6x - 5y \leq 30, \quad x \geq 0, \quad y \geq 0.$$



Wrong side of each inequality line shaded; all that is left unshaded is C

- (a) A (b) B
 (d) D (e) E

26. (5 pts.) Healthy Deli makes super-healthy soups by mixing three stocks.

- Stock A has 500 calories, 6 grams of protein and 90 mg of salt per cup.
- Stock B has 600 calories, 7 grams of protein and 50 mg of salt per cup.
- Stock C has 400 calories, 5 grams of protein and 70 mg of salt per cup.

You want to make some soup for lunch, with at least 500 calories and at most 60 mg of salt. You want to maximize your protein. Which is the correct set of constraints and the objective function for this problem if A, B and C denote the cups of each stock in your soup?

(a) ~~$500A + 600B + 400C \geq 500$
 $90A + 50B + 70C \geq 60$
 $A \geq 0, B \geq 0, C \geq 0$
 objective $6A + 7B + 5C$~~

(b) ~~$500A + 600B + 400C \leq 500$
 $90A + 50B + 70C \leq 60$
 $A \geq 0, B \geq 0, C \geq 0$
 objective $6A + 7B + 5C$~~

(c) ~~$500A + 600B + 400C = 500$
 $90A + 50B + 70C \geq 60$
 $A \geq 0, B \geq 0, C \geq 0$
 objective $6A + 7B + 5C$~~

(d) ~~$500A + 600B + 400C \geq 500$
 $90A + 50B + 70C \leq 60$
 $A \geq 0, B \geq 0, C \geq 0$
 objective $6A + 7B + 5C$~~

(e) ~~$500A + 600B + 400C \leq 500$
 $90A + 50B + 70C \geq 60$
 $A \geq 0, B \geq 0, C \geq 0$
 objective $6A + 7B + 5C$~~

Calories: $500A + 600B + 400C \geq 500$
 Salt: $90A + 50B + 70C \leq 60$

Name: _____

27. (5 pts.) In the 1980 presidential election, Reagan (R) and Carter (C) played a game where each one made a statement. If both made a true statement, Reagan picked up 10 percentage points (and Carter lost 10 percentage points) in the next poll. If Reagan's statement was true and Carter's was false, Reagan picked up 5 points. If Reagan's statement was false and Carter's was true, Carter picked up 3 points. If both statements were false, Reagan picked up 6 points. Which of the following is the pay-off matrix (with Reagan as row player) for this zero-sum game?

(a)

	T	F
T	10	5
F	-3	6

(b)

	T	F
T	10	-3
F	5	6

(c)

	T	F
T	10	6
F	-3	5

(d)

	T	F
T	-10	-5
F	3	-6

(e)

	T	F
T	-10	3
F	-5	-6

		C	
		T	F
R	T	+10	+5
	F	-3	+6

28. (5 pts.) The following is the pay-off matrix for the row player in a zero-sum game:

Saddle point →

1	3	2
0	-1	3
-3	-2	5

min 1 } Max of row mins is 1
 min -1
 min -3

max 1 } min of col maxs is 1
 max 3
 max 5

Which of the following is true?

- (a) There is no saddle point in this matrix. FALSE
- (b) The game is strictly determined with a value of 3. F
- (c) The game is strictly determined with a value of 5. F
- (d) The game is strictly determined with a value of 0. F
- (e) The game is strictly determined with a value of 1. T

r_1 : row player's best strategy
 c_1 : col " " "
 Value: 1