Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2018

Name: SOLUTIONS

Instructors: Basit & Galvin

Exam 3 Practice (actual exam from Fall 2017)

April 19, 2018

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an × through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Tot.	______

Multiple Choice

1. (5 pts.) In Camelot the average temperature at noon is 80°F. In a particular week, the temperatures at noon on the first six days were 78, 79, 81, 78, 79 and 80 (degrees Fahrenheit). What did the temperature need to be on the seventh day of that week in order for the average to indeed by 80° for that week?

~~(a)~~ 85 (b) 84 (c) 83 (d) 82 (e) 81

$$\frac{78+79+81+78+79+80+x}{7} = 80$$

$$475 + x = 560$$

$$x = 85$$

2. (5 pts.) In Camelot, in a particular week the temperatures at noon were 76, 81, 83, 78, 83, 80 and 79 (degrees Fahrenheit). The mean of these seven numbers is 80 – you do not have to verify this. To the nearest three decimal places, what was the (population) *standard deviation* (not variance) of the temperatures over those seven days?

(a) 6.325 (b) 0 ~~(c)~~ 2.390 (d) 5.714 (e) 40

x	$(x-\mu)$	$(x-\mu)^2$
76	-4	16
81	1	1
83	3	9
78	-2	4
83	3	9
80	0	0
79	-1	+ 1
		40

$\mu = 80$

$$\sigma^2 = \frac{40}{7} \approx 5.7143$$

$$\sigma \approx 2.390 \left(= \sqrt{40/7}\right)$$

3. (5 pts.) In a tray of 10 Halloween cookies, 3 have peanuts in them and 7 don't. Four are selected at random and the number of selected cookies containing peanuts is noted. Let X be the number of cookies containing peanuts. Find the probability distribution of X.

(a)

x	$P(x)$
0	1/8
1	3/8
2	3/8
3	1/8

~~(b)~~

x	$P(x)$
0	1/6
1	1/2
2	3/10
3	1/30

(c)

x	$P(x)$
0	1/4
1	1/4
2	1/4
3	1/4

(d)

x	$P(x)$
0	1/6
1	1/6
2	1/10
3	1/30

(e)

x	$P(x)$
0	1/30
1	1/6
2	1/2
3	3/10

k	$P(X=k)$
0	$C(3,0)\,C(7,4)\,/\,C(10,4) = 35/210 = 1/6$
1	$C(3,1)\,C(7,3)\,/\,C(10,4) = 105/210 = 1/2$
2	$C(3,2)\,C(7,2)\,/\,C(10,4) = 63/210 = 3/10$
3	$C(3,3)\,C(7,1)\,/\,C(10,4) = 7/210 = 1/30$

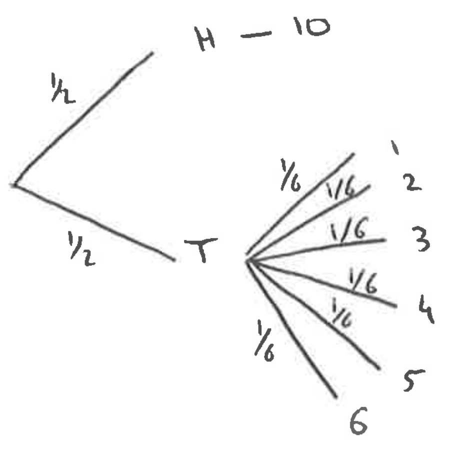
4. (5 pts.) Your flaky uncle Bob gives you the following present for your birthday. First he has you flip a coin. If it's heads he gives you \$10. If it's tails he has you roll a die, and he gives you a number of dollars equal to whatever the die shows. How much money do you expect to make from this game? (Hint: it might be helpful to make a tree diagram.)

(a) \$8.50 (b) \$13.50 (c) \$1.75 (d) \$4.43 ~~(e)~~ \$6.75

$$E[X] = \frac{1}{2}\times 10 + \frac{1}{6}\times\frac{1}{2}\times 1 + \frac{1}{6}\times\frac{1}{2}\times 2 + \frac{1}{6}\times\frac{1}{2}\times 3 + \frac{1}{6}\times\frac{1}{2}\times 4 + \frac{1}{6}\times\frac{1}{2}\times 5 + \frac{1}{6}\times\frac{1}{2}\times 6$$

$$= \frac{1}{2}\times 10 + \frac{1}{12}\times(1+2+3+4+5+6)$$

$$= 6.75$$



5. (5 pts.) The Flyers and the Rangers compete in the NHL playoffs in a best-of-five series, so the team that wins three games is the winner. (As soon as one team reaches its third win, the series is over and that team wins the series.) In any given game, the probability that the Flyers win is 3/4. Find the probability that the Flyers win in *exactly* four games. (Warning: you have to exclude the possibility that they win in three games!) The following answers are rounded to one decimal place.

(a) $27/256 = 10.5\%$ (b) $27/64 = 42.2\%$ (c) $3/4 = 75\%$

~~(d)~~ $81/256 = 31.6\%$ (e) $3/16 = 18.8\%$

There are three possible sequences of wins/losses

→ LWWW, WLWW, WWLW

the probability of each is $\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) = \frac{27}{256}$

so total probability is $3 \times \frac{27}{256} = \frac{81}{256}$

$p = \frac{3}{4}$, $q = \frac{1}{4}$, $n = 4$

$P(3 \text{ wins}) = \binom{4}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) = \frac{27}{64}$, but this includes WWWL, which we have to exclude

so the answer is $\frac{27}{64} - \frac{27}{256} = 81/256$

6. (5 pts.) At Canta Ford Autos the manager knows that 20% of the people that enter the dealership wind up buying a car. If 10 (unrelated) people come on a particular day, what is the probability that at most 3 will buy cars? (Round percentages to two decimal places.)

(a) 58.65% (b) 74.14% (c) 46.83% ~~(d)~~ 87.91% (e) 64.31%

X = # people that buy cars

→ X is Binomial with $n = 10$, $p = 0.2$

$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \binom{10}{0}(0.2)^0(0.8)^{10} + \binom{10}{1}(0.2)^1(0.8)^9 + \binom{10}{2}(0.2)^2(0.8)^8 + \binom{10}{3}(0.2)^3(0.8)^7$

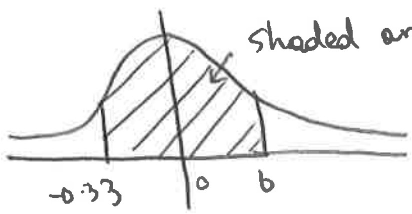
$\approx 0.8791 = 87.91\%$

7. (5 pts.) Suppose a random variable X has the standard normal distribution. If $P(-0.33 \le X \le b) = 0.3935$, what is b?

(a) 0.27 ~~(b)~~ 0.72 (c) 1.13

(d) 1.25 (e) 0.63

Shaded area $= 0.3935$



$0.3935 = A(-0.33) + A(b)$

$\rightarrow 0.3935 = A(0.33) + A(b)$ (because negative is same as positive)

$\rightarrow A(b) = 0.3935 - 0.1293$

$= 0.2642$

$\rightarrow b = +0.72$ (positive because on right of mean)

8. (5 pts.) In 2017, the average SAT score for students admitted to St. Patrick's college was 1479 with standard deviation 50. Suppose the distribution of SAT scores can be modelled by a normal distribution and let X be the score of a random student. Find $P(X \le 1390)$.

~~(a)~~ 0.0375 (b) 0.4625 (c) 0.9625

(d) 0.9250 (e) 0.7258

z-score of 1390

$= \frac{1390 - 1479}{50} = -1.78$

$P(X \le 1390) = P(Z \le -1.78)$

$= P(Z \ge 1.78)$

$= 0.5 - A(1.78) = 0.5 - 0.4625$

$= 0.0375$

9. (5 pts.) Which of the following points satisfy all the inequalities below: (Remember to take note of $\leq$ vs $<$ and $\geq$ vs $>$.)

$$\begin{aligned} x + y &\geq 5 \\ 4x - 3y &\leq 4 \\ x > 0,\ y &\leq 6 \end{aligned}$$

(a) (0, 6) (b) (1, 7) (c) (1, 3)

(d) (5, 1) ~~(e)~~ (2, 3)

Just plug in values and check.

10. (5 pts.) Alice has invited the Mad Hatter, the Dormouse, and the March Hare to a tea party. She wants to brew tea and bake cookies for the party, but has only 2 hours to do so. It takes her 10 minutes to brew a cup of tea and 15 minutes to bake each cookie (she can only brew one cup of tea or bake one cookie at a time). She also wants to make sure there are enough cups of tea and cookies for everyone at the party to have at least one thing to eat or drink. Let x be the number of cups of tea and y be the number of cookes she can bake. Which of the following inequalities properly describe all constraints on Alice?

~~(a)~~ $x + y \geq 4$; $10x + 15y \leq 120$; $x \geq 0$; $y \geq 0$.

(b) $x + y \geq 4$; $15x + 10y \leq 120$; $x \geq 0$; $y \geq 0$.

(c) $x + y \leq 4$; $10x + 15y \leq 120$; $x \geq 0$; $y \geq 0$.

(d) $x + y \geq 4$; $15x + 10y \leq 120$.

(e) $x + y \geq 4$; $10x + 15y \leq 120$.

Partial Credit

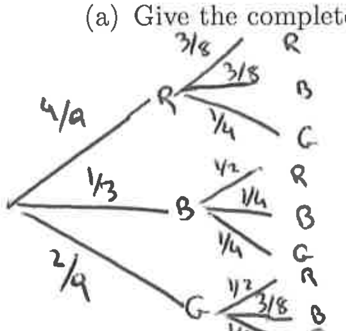
You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) Alice plays the following game at the carnival. She gets to pick (without replacement) two balls from a hat with 4 red, 3 blue and 2 green balls. Each ball is worth a certain amount of money.

- A red ball is worth \$2.
- A blue ball is worth \$1.
- A green ball is worth \$0.

Her winnings are the sum of the amounts the balls she picks are worth. Let X be the amount Alice wins.

(a) Give the complete probability distribution for X. (A tree diagram might help.)



k	$P(X=k)$
0	$2/9 \times 1/8 = 2/72$
1	$1/12 + 6/72 = 1/6$
2	$1/9 + 1/12 + 1/9 = 11/36$
3	$1/6 + 1/6 = 1/3$
4	$1/6$

(b) What is the expected value of X (assuming that it doesn't cost any money to play the game)?

$$0(1/36) + 1(1/6) + 2(11/36) + 3(1/3) + 4(1/6)$$

$$= \frac{1}{6} + \frac{11}{18} + 1 + \frac{2}{3} = \frac{44}{18} = \frac{22}{9} \approx 2.44$$

(c) Suppose now that the carnival decided to charge \$3 to play the game. On a given day 100 people play the game. How much money (up to 2 decimal places) should the carnival expect to make?

Profit per game $= 3 - 2.44 = 0.56$

Profit for 100 games $= 0.56 \times 100$

$= \$56$

12. (10 pts.) In an experiment, I roll a four sided die (with sides numbered 1, 2, 3 and 4) and a three sided die (with sides numbered 1, 2 and 3). Let X be the sum of the numbers that come up.

(a) What are the values that X can take?

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7

Values of $X = \{2, 3, 4, 5, 6, 7\}$

(b) Find the probability distribution of X.

k	$P(X=k)$
2	$1/12$
3	$2/12 = 1/6$
4	$3/12 = 1/4$
5	$3/12 = 1/4$
6	$2/12 = 1/6$
7	$1/12$

(c) Compute $E[X]$ and $\sigma(X)$. (Round your answers to two decimal places.)

k	$P(X=k)$	$(k-\mu)^2$
2	$1/12$	$(-2.5)^2 = 6.25$
3	$1/6$	$(-1.5)^2 = 2.25$
4	$1/4$	$(-0.5)^2 = 0.25$
5	$1/4$	$(0.5)^2 = 0.25$
6	$1/6$	$(1.5)^2 = 2.25$
7	$1/12$	$(2.5)^2 = 6.25$

$$\sigma^2 = 6.25 \times \frac{1}{12} + 2.25 \times \frac{1}{6} + 0.25 \times \frac{1}{4} + 0.25 \times \frac{1}{4} + 2.25 \times \frac{1}{6} + 6.25 \times \frac{1}{12}$$

$$= 1.9166$$

$$\rightarrow \sigma = \sqrt{1.9166} = 1.38$$

$$\mu = E[X] = 2\left(\frac{1}{12}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{4}\right) + 5\left(\frac{1}{4}\right) + 6\left(\frac{1}{6}\right) + 7\left(\frac{1}{12}\right)$$

$$= 4.5$$

13. (10 pts.) In a basketball competition against the Red Queen, Alice is required to attempt 20 free throws. The probability that she makes a throw is 0.7, independent of other throws.

(a) Find the probability that Alice makes exactly 17 of the throws? You do not need to simplify your answer.

$$\binom{20}{17}(0.7)^{17}(0.3)^{3}$$

(b) Find the probability that she makes at least 17 of the throws? You do not need to simplify your answer.

$$\binom{20}{17}(0.7)^{17}(0.3)^{3} + \binom{20}{18}(0.7)^{18}(0.3)^{2} + \binom{20}{19}(0.7)^{19}(0.3)^{1} + \binom{20}{20}(0.7)^{20}(0.3)^{0}$$

(c) Compute the mean and standard deviation of X, where X is the number of throws that Alice makes. (Round your answers to two decimal places.)

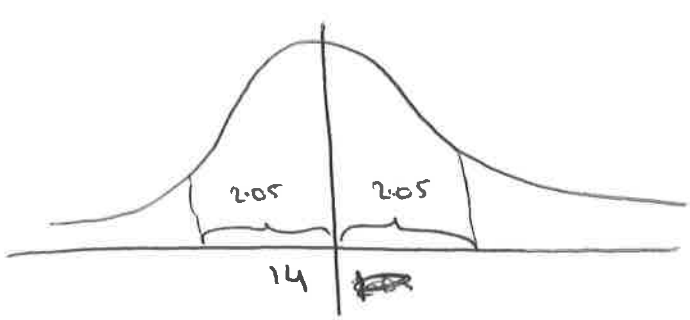
$p = 0.7$
$q = 0.3$
$n = 20$

$\mu = E[X] = np = 20 \times 0.7 = 14$

$\sigma = \sqrt{npq} = \sqrt{20(0.7)(0.3)}$

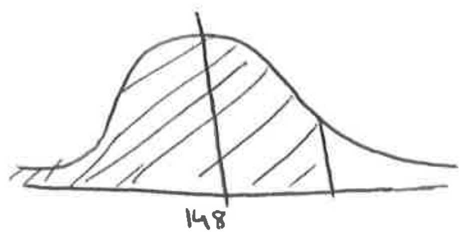
≈ 2.05

(d) Draw the normal curve that best approximates the distribution of X. Make sure to label the mean and standard deviation.



14. (10 pts.) The scores on a certain standardized test are normally distributed with a mean of 148 and a standard deviation of 16.

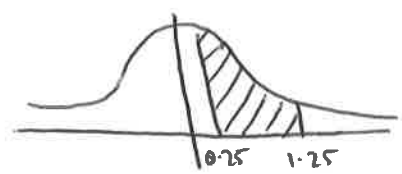
(a) Sketch a normal curve corresponding to this distribution and shade the area corresponding to the probability that someone scores 164 or less. Be sure to label where the mean is in your sketch.



(b) A score is selected at random. Find the probability that it is between 152 and 168.

z-score of 152 = $\frac{152-148}{16} = 0.25$

z-score of 168 = $\frac{168-148}{16} = 1.25$

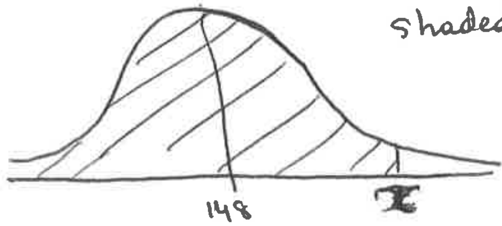


$P(0.25 \le Z \le 1.25) = A(1.25) - A(0.25)$

$= 0.3944 - 0.0987$

$= 0.2957$

(c) Emily took the exam and was told that 99% of all scores were less than or equal to hers. What was her score? Explain your answer.



shaded region is 0.99

$z = \frac{x - 148}{16}$

$P(Z \le z) = 0.5 + A(z) = 0.99$

$\rightarrow A(z) = 0.49$

$\rightarrow z = 2.33$

$\rightarrow x = 2.33 \times 16 + 148$

$= \underline{185.28}$

15. (10 pts.) Consider the following set of inequalities:

$$\begin{aligned} l_1 &: x - y \leq 1 \\ l_2 &: x + y \geq 3 \\ l_3 &: x \geq 0 \\ l_4 &: y \geq 0. \end{aligned}$$

Sketch the inequalities, label the lines and label the intercepts for each of them.