Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2018

Name: SOLUTIONS

Instructors: Basit/Galvin

Exam 1 — practice (actual Exam 1 from fall 2017)

February 2, 2018.

This exam is in two parts on 9 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

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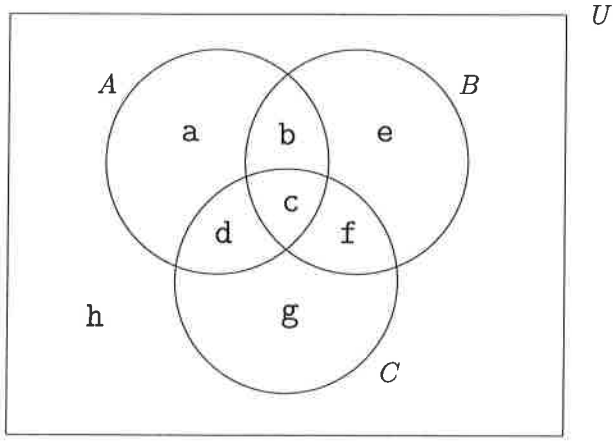
MC. _____
11. _____
12. _____
13. _____
14. _____

Tot. _____

Multiple Choice

1. (5 pts.) In the following Venn diagram, which of the following is equal to $(A \cup B) \cap C'$? (Note: C', not C.)



(a) $\{a, b, e, g, h\}$ (b) $\{a, e\}$ (c) $\{c, d, f\}$

(d) $\{a, b, e\}$ (e) $\{h\}$

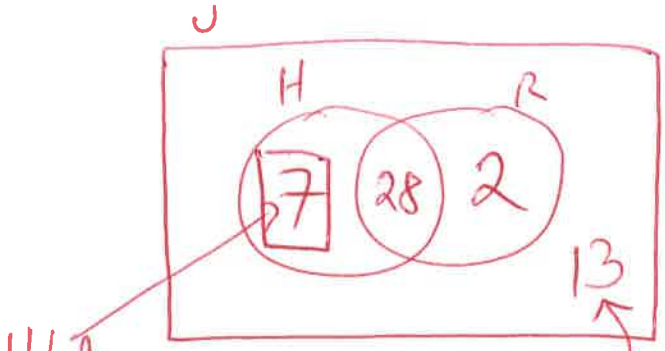
$A \cup B \rightarrow \{a, b, c, d, e, f\}$

$C' \rightarrow \{a, b, e, h\}$

$(A \cup B) \cap C' \rightarrow \{a, b, e\}$

2. (5 pts.) In a small school, the sixth grade class has 50 students. They have a history club and a reading club, and students are allowed to be in one, both or neither club. If 35 students are in the history club, 30 students are in the reading club and 28 students are in both clubs, how many students are in the history club but not the reading club? (A Venn diagram might be helpful.)

(a) 7 (b) 2 (c) 8 (d) 10 (e) 13



H but not R

$n(U) = 50$

$n(H) = 35$

$n(R) = 30$

$n(H \cap R) = 28$

$28 + 7 + 2 = 37$

$50 - 37 = 13$

3. (5 pts.) A euchre deck consists of 24 cards, namely the 9, 10, J, Q, K and A of each suit (clubs, diamonds, hearts and spades). In how many ways can a person choose one card from each suit?

(a) $C(24,4)$ (b) $P(24,4)$ (c) 6^4

(d) $4 \cdot 6^4$ (e) $4 \cdot C(24,4)$

6 choices for a club
then 6 choices for a diamond
then 6 " " " heart
then 6 " " " spade

$6 \times 6 \times 6 \times 6 = 6^4$ choices

4. (5 pts.) A license plate in a certain state features 3 letters (repetition allowed) followed by 3 digits (repetition not allowed). How many different license plates are possible?

(a) $26^3 \cdot P(10,3)$ (b) $26^3 \cdot C(10,3)$ (c) $C(26,3) \cdot P(10,3)$

(d) $P(26,3) \cdot P(10,3)$ (e) $26^3 \cdot 10^3$

$26 \times 26 \times 26 \times 10 \times 9 \times 8 = 26^3 P(10,3)$

first letter, second, third, first number, second, third

5. (5 pts.) Three couples line up for a picture. In how many ways can this be done if the members of each couple must stand next to each other?

(a) 8 (b) 120 (c) 14 (d) 720 (e) 48

6 choices for first spot in picture
1 choice for second (partner of person in first)
4 " s " third spot
1 " " fourth "
2 " s " fifth "
1 " " sixth

$6 \times 1 \times 4 \times 2 \times 1 = 48$

6. (5 pts.) In St. Patrick's College, all first year students are required to take 1 Math and 3 Philosophy courses. They can pick any course they like from among 4 Math and 5 Philosophy courses. How many different ways can a student pick his courses?

(a) $4! \cdot 5!$ (b) $C(4,1) \cdot C(5,3)$ (c) $P(4,1) + P(5,3)$

(d) $C(4,1) + C(5,3)$ (e) $P(4,1) \cdot P(5,3)$

C(4,1) for math
C(5,3) for philosophy
} multiply, because choosing math course and philosophy courses

order doesn't matter

7. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A Poker hand consists of 5 cards.

How many Poker hands have all 5 cards from the same suit?

(a) $4 \cdot C(13,5)$ (b) $4 \cdot P(13,5)$ (c) $C(13,5) + C(4,1)$

(d) $C(13,5)$ (e) $P(13,5)$

4 choices for the suit, and then $C(13,5)$ choices for the specific five cards (order doesn't matter), so

$4 \times C(13,5)$

8. (5 pts.) John, Ryan, Emily and Anna want to play a game of Poker. How many ways are there to deal 5 cards to each of the players?

(a) $4 \cdot C(52,5)$ (b) $C(52,5) \cdot C(47,5) \cdot C(42,5)$

(c) $4 \cdot P(52,5)$ (d) $P(52,5) \cdot P(47,5) \cdot P(42,5) \cdot P(37,5)$

(e) $C(52,5) \cdot C(47,5) \cdot C(42,5) \cdot C(37,5)$

First give John 5 cards: $C(52,5)$ options

then give Ryan 5 cards: $C(47,5)$ options (only 47 cards left in deck)

then $C(42,5)$ options for Emily,

then $C(37,5)$ options for Anna

9. (5 pts.) Suppose I roll a six sided die 5 times and record the resulting sequence of numbers. In how many ways can I get exactly three sixes? (Dont forget that there are five rolls.)

(a) $C(5,3)$ (b) $P(5,3)$ (c) $C(5,3)\cdot 6^3$

(d) $C(5,3)\cdot 5^2$ (e) 6^3

First decide in which 3 of the 5 rolls the sixes occur, $C(5,3)$ options.

Then decide the non-6 numbers that appeared in the other 2 positions, $5\times 5 = 5^2$ options

10. (5 pts.) The Irish senate consists of 104 members, 4 from each of the 26 counties. A committee of 5 senators is to be formed, in which each of the five members must represent different county. In how many different ways can such a committee be formed?

(a) $C(26,5)\cdot 5^4$ (b) $C(104,5)/5^4$ (c) $C(26,5)\cdot C(5,4)$

(d) $C(104,5)-4^5$ (e) $C(26,5)\cdot 4^5$

First select the 5 counties that will get represented in the committee, $C(26,5)$ options

For each chosen county, have 4 choices for the senator. So $4\times 4\times 4\times 4\times 4 = 4^5$ options to complete.

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (15 pts.) A club has 6 men and 7 women. For each of the following questions, give a numerical answer (e.g. if the answer should be $C(4,2)$, write 6.) These questions should be assumed to be independent of each other.

(a) In how many ways can they choose a president and a vice president?

13 members, so $P(13,2) = 13 \times 12 = \boxed{156}$

(Order of selection matters)

(b) In how many ways can they choose an executive committee of 3 people?

$C(13,3) = \dfrac{13 \times 12 \times 11}{3!} = \boxed{286}$

(Order doesn't matter)

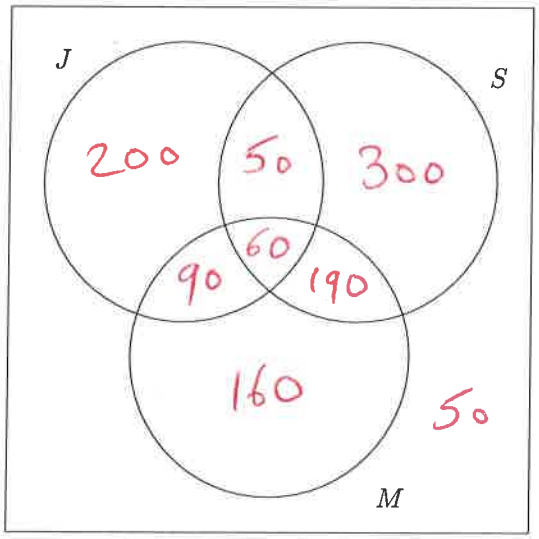
(c) In how many ways can they choose a dance committee consisting of 2 men and 2 women?

$C(6,2) \times C(7,2) = \boxed{315}$

12. (10 pts.) A certain college has 1100 students. We have the following information about the clubs that they belong to.

- 400 belong to the juggling club.
- 600 belong to the science club.
- 500 belong to the Mock Trial club
- 110 belong to both the juggling club and the science club.
- 250 belong to both the science club and the Mock Trial club.
- 150 belong to both the juggling club and the Mock Trial club.
- 60 belong to all three clubs.

Fill in **all** regions of the following Venn diagram, where J represents the juggling club, S represents the science club and M represents the Mock Trial club.



Start from inside (J∩S∩M)
and work out.

13. (15 pts.) I have a standard coin that comes up heads or tails each time I toss it. Suppose I toss the coin 12 times and note down the sequence of heads and tails that shows up.

Note: In the following three parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k).

(a) How many different sequences of heads and tails are possible?

$\underbrace{2 \times 2 \times 2 \times \ldots \times 2}_{12 \text{ times}} = \boxed{2^{12}}$

(b) How many of the sequences have at least 9 and no more than 11 heads?

$\boxed{C(12,9) + C(12,10) + C(12,11)}$

$C(12,9)$: exactly 9 heads; $C(12,10)$: exactly 10; $C(12,11)$: exactly 11

(c) In how many ways can I get a total of 5 heads with the first and last toss being heads?

H _ _ _ _ _ _ _ _ _ _ H

Need to make sure there are exactly 3 heads here, so

$\boxed{C(10,3)}$ options.

14. (10 pts.)
A bag contains 9 colored marbles, of which 5 are red and 4 are blue marbles. I plan to pick 3 marbles from the bag.

Note: In the following three parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k).

(a) What is the total number of ways 3 marbles can be selected?

$C(9,3)$

(b) If I pick 3 marbles, in how many ways can I get all red marbles or all blue marbles?

$C(5,3)$ choices with all red
$C(4,3)$ choices with all blue

So $C(5,3) + C(4,3)$ total options

(c) In how many ways can I get at least one red and at least one blue marble among the 3 that I select?

Get at least one red, at least one blue by

Either getting 1 red, 2 blue, in $C(5,1)C(4,2)$ ways

OR getting 2 red, 1 blue, in $C(5,2)C(4,1)$ ways,

So $C(5,1)C(4,2) + C(5,2)C(4,1)$ options

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