Conditional Probability

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For example, based on a .292 batting average for 2016, we might assign probability 29% to Kris Bryant having a hit in his first at-bat of 2017.

But suppose (closer to opening day), we learn that the pitcher Bryant will be facing in his first at-bat will be left-handed. We might want to (indeed we should) use this new information to re-assign the probability.

Conditional Probability

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Additional information may change the **sample space** and the **successful event subset.**

Conditional Probability

Example Let us consider the following experiment: A card is drawn at random from a standard deck of cards. Recall that there are 13 hearts, 13 diamonds, 13 spades and 13 clubs in a standard deck of cards.

- Let H be the event that a heart is drawn,
- let R be the event that a red card is drawn and
- let F be the event that a face card is drawn, where the face cards are the kings, queens and jacks.

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(a) If I draw a card at random from the deck of 52, what is $\mathbf{P}(H)$?

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(a) If I draw a card at random from the deck of 52, what is $\mathbf{P}(H)$? $\frac{13}{52} = 25\%$.

Conditional Probability

(b) If I draw a card at random, and without showing you the card, I tell you that the card is red, then what are the chances that it is a heart?

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(b) If I draw a card at random, and without showing you the card, I tell you that the card is red, then what are the chances that it is a heart? $\frac{13}{26} = 50\%$. Because you know that the card drawn is red, the Sample Space of all possible outcomes has size 26 (there are 26 red cards), not 52. Of the 26 red cards, 13 are hearts, so there are 13 successful outcomes.

Conditional Probability

(b) If I draw a card at random, and without showing you the card, I tell you that the card is red, then what are the chances that it is a heart? $\frac{13}{26} = 50\%$. Because you know that the card drawn is red, the Sample Space of all possible outcomes has size 26 (there are 26 red cards), not 52. Of the 26 red cards, 13 are hearts, so there are 13 successful outcomes.

Here we are calculating the probability that the card is a heart given that the card is red. This is denoted by $\mathbf{P}(H|R)$, where the vertical line is read as "given". Notice how the probability changes with the prior information. Note also that we can think of the prior information as restricting the sample space for the experiment in this case. We can think of all red cards or the set R as a **reduced sample space**.

Conditional Probability

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There are 6 red face cards and 26 red cards so $\mathbf{P}\left(F \middle| R\right) = \dfrac{6}{26} = \dfrac{3}{13}$. *Notice that in this case, the probability does not change even though both the sample space and the event space do change.*

Conditional Probability

Recap: the probability that the card is a heart given (the prior information) that the card is red is denoted by

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Note that

$$\mathbf{P}\left(H \middle| R\right) = \frac{n(H \cap R)}{n(R)} = \frac{n(H \cap R)/n(S)}{n(R)/n(S)} = \frac{\mathbf{P}(H \cap R)}{\mathbf{P}(R)}.$$

This probability is called the **conditional probability of H given R.**

Conditional Probability

Definition: If A and B are events in a sample space S, with $\mathbf{P}(B) \neq 0$, the **conditional probability** that an event A will occur, given that the event B has occurred is given by

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From our example above, we saw that sometimes $\mathbf{P}\left(A \middle| B\right) = \mathbf{P}(A)$ and sometimes $\mathbf{P}\left(A \middle| B\right) \neq \mathbf{P}(A)$. When $\mathbf{P}(A|B) = \mathbf{P}(A)$, we say that the events A and B are *independent*. We will discuss this in more detail in the next section.

Calculating Conditional Probabilities

Example Consider the data, in the following table, recorded over a month with 30 days:

Weather

Mood		S	NS
	G	9	6
	NG	1	14

On each day I recorded, whether it was sunny, (S), or not, (NS), and whether my mood was good, G, or not (NG).

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On each day I recorded, whether it was sunny, (S), or not, (NS), and whether my mood was good, G, or not (NG).

(a) If I pick a day at random from the 30 days on record, what is the probability that I was in a good mood on that day, $\mathbf{P}(G)$? The sample space is the 30 days under discussion. I was in a good mood on $9 + 6 = 15$ of them so $\mathbf{P}(G) = \frac{15}{30} = 50\%$.

Calculating Conditional Probabilities

(b) What is the probability that the day chosen was a Sunny day, $\mathbf{P}(S)$?

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The sample space is still the 30 days under discussion. It was sunny on $9 + 1 = 10$ of them so $\mathbf{P}(S) = \frac{10}{30} \approx 33\%$.

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(c) What is $\mathbf{P}\left(G \middle| S\right)$?

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(c) What is $\mathbf{P}\left(G \middle| S\right)$? $\mathbf{P}\left(G \middle| S\right) = \frac{\mathbf{P}(G \cap S)}{\mathbf{P}(S)}$. Hence we need to calculate $\mathbf{P}(G \cap S)$. Here the sample space is still the 30 days: $G \cap S$ consists of sunny days in which I am in a good mood and there were 9 of them. Hence $\mathbf{P}(G \cap S) = \frac{9}{30}$. Therefore $\mathbf{P}\left(G \middle| S\right) = \frac{\frac{9}{30}}{\frac{10}{30}} = \frac{9}{10} = 90\%$.

Calculating Conditional Probabilities

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Note that if we discover that $\mathbf{P}\left(A \middle| B\right) \neq \mathbf{P}(A)$, it does not necessarily imply a cause-and-effect relationship. In the example above, the weather might have an effect on my mood, however it is unlikely that my mood would have any effect on the weather.

Calculating Conditional Probabilities

Example Of the students at a certain college, 50% regularly attend the football games, 30% are first-year students and 40% are upper-class students (i.e., non-first years) who do not regularly attend football games.

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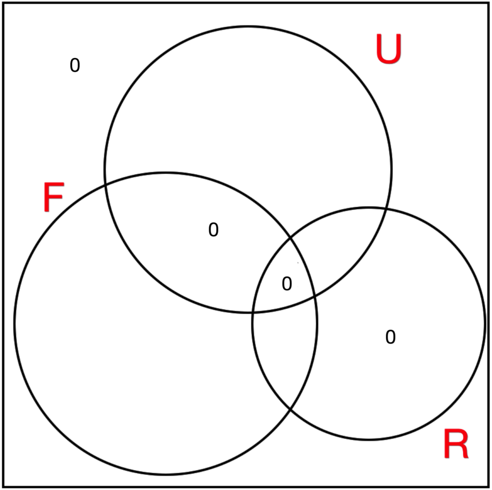
(a) What is the probability that a student selected at random is both is a first-year student and regularly attends football games?

We could use an algebra approach, or a Venn diagram approach. We'll do the latter. Let R be the set of students who regularly attend football games; let U be the set of upper-class students, and let F be the set of first-year students. Note in this example that U does not stand for the UNIVERSAL SET (if you want a relevant universal set it is $F \cup U$). We are given $\mathbf{P}(R) = 50\%$; $\mathbf{P}(F) = 30\%$; $\mathbf{P}(U \cap R') = 40\%$.

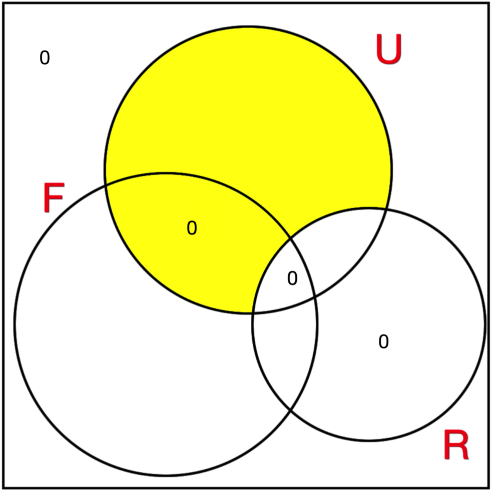
Calculating Conditional Probabilities



We know $U \cup F$ is everybody, $U \cap F = \emptyset$ and $R \subset U \cup F$. Therefore we can fill in four 0's as indicated.

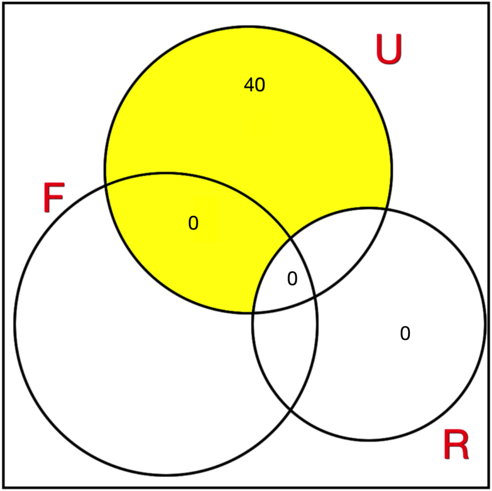
Calculating Conditional Probabilities

Next identify what you are given.



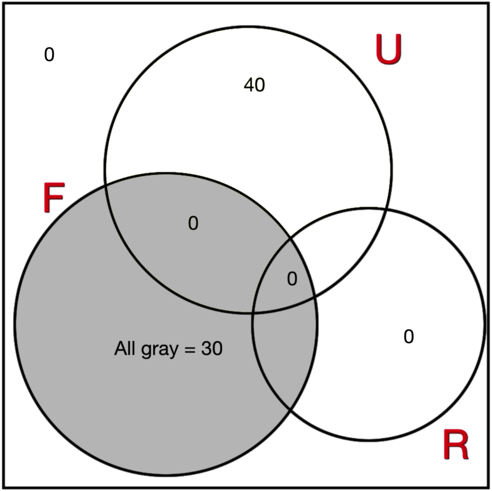
The yellow region is $U \cap R'$ and we know $\mathbf{P}(U \cap R') = 40\%$.

Calculating Conditional Probabilities



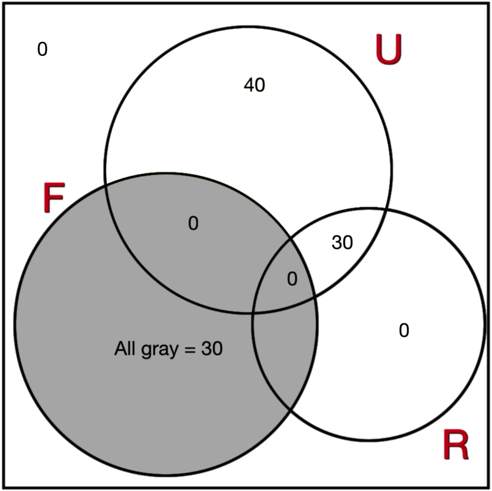
We also know the values for the disk R and the disk F.

Calculating Conditional Probabilities



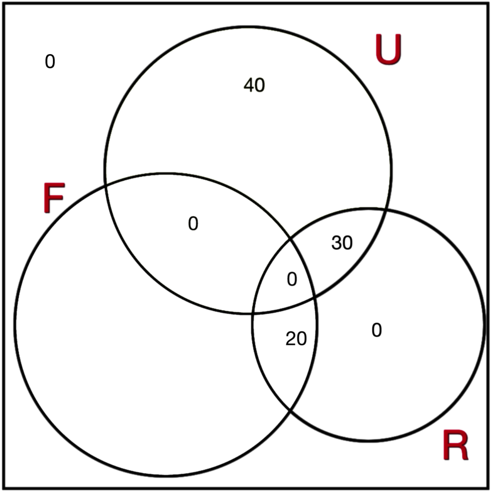
Since $F \cup U$ is everybody, $\mathbf{P}(F \cup U) = 100$. From the Inclusion-Exclusion Principle we see we can work out the unknown bit of U.

Calculating Conditional Probabilities



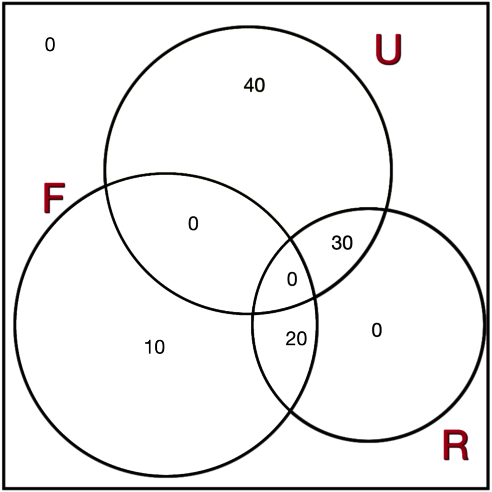
Since n(F) + n(U) = 100

Calculating Conditional Probabilities



Since $\mathbf{P}(R) = 50$ we can fill in the last bit of R.

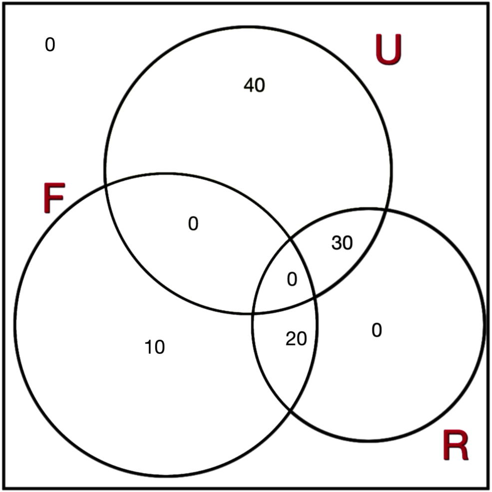
Calculating Conditional Probabilities



Since $\mathbf{P}(F) = 30$ we can fill in the last bit of F. Now we can give the answer to part a): $\mathbf{P}(F \cap R) = .2$

Calculating Conditional Probabilities

(b) What is the conditional probability that the person chosen attends football games, given that he/she is a first year student?



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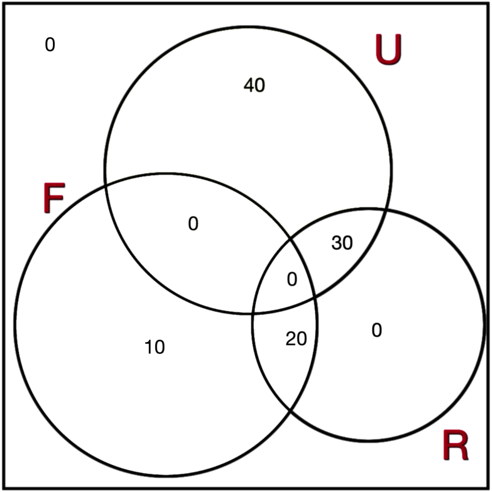
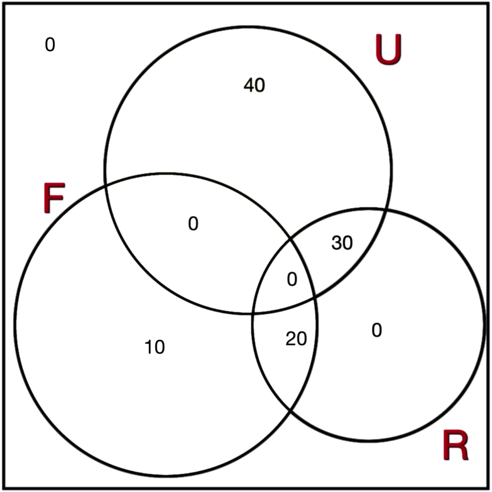


$$\mathbf{P}\left(R \middle| F\right) = \frac{\mathbf{P}(R \cap F)}{\mathbf{P}(F)} = \frac{0.2}{0.3} \approx 67\%.$$

Calculating Conditional Probabilities

(c) What is the conditional probability that the person is a first year student given that he/she regularly attends football games?



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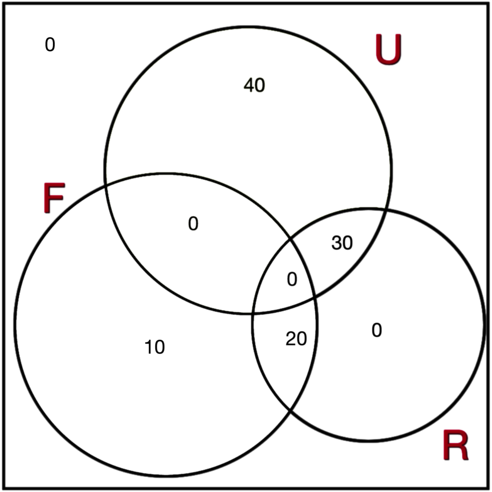

$$\mathbf{P}\left(F \middle| R\right) = \frac{\mathbf{P}(R \cap F)}{\mathbf{P}(R)} = \frac{0.2}{0.5} = 40\%.$$

Calculating Conditional Probabilities

Example If S is a sample space, and E and F are events with

$$\mathbf{P}(E) = .5, \quad \mathbf{P}(F) = .4 \quad \text{and} \quad \mathbf{P}(E \cap F) = .3,$$

(a) What is $\mathbf{P}\left(E \middle| F\right)$?

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(b) What is $\mathbf{P}\left(F \middle| E\right)$?

$$\mathbf{P}\left(F \middle| E\right) = \frac{\mathbf{P}(E \cap F)}{\mathbf{P}(E)} = \frac{0.3}{0.5} = 60\%.$$

A formula for $\mathbf{P}(E \cap F)$.

We can rearrange the equation

$$\mathbf{P}\left(E \middle| F\right) = \frac{\mathbf{P}(E \cap F)}{\mathbf{P}(F)}$$

to get

$$\boxed{\mathbf{P}(F)\mathbf{P}\left(E \middle| F\right) = \mathbf{P}(E \cap F).}$$

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This formula gives us a **multiplicative formula** for $\mathbf{P}(E \cap F)$. In addition to giving a formula for calculating the probability of two events occurring simultaneously, it is very useful in calculating probabilities for **sequential events**.

$$\mathbf{P}(E \cap F) = \mathbf{P}(E)\mathbf{P}\left(F \middle| E\right).$$

The probability that E and then F will occur is the probability of E times the probability that F happens given that E has happened.

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$\mathbf{P}(E) = \mathbf{P}\left(E \middle| F\right) \cdot \mathbf{P}(F) = 0.2 \cdot 0.3 = 0.06 = 6\%.$

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Example: The probability that it will be 30°F or below tomorrow morning is 0.5. When the temperature is that low, the probability that my car will not start is 0.7. What is the probability that tomorrow morning it will be 30°F or below **and** my car will not start?

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Let N be the event "my car will not start" and let T be the event the "maximum temperature tomorrow will be 30°F or below". Then
$\mathbf{P}(T \cap N) = \mathbf{P}(T) \cdot \mathbf{P}\left(N \middle| T\right) = 0.5 \cdot 0.7 = 0.35 = 35\%.$

Tree Diagrams.

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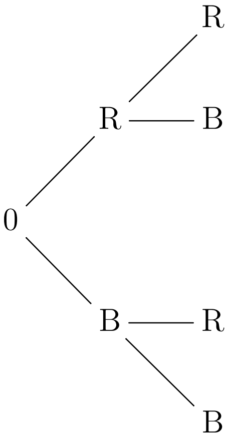
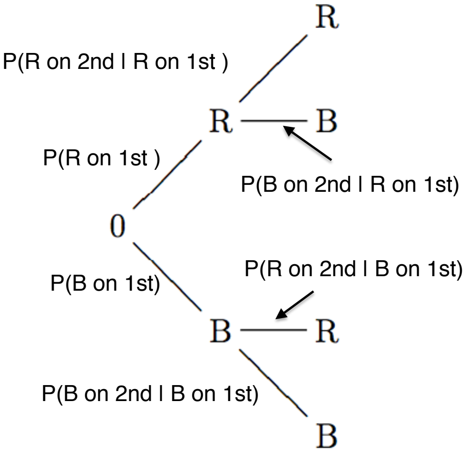
We can draw a tree diagram to represent the possible outcomes of the above experiment and label it with the appropriate conditional probabilities as shown (where 1st denotes the first draw and 2nd denotes the second draw):

Tree Diagrams.





Tree Diagrams.

(a) Fill in the appropriate probabilities on the tree diagram on the left above (note: the composition of the bag changes when you do not replace the first ball drawn).

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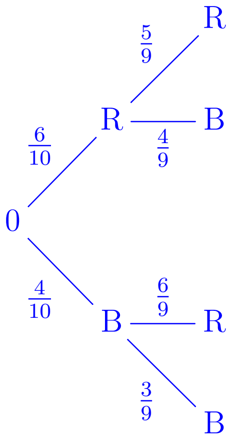

$$\mathbf{P}(R) = \frac{6}{10}; \mathbf{P}(B) = \frac{4}{10};$$

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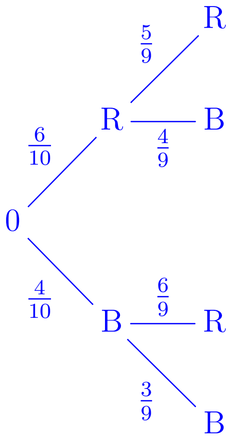

$\mathbf{P}(R) = \frac{6}{10}$; $\mathbf{P}(B) = \frac{4}{10}$;

Because you did not replace the ball, there are only 9 balls at the second step.

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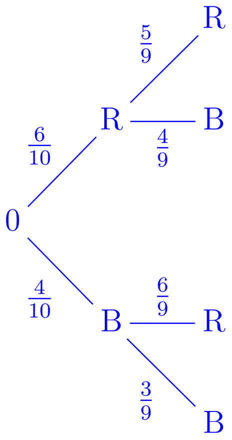

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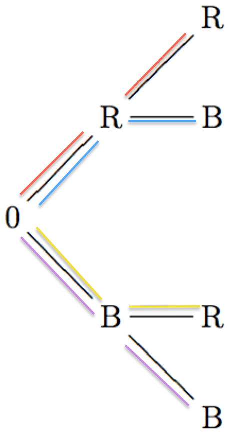
Because you did not replace the ball, there are only 9 balls at the second step.

If you are at the R node after step 1, there are 5 red balls and 4 black ones.

If you are at the B node after step 1, there are 6 red balls and 3 black ones.

Tree Diagrams.

Note that each path on the tree diagram represents one outcome in the sample space.



Outcome	Probability
RR(red path)	
RB(blue path)	
BR(yellow path)	
BB(purple path)	

Tree Diagrams.

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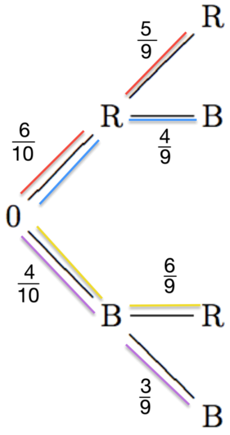
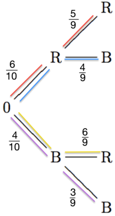
Outcome	Probability
RR(red path)	$\frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90}$
RB(blue path)	$\frac{6}{10} \cdot \frac{4}{9} = \frac{24}{90}$
BR(yellow path)	$\frac{4}{10} \cdot \frac{6}{9} = \frac{24}{90}$
BB(purple path)	$\frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90}$



Tree Diagrams.

To find the probability of an event, we identify the outcomes (paths) in that event and add their probabilities.



What is the probability that both marbles are red?

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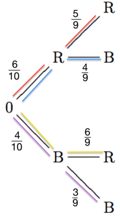

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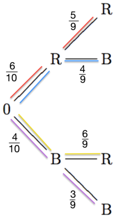

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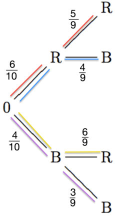

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$$\frac{6}{10} \cdot \frac{4}{9} + \frac{4}{10} \cdot \frac{3}{9} = \frac{24}{90} + \frac{12}{90} = \frac{36}{90} = 40\%.$$

Tree Diagrams.

Summary of “rules” for drawing tree diagrams

1. The branches emanating from each point (that is branches on the immediate right) must represent all possible outcomes in the next stage of classification or in the next experiment.

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Tree Diagrams.

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3. To calculate the probability of an event E, collect all paths in the event E, calculate the probability for each such path and then add the probabilities of those paths.

Tree Diagrams.

Example 1 A box of 20 apples is ready for shipment, four of the apples are defective. An inspector will select at most four apples from the box. He selects each apple randomly, one at a time, inspects it and if it is not defective, sets it aside. The first time he selects a defective apple, he stops the process and the box will not be shipped. If the first four apples selected are good, he replaces the 4 apples and ships the box.

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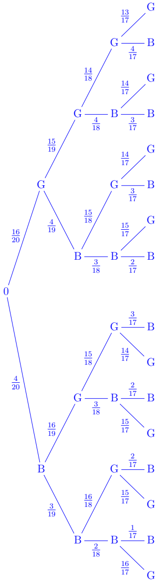
Tree Diagrams.



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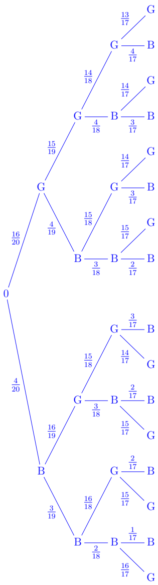

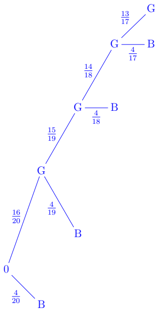

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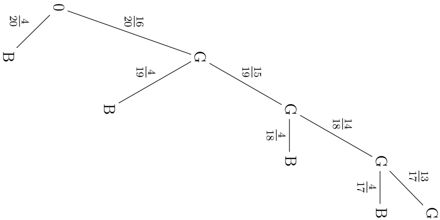
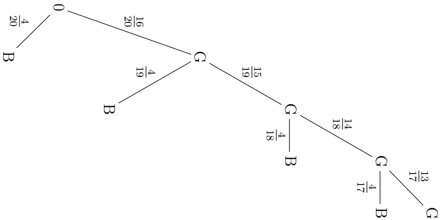


(b) What is the probability that the box is shipped?

Tree Diagrams.



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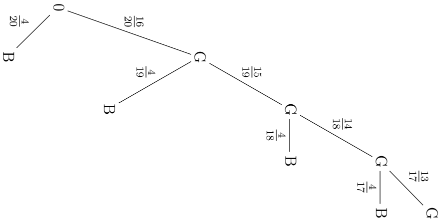

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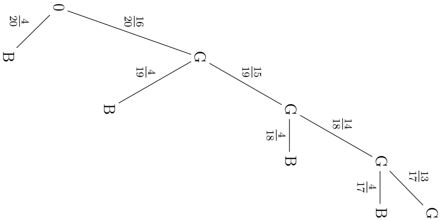

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$$\frac{4}{20} + \frac{16}{20} \cdot \frac{4}{19} \approx 0.37.$$

Tree Diagrams.

Example In a certain library, twenty percent of the fiction books are worn and need replacement. Ten percent of the non-fiction books are worn and need replacement. Forty percent of the library's books are fiction and sixty percent are non-fiction. What is the probability that a book chosen at random needs repair? Draw a tree diagram representing the data.

Tree Diagrams.

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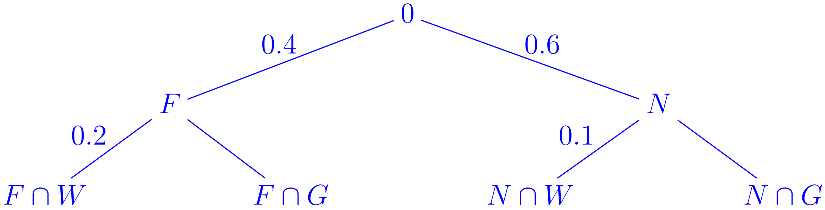
Let F be the subset of fiction books and let N be the subset of non-fiction books. Let W be the subset of worn books and let G be the subset of non-worn books.

Tree Diagrams.

What we are given.



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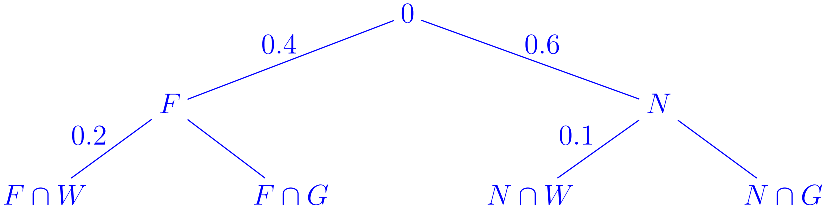

Since the probabilities at each node need to add up to one, we can complete the tree as follows

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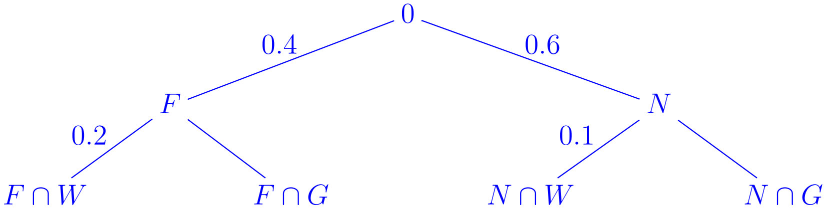

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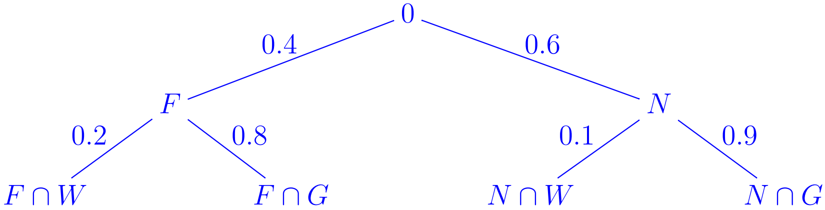


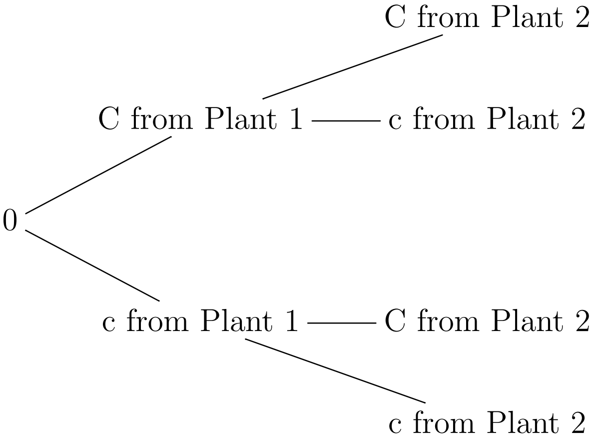
Tree Diagrams.

Example (Genetics) Traits passed from generation to generation are carried by genes. For a certain type of pea plant, the color of the flower produced by the plant (either red or white) is determined by a pair of genes. Each gene is of one of the types C (dominant gene) or c (recessive gene). Plants for which both genes are of type c (said to have genotype cc) produce white flowers. All other plants — that is, plants of genotypes CC and Cc — produce red flowers. When two plants are crossed, the offspring receives one gene from each parent. If the parent is of type Cc, both genes are equally likely to be passed on.

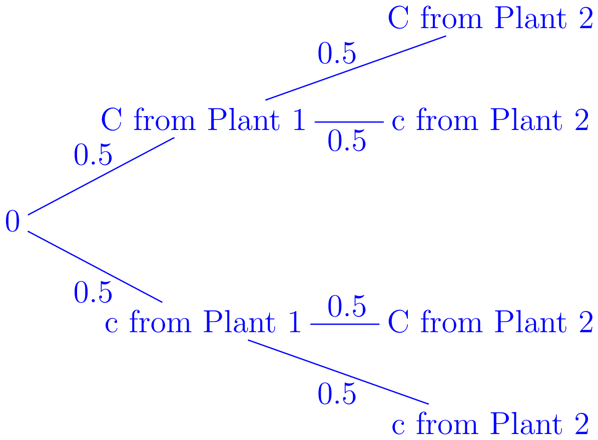
Tree Diagrams.

I. Suppose you cross two pea plants of genotype Cc,

I(a) Fill in the probabilities on the tree diagram below.



Tree Diagrams.



Tree Diagrams.

I(b) What is the probability that the offspring produces white flowers?

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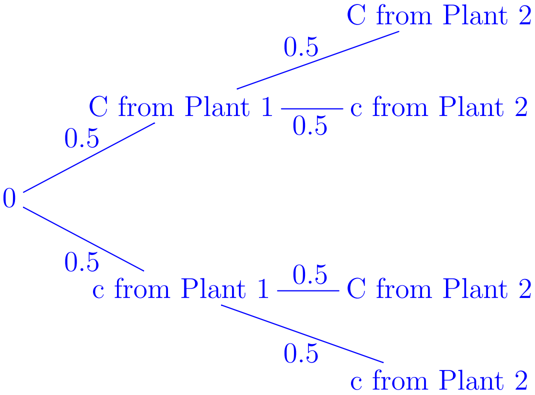


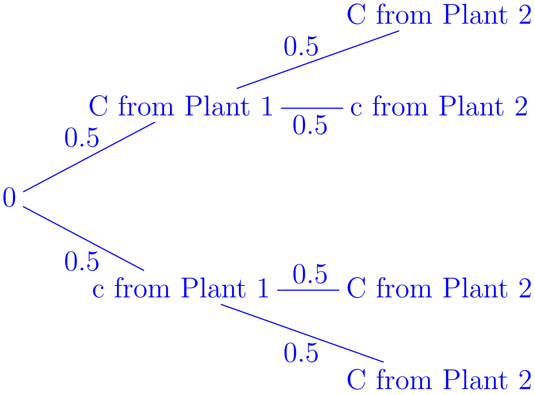
You only get white flowers from path “c from Plant 1” to “c from Plant 2” so the answer is $0.5 \cdot 0.5 = 0.25 = 25\%$.

Tree Diagrams.

I(c) What is the probability that the offspring produces red flowers?

Tree Diagrams.

I(c) What is the probability that the offspring produces red flowers?

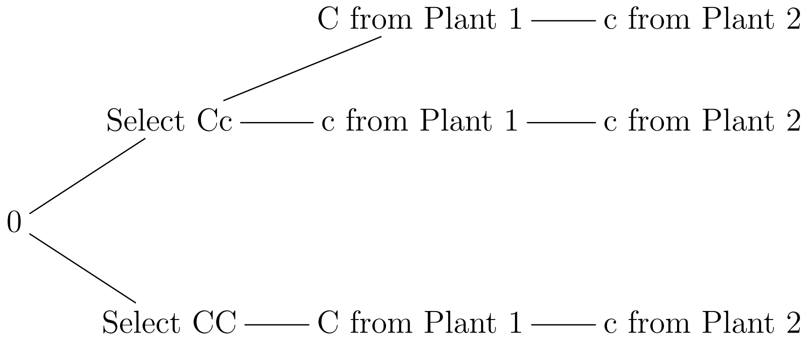


The answer is either $1 - \mathbf{P}(cc) = 75\%$ or $\mathbf{P}(cC) + \mathbf{P}(Cc) + \mathbf{P}(CC) = 75\%$.

Tree Diagrams.

II. Suppose you have a batch of red flowering pea plants, of which 60% have genotype Cc and 40% have genotype CC. You select one of these plants at random and cross it with a white flowering pea plant.

II(a) What is the probability that the offspring will produce red flowers (use the tree diagram below to determine the probability).



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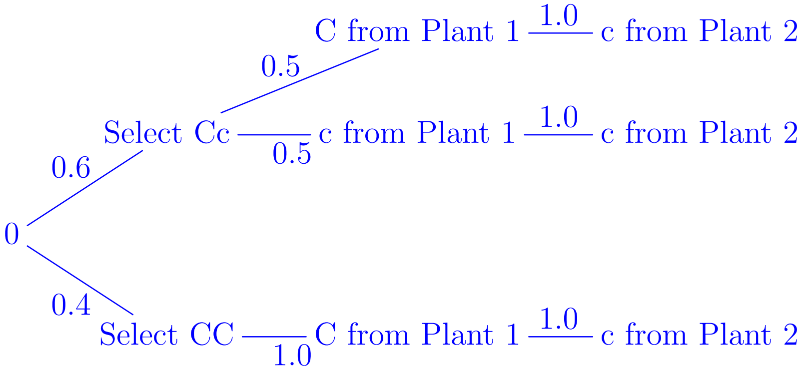

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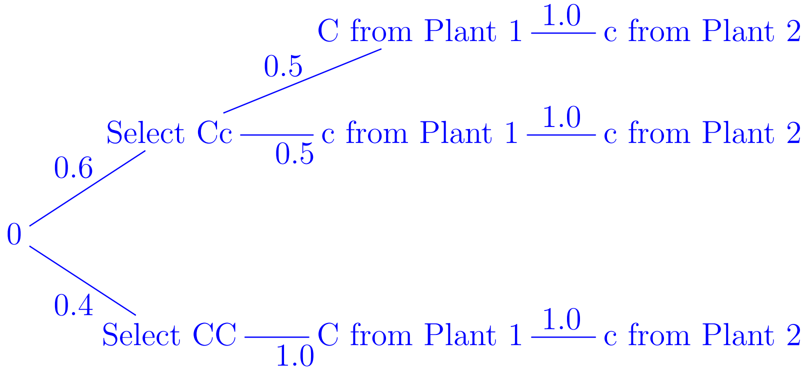

You get red flowers unless you get cc: $0.6 \cdot 0.5 \cdot 1.0 = 0.3$, so the requested probability is $1 - 0.3 = 0.7$. You can also get it as

$$0.6 \cdot 0.5 \cdot 1.0 + 0.4 \cdot 1.0 \cdot 1.0 = 0.3 + 0.4 = 0.7$$