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Repetitions in the list or changes in the order of presentation do not change the set. For example, the following lists describe the same set:

$$\{5, 5, 6, 7\} = \{5, 6, 7\} = \{6, 5, 7\}.$$

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When describing the elements of a set, we should be careful that there is **no ambiguity in our description** and the set is well defined.

For example, to talk about “the set of the 15 sexiest men of the twenty-first century” does not make sense, since the description is subject to personal opinion and different people may produce different sets from this description.

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On the other hand, the **set** of “men named ‘sexiest man alive’ by *People* magazine between 2001 and 2015” is unambiguous.

Notation

We read the notation “ $5 \in D$ ” as “*5 is an element of D*”.

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Equal Sets We say two sets are **equal** if they consist of exactly the same elements.

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These two sets are equal and have a single element. We have $A = B = \{\text{Barry Bonds}\}.$

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When we place an element after the dots, as in

$K = \{2, 4, 6, \dots, 100\}$, this indicates that we are talking about the finite set of even numbers greater than 1 and less than or equal to 100 (the last element on the list is 100).

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NB: This notation assumes we have an implicit agreement as to the formula for the remaining terms. For example E might have been $\{2, 4, 6, 12, 14, 16, 22, 24, 26, \dots\}$.

Set-Builder Notation

A description of the set D above may also be written using set-builder notation:

$$D = \{x \mid x \text{ is an integer between 5 and 7 inclusive}\}$$

or

$$D = \{x \mid x \text{ is an integer and } 5 \leq x \leq 7\}.$$

Here the symbol \mid is read as “such that” and the upper mathematical sentence above reads as “ D is equal to the set of all x such that x is an integer between 5 and 7 inclusive”.

The Empty Set

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$\{\text{all major league baseball players who got more than 80 home runs in a single season}\} = \emptyset$.

$\{\text{Years in which Jason Biggs won Best Actor Oscar}\} = \emptyset$.

Subsets

A **Subset** of a set A is a collection of elements of A . We have B is a subset of A , denoted by $B \subseteq A$, if every element of B is also an element of A . We say that B is a **proper subset** of A if $B \subseteq A$, but $B \neq A$.

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Is $D \subseteq B$? Yes. $2 \in B$ and $6 \in B$ so every element in D is in B .

Is $D \subseteq C$?

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Is D a proper subset of B ? Yes: For example $8 \in B$ but $8 \notin D$.

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Two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

The collection of all the subsets of a set is called the **power set**. For example, the power set of $\{a, b, c\}$ has eight elements: \emptyset , $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and $\{a, b, c\}$

Universal Sets

Sometimes we wish to restrict our attention to a particular set, called a *universal set* and usually denoted by U . For example, in doing a voter polling survey you might want to restrict attention to likely voters. In this case the universal set would be likely voters (however you defined it). Or you might want to survey likely Republican voters which would give you a different universal set.

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If we let $R = \{\text{students in the class who like Rap music}\}$,

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Note To avoid ambiguity in the definition of such sets, it is common in surveys such as this to restrict answers to the given questions to “yes” and “no”; (e.g. “Do you like Rap music? Yes No (circle one)”). In doing this the resulting sets are well defined, but of course we have ignored the tastes of those who like some rap music but not all.

Union

Given two sets, A and B , we define their **union**, denoted $A \cup B$, to be the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Important note: when we say “or” we **always** mean “inclusive or”. So if $a \in A$ and $a \in B$, then $a \in A \cup B$.

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Notes:

- ▶ $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
- ▶ $A \cup B = B \cup A$.
- ▶ $A \cup \emptyset = \emptyset \cup A = A$.

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Union of 3 sets If A and B and C are sets, their union $A \cup B \cup C$ is the set whose elements are those objects which appear in at least one of A **or** B **or** C .

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Intersections

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Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. List the elements of the set $A \cap B \cap C$.
 $A \cap B = \{2, 4\}$ so $A \cap B \cap C = \{4\}$.

Universal set and complements

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$$A^c = \{5, 6, 7, 8, 9, 10\}, B \cup C = \{2, 3, 4, 5, 6, 8\},$$
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Give a verbal description of the set $(R \cup E \cup C)' = (R \cup E \cup C)^c$. Are you in this set?

The people in the class who liked none of rap, classical or 80's music.

Properties of the empty set and complements

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Complements have the following properties:

$$A \cap A' = \emptyset, \quad (A')' = A \quad \text{and} \quad A \cup A' = U$$

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For example if

$A = \{\text{All major league baseball players who hit more than 700 home runs in their career}\} = \{\text{Barry Bonds, Hank Aaron, Babe Ruth}\}$ and

$B = \{\text{All major league baseball players with a career batting average greater than .350}\} = \{\text{Ty Cobb, Rogers Hornsby, Joe Jackson}\}$, then A and B are disjoint, i.e.

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$B \cap C = \emptyset$, so B and C are disjoint.

A and B have elements in common, so they are not disjoint. Also, A and C are not disjoint.