Finite Mathematics (Math 10120), Spring 2017

Quiz 6, Monday May 1

Solutions

R2-D2 and C-3PO play a game in which R2-D2 picks either 1 or 2 and C-3PO picks either 2 or 3. If the sum of the two picked numbers is even then R2-D2 pays that sum to C-3PO (so R2-D2 *loses* money). If the sum of the two picked numbers is odd then C-3PO pays that sum to R2-D2 (so R2-D2 *wins* money).

1. (3 pts) Which of the following gives the payoff matrix for this game? (Note R2-D2 is the row player and C-3PO is the column player.)

(a)

	2	3
1	-3	4
2	4	-5

(b)

	2	3
1	3	-4
2	-4	5

(c)

	2	3
1	-3	-4
2	-4	-5

(d)

	2	3
1	3	4
2	4	5

(e)

	2	3
1	4	-3
2	-5	4

Solution.

- If R2-D2 picks 1 and C-3PO picks 2 then the sum $1+2=3$ is odd. In this case, R2-D2 *wins* that sum, and so the corresponding entry of the payoff matrix is 3.
- If R2-D2 picks 1 and C-3PO picks 3 then the sum $1+3=4$ is even. In this case, R2-D2 *loses* that sum, and so the corresponding entry of the payoff matrix is -4.
- If R2-D2 picks 2 and C-3PO picks 2 then the sum $2+2=4$ is even. In this case, R2-D2 *loses* that sum, and so the corresponding entry of the payoff matrix is -4.
- If R2-D2 picks 2 and C-3PO picks 3 then the sum $2+3=5$ is odd. In this case, R2-D2 *wins* that sum, and so the corresponding entry of the payoff matrix is 5.

Therefore the correct answer is (b).

2. (2 pts) Is the game strictly determined? If so, what are the saddle points and the value of the game?

Solution. The maxmin of the rows (the maximum of the row minima) is -4 and the minmax of the columns (the minimum of the column maxima) is 3. These are different, so the game is not strictly determined.

3. (5 pts) Suppose R2-D2 plays the mixed strategy $\begin{bmatrix} 0.2 & 0.8 \end{bmatrix}$ and C-3PO plays the mixed strategy $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$. What is the long run expected value of the game in this case?

Solution. Perform the expected value computation:.

$$\mu = \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = (0.2)(3)(0.5) + (0.2)(-4)(0.5) + (0.8)(-4)(0.5) + (0.8)(5)(0.5) = 0.3$$

This is the long run expected value of the game for these two mixed strategies.