Finite Mathematics (Math 10120), Spring 2017

Quiz 4, Friday March 31

Solutions

1. (5 pts) I play a casino game in which I have a 10% chance of winning \$5; I have a 2/10 chance of winning \$3; I have a probability .3 of winning \$2; and the remaining possibility is that I lose \$2.

   Let Y be my winnings, in dollars, in a play of this game (note that $Y = -2$ if I lose \$2). Calculate the expected value $E(Y)$ of Y, and calculate the probability that I win more than $E(Y)$ dollars.

   Solution: Here is the probability distribution table for this experiment.

value of Y	5	3	2	-2
probability of that value	0.1	0.2	0.3	0.4

   Notice that we can calculate $\mathbf{P}(Y = -2) = 0.4$ from the fact that the sum of the probabilities should be 1.

   We get that

$$E(Y) = 5 \times 0.1 + 3 \times 0.2 + 2 \times 0.3 - 2 \times 0.4 = 0.9.$$

   The probability that I win more than $E(Y)$ dollars is the probability that I win more than 0.9, which is the probability that I win either 5 or 3 or 2, which is $0.1 + 0.2 + 0.3 = 0.6$.

2. (5 pts) I've just gotten off a flight from Chicago to South Bend. There are four bags in the hold, two of which are mine. The bags will come out on the luggage carousel in a random order. Let X be the number of bags that have come out the moment the second of my bags comes out. For example, if the first two bags out are mine then $X = 2$; if mine are the last two to come out then $X = 4$. The other possible value for X is 3.

   Find the probability distribution of X. That is, find $\mathbf{P}(X = 2)$, $\mathbf{P}(X = 3)$ and $\mathbf{P}(X = 4)$. **Hint**: draw a tree diagram, branching on whether the first bag, second bag, third bag etc. is one of mine or not.

x	$\mathbf{P}(X = x)$
2	1/6
3	2/6 or 1/3
4	3/6 or 1/2

   Solution: The only way to have $X = 2$ is to have mine be the first two bags out; this happens with probability $(2/4)(1/3) = 1/6$. We have $X = 3$ either if my bags come out first and third (probability $(2/4)(2/3)(1/2) = 1/6$) or if my bags come out second and third (probability $(2/4)(2/3)(1/2) = 1/6$), so $\mathbf{P}(X = 3) = (1/6) + (1/6) = 1/3$. We have $X = 4$ either if my bags come out first and fourth (probability $(2/4)(2/3)(1/2)(1) = 1/6$) or if my bags come out second and fourth (probability $(2/4)(2/3)(1/2)(1) = 1/6$), or if my bags come out third and fourth (probability $(2/4)(1/3)(1)(1) = 1/6$)so $\mathbf{P}(X = 4) = (1/3) + (1/3) + (1/3) = 1/2$.