

Name: SOLUTIONS

MATH 10120 Finite Mathematics
Final Exam - practice version 2
May 1, 2017

Instructor: D. Galvin and G. Conant

- Be sure that you have all 16 pages of the exam.
- The exam lasts for 2 hrs.
- There are 30 multiple choice questions, each worth 5 points.
- You may use a calculator.
- The Honor Code is in effect for this exam.
- A table of areas under the standard normal curve will be given out with the exam.

May the force be with you!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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Name: _____

Instructor: D. Galvin and G. Conant

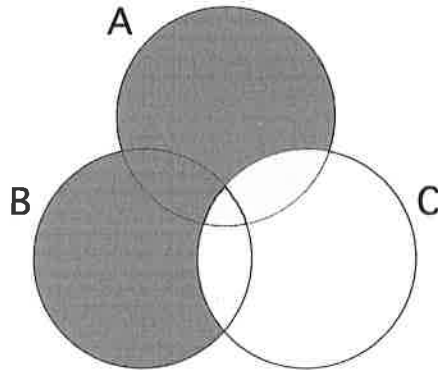
Multiple Choice

1. (5 pts.) Let A and B be sets such that $n(B) = 12$, $n(A \cap B) = 8$ and $n(A \cup B) = 21$. Find $n(A)$.

- (a) 13 (b) 17 (c) 41 (d) 1 (e) 9

$$\begin{aligned}n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\21 &= n(A) + 12 - 8 \\17 &= n(A)\end{aligned}$$

2. (5 pts.) Which of the following corresponds to the area shaded in gray in the following Venn diagram? (Remember that the notation $()'$ refers to the complement of the set $()$).



- (a) $(A \cup B)' \cap C$ (b) $(A \cap B) \cap C$ (c) $(A \cap B)' \cup C$
(d) $(A \cup B) \cap C'$ (e) $(A \cap B) \cap C'$

Name: _____

Instructor: D. Galvin and G. Conant

3. (5 pts.) Let $A = \{a, b, c, d, e, f, g, h, i\}$. Not including the empty set and A itself, how many subsets does A have?

- (a) 524 (b) 484 (c) 7 (d) 254 (e) 510

9 elements, so $2^9 = 512$ subsets in all.

Discounting ϕ, A , get 510

4. (5 pts.) Let M be the set of Notre Dame students enrolled in a math class, let B be the set of Notre Dame students enrolled in a biology class, and let E be the set of Notre Dame students enrolled in an engineering class. Let the universal set, U , be the set of all Notre Dame students. In words, what is $(M \cup B)' \cap E$?

- (a) Notre Dame students **not** enrolled in engineering **or** math **or** biology
(b) Notre Dame students enrolled in engineering and **either** math **or** biology but **not** both
(c) Notre Dame students enrolled in engineering **and** math **and** biology
(d) Notre Dame students enrolled in engineering but **not** math and **not** biology.
(e) Notre Dame students enrolled in engineering

Name: _____

Instructor: D. Galvin and G. Conant

5. (5 pts.) This exam contains 30 multiple choice problems, each of which has five choices. In how many ways can the exam be answered?

- (a) 30^5 (b) 5^{30} (c) 30×5 (d) $P(30, 5)$ (e) $C(30, 5)$

6. (5 pts.) Three couples go to a movie theatre. They sit in consecutive seats such that each couple is seating together, that is, each person is seating next to his/her partner. If there are 6 seats available, in how many ways can they sit?

- (a) 24 (b) 48 (c) 720 (d) 8 (e) 1

$6 \times 1 \times 4 \times 1 \times 2 \times 1$
↑ ↑ ↑ ↑ → last two
first seat third seat 4^{th} must be partner of 3rd
second person must be partner of first

Name: _____

Instructor: D. Galvin and G. Conant

7. (5 pts.) The 10 members of the "The Shirt" committee have to choose one president and two vice presidents (these have to be three different people). In how many ways can they choose these officers? [Note: here is no distinction between the two vice presidents; there isn't a "first" VP and a "second" VP]

(a) 36

(b) 360

(c) 45

(d) 120

(e) 720

$$10 \times C(9, 2)$$

↑ president ↑ two vice-presidents

8. (5 pts.) A university wants to assign a three digit number to each classroom of a new building. They can use the digits {1, 2, 3, 4, 5} but they **cannot** use any digit more than once. How many classroom numbers can they assign if the numbers have to be **less than** 250?

(a) 10

(b) 24

(c) 60

(d) 12

(e) 21

2 choices for first digit (1, 2)

If 1 chosen first: ~~4~~ for second ~~not~~ } 12 in all
3 for third

If 2 chosen first: 3 for second (not allowed to start 25-)
3 for third } 9 in all
5

Total: 12 + 9

Name: _____

Instructor: D. Galvin and G. Conant

9. (5 pts.) The University wants to select 4 students for a feedback survey. They want **all four** students from either Prof. Galvin's section or Prof. Conant's section. Prof. Galvin has 40 students in his class and Prof. Conant has 30 students in his class. How many selections are possible?

(a) 2,851,080

(b) 18,795

(c) 2,504,542,950

(d) 22,005,480

(e) 916,895

$$C(40, 4) + C(30, 4)$$

10. (5 pts.) The sample space of an experiment is $\{1, 2, 3, 4, 5\}$. The probability that the outcome is an even number is 0.4, the probability that the outcome is 1 is 0.25 and the probability that the outcome is 5 is 0.15. What is the probability that the outcome is 3?

(a) 0.2

(b) 0.8

(c) 0.4

(d) 0.6

(e) Cannot be determined with the given information.

x	1	2	3	4	5
$P(x)$.25		?		.15

Sum to .4

$$? = 1 - .25 - .15 - .4$$

Name: _____

Instructor: D. Galvin and G. Conant

11. (5 pts.) A dice is rolled twice and the numbers on the uppermost faces are recorded. Consider the three events:

- E : At least one number is odd
 F : exactly one of the numbers is 2
 G : Both numbers are even

Which of the following are a pair of mutually exclusive events?

- (a) E and G' (b) E and F' (c) F and G (d) E and G (e) E and F

12. (5 pts.) A dice is rolled three times. What is the probability that either the first roll is a six, or both of the last two rolls are sixes?

- (a) $\frac{41}{216}$ (b) $\frac{6}{216}$ (c) $\frac{36}{216}$ (d) $\frac{42}{216}$ (e) $\frac{1}{216}$

$$P(\text{First is } 6) = \frac{1}{6}$$

$$P(\text{both last two are } 6) = \frac{1}{36}$$

$$P(\text{Either one or other}) = P(\text{First is } 6) + P(\text{last two are } 6) \\ - P(\text{all three are } 6)$$

$$= \frac{1}{6} + \frac{1}{36} - \frac{1}{216}$$

Name: _____

Instructor: D. Galvin and G. Conant

13. (5 pts.) Three people are shooting at a target. The probabilities that they hit the target are 0.5, 0.6, and 0.8, respectively. Find the probability that they all miss the target.

- (a) 4% (b) 1% (c) 96% (d) 76% (e) 24%

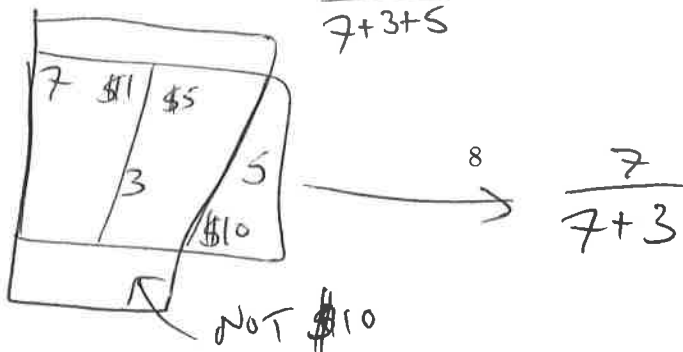
$$P = (1 - 0.5)(1 - 0.6)(1 - 0.8) = .04$$

14. (5 pts.) A wallet contains seven \$1 bills, three \$5 bills, and five \$10 bills. A bill is selected at random from the wallet. Find the probability that the bill is a \$1 bill given that it is not a \$10 bill.

- (a) $\frac{7}{15}$ (b) $\frac{4}{15}$ (c) $\frac{7}{12}$ (d) $\frac{7}{10}$ (e) $\frac{4}{5}$

$$P(\$1 / \text{NOT } \$10) = \frac{P(\$1 \text{ AND NOT } \$10)}{P(\text{NOT } \$10)}$$
$$= \frac{\frac{7}{7+3+5}}{\frac{7+3}{7+3+5}} = \frac{7}{10}$$

OR:



Name: _____

Instructor: D. Galvin and G. Conant

15. (5 pts.) Visiting the neonatal wing of the hospital, Dr. Jones noticed that a set of triplets had been born the night before. Asking about them, she was told that at least two of the babies were boys. Given this information, what is the probability that all three are boys? [Assume that within a set of triplets, each baby is equally likely to be a boy or a girl, independently of the sex of the others.]

(a) $\frac{1}{4}$

(b) $\frac{1}{8}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$

(e) $\frac{3}{4}$

8 equally likely possibilities:

BBB	} 4 have ≥ 2 boys. of those 4, 1 has all 3 boys.
BGB	
BBG	
GBB	
GGB	
G BG	
BGG	
GGG	

So conditional probability is $\frac{1}{4}$

16. (5 pts.) I have 6 keys on my keyring. Two of them open my office door, the other four do not. I try to open my office door using randomly selected keys from my keyring, never trying the same key twice. What is the probability that I succeed in opening the door using the third key that I try?

(a) $\frac{3}{5}$

(b) $\frac{1}{5}$

(c) $\frac{1}{8}$

(d) $\frac{1}{3}$

(e) $\frac{4}{27}$

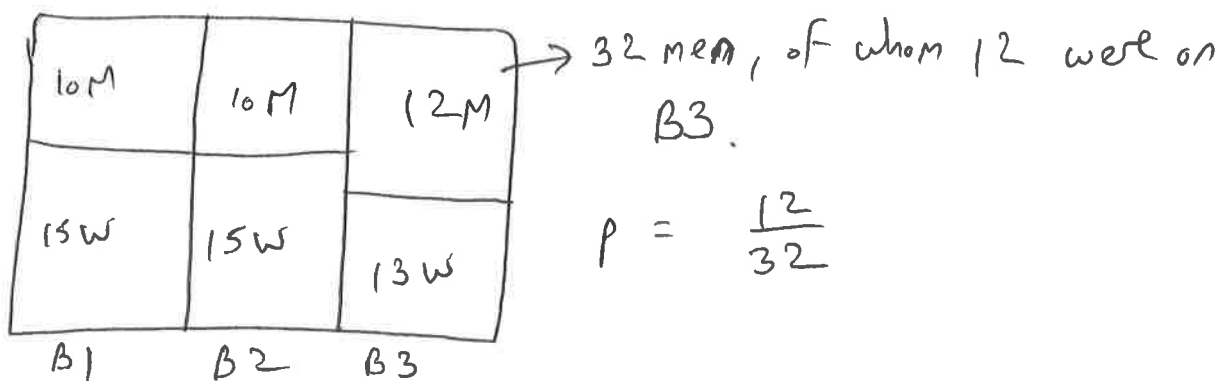
$$\begin{aligned} P(\text{Succeed on try 3}) &= P(\text{fail on first}) P(\text{fail on second} \mid \text{fail on first}) \\ &\quad P(\text{Succeed on try 3} \mid \text{fail on first 2}) \\ &= \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{1}{5} \end{aligned}$$

Name: _____

Instructor: D. Galvin and G. Conant

17. (5 pts.) Three buses, B1, B2 and B3, arrive at the South Bend Transpo Station at the same time. B1 arrives with 10 men and 15 women; B2 arrives with 10 men and 15 women; B3 arrives with 12 men and 13 women. A passenger is chosen at random. What is the probability that the passenger was in B3, given that he is a man?

- (a) 0.48 (b) 0.16 (c) 0.426 (d) 0.375 (e) 0.315



18. (5 pts.) The number of customers waiting in line at the express checkout of Martins Supermarket was counted at the beginning of each 3-min interval between 9a.m. and noon on Saturday. The data is as follows:

# customers	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	4	2	7	14	8	10	6	3	4	1

What is the relative frequency of the outcome "5 customers"?

- (a) $\frac{5}{55}$ (b) $\frac{5}{60}$ (c) $\frac{8}{55}$ (d) $\frac{8}{60}$ (e) $\frac{2}{60}$

8

$$1 + 4 + 2 + 7 + 14 + 8 + 10 + 6 + 3 + 4 + 1$$

Name: _____

Instructor: D. Galvin and G. Conant

19. (5 pts.) A game consists of rolling a die. If the number that shows up is even, the player wins, in dollars, the number shown. If the number that shows up is odd, the player loses, in dollars, the number shown. What is the expected value for the amount of money the player would win? [Note: A negative number means the player loses money.]

- (a) \$0.5 (b) \$0 (c) -\$3.5 (d) -\$0.16 (e) \$3

$$2\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) - 1\left(\frac{1}{6}\right) - 3\left(\frac{1}{6}\right) - 5\left(\frac{1}{6}\right) = \frac{3}{6}$$

20. (5 pts.) In a collection of 10 electronic components, three are defective. Two are selected at random and the number of defective components is noted. Let X be the number of defective components. Compute the probability distribution of X .

(a)

X	$P(X)$
0	$3/50$
1	$21/50$
2	$21/50$

(b)

X	$P(X)$
0	0
1	$3/10$
2	$9/100$

(c)

X	$P(X)$
0	$21/45$
1	$21/45$
2	$3/45$

(d)

X	$P(X)$
0	$21/45$
1	$21/45$
2	$6/45$

(e)

X	$P(X)$
0	$21/50$
1	$21/50$
2	$8/50$

$$P(X=0) = \frac{C(7,2)}{C(10,2)} = \frac{21}{45}$$

$$P(X=1) = \frac{7 \times 3}{C(10,2)} = \frac{21}{45}$$

~~$\rightarrow \times 2$ because (lose one two ways)~~

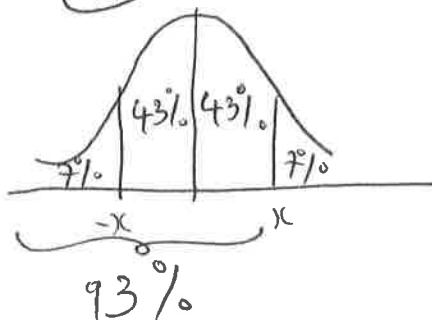
$$P(X=2) = \frac{C(3,2)}{C(10,2)} = \frac{3}{45}$$

Name: _____

Instructor: D. Galvin and G. Conant

23. (5 pts.) Suppose that on a certain standardized test given nationally, the mean score is 760 with a standard deviation of 40. A statistician reports that 86% of all people taking the test received a score that was within x points of the mean. What is x (rounded to the nearest whole number)?

- (a) 59 (b) 43 (c) 62 (d) 40 (e) 118



From table, $P(Z \leq 1.48) =$ ~~0.9306~~ $.9306$

~~x has score 1.48~~

$$\begin{aligned} x &= 1.48 \text{ standard deviations} \\ &= 1.48 \times 40 \end{aligned}$$

24. (5 pts.) A dice is rolled 7 times. Let X be the number of sixes that come up. Estimate $P(2 \leq X \leq 4)$.

- (a) 20% (b) 33% (c) 18% (d) 50% (e) 37%

$$\begin{aligned} X &= \text{Binomial}, \quad n = 7 \\ p &= \frac{1}{6} \\ q &= \frac{5}{6} \end{aligned}$$

$$\begin{aligned} P(2 \leq X \leq 4) &= \binom{7}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 + \binom{7}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4 \\ &\quad + \binom{7}{4} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3 \end{aligned}$$

$$\approx .33$$

Name: _____

Instructor: D. Galvin and G. Conant

25. (5 pts.) A student earns \$13 per hour working for the Computer Center's help line and \$9.50 per hour working at the Copy Center. Because of her course load, she limits her work to 25 hours per week. She wants to earn at least \$275 each week. Express this information as linear inequalities.

(a) $13x + 9.5y \leq 275$
 $x + y \geq 25$

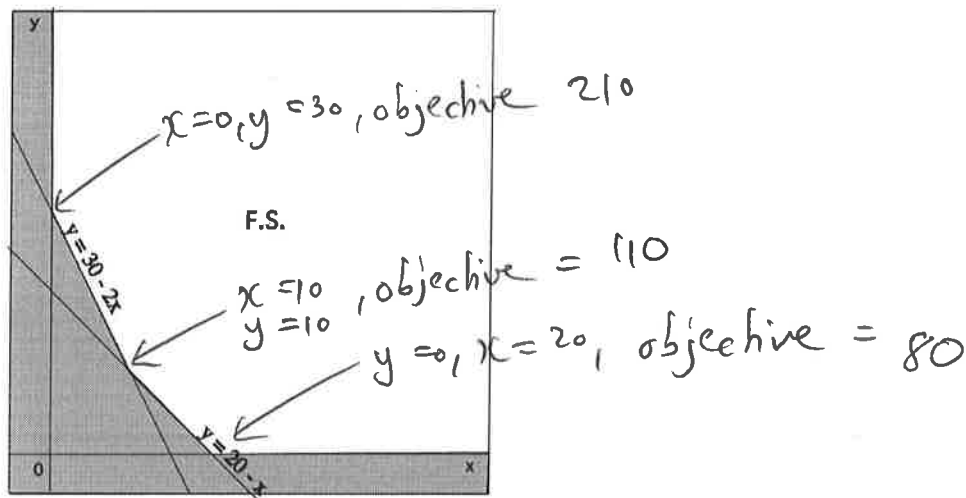
(b) $13x + 9.5y \leq 275$
 $x + y \leq 25$
 $x \geq 0, y \geq 0$

(c) $13x + 9.5y \geq 275$
 $x + y \leq 25$

~~(d)~~ $13x + 9.5y \geq 275$
 $x + y \leq 25$
 $x \geq 0, y \geq 0$

(e) $13x + 9.5y \geq 275$
 $x + y \geq 25$
 $x \geq 0, y \geq 0$

26. (5 pts.) Find the minimum of the objective function $4x + 7y$ on the feasible set given below:



(a) 70

(b) 250

(c) 80

(d) 110

(e) 210

Name: _____

Instructor: D. Galvin and G. Conant

27. (5 pts.) Raphael (R) and Chris (C) play the rock-paper-scissors game. Nothing happens if they both show the same shape. If one player chooses rock and the other chooses scissors, then the player who chose rock wins \$5. If one player chooses scissors and the other chooses paper, then the player who chose scissors wins \$3. If one player chooses paper and the other chooses rock, then the player who chose paper wins \$1. Find the pay-off matrix for Raphael (R). [Note: as usual, the rows indicate R's choices and the columns indicate C's choices.]

(a)

	Rock	Paper	Scissors
Rock	0	-1	5
Paper	1	0	-3
Scissors	-5	3	0

(b)

	Rock	Paper	Scissors
Rock	0	-5	1
Paper	5	0	3
Scissors	-1	-3	0

(c)

	Rock	Paper	Scissors
Rock	0	5	-1
Paper	-5	0	3
Scissors	1	-3	0

(d)

	Rock	Paper	Scissors
Rock	0	-3	5
Paper	3	0	-1
Scissors	-5	1	0

(e)

	Rock	Paper	Scissors
Rock	0	1	-5
Paper	-1	0	3
Scissors	5	-3	0

28. (5 pts.) Rosita (R) and Carlos (C) play a zero-sum game with pay-off matrix for Rosita given by:

	C1	C2	row min
R1	1	3	1
R2	5	2	2

Which of the following statements is FALSE?

column max 5 3 min

} different, so game is not strictly

(a) \neg If both players play each of their options equally likely, on average Rosita wins 2.75

(b) \neg If Rosita is equally likely to play each of her options, then it is better for Carlos to play C2 than C1

(c) \neg If Carlos plays C2, then it is better for Rosita to play R1 than R2

definitely

(d) This is a strictly determined game

(e) \neg The entry in the row 1, column 2 position of the payoff matrix is not a saddle point

Name: _____

Instructor: D. Galvin and G. Conant

29. (5 pts.) Rob (R) and Chad (C) play a zero-sum, two person game. The payoff matrix for the game is:

$$\begin{bmatrix} 4 & -3 \\ -2 & 3 \end{bmatrix}$$

If Rob (playing rows) uses the strategy $[0.4 \ 0.6]$ and Chad uses the strategy $\begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$, what is the (expected) value of the game?

- (a) 0.06 (b) 0.36 (c) 0.16 (d) 0 (e) 0.46

$$(0.4)(4)(0.7) + (0.4)(-3)(0.3) + (0.6)(-2)(0.7) + (0.6)(3)(0.3) = -0.46$$

30. (5 pts.) Rusty (R) and Crusty (C) play the following game: they both shout out a number, either 1 or 2. If they both shout out 1, Rusty wins 2 points. If they both shout out 2, Crusty wins 1 point. If Rusty says 1 and Crusty says 2, Rusty wins 1 point, and if Rusty says 2 and Crusty says 1, Rusty wins 5 points. If Crusty plays the strategy $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, which of the following is the best counter-strategy for Rusty?

- (a) $[1 \ 0] \rightarrow$ Rusty wins 1 (b) $[.2 \ .8] \rightarrow$ Rusty loses .6
(c) $[0 \ 1] \rightarrow$ Rusty loses 1 (d) $[.5 \ .5] \rightarrow$ Rusty breaks even
(e) Rusty has no good counter-strategy \leftarrow there is always a good counter-strategy

