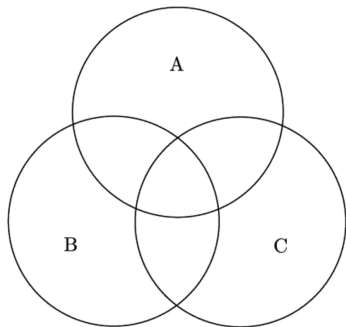


## A puzzler about Inclusion-Exclusion

The inclusion-exclusion formula is

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

What would it become if we had *three* sets?



I.e.,  $n(A \cup B \cup C) = \dots?$

## Answer

$$\begin{aligned}n(A \cup B \cup C) &= \\n(A) + n(B) + n(C) &- n(A \cap B) - n(A \cap C) - n(B \cap C) \\+ n(A \cap B \cap C) &\end{aligned}$$

In general, to find the number of elements in the union of  $n$  sets,

- ▶ add up the sizes of the sets
- ▶ subtract off the sizes of intersections, taken two at a time
- ▶ add back the sizes of intersections, taken three at a time
- ▶ etc.