4/14/2016 is a bad date.

Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2016

Name:_____

Instructors: Bhattacharya/Galvin

Exam 3

April 14, 2016

This exam is in two parts on 12 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC. _____
11. _____
12. _____
13. _____
14. _____

Tot. _____

Multiple Choice

1. (5 pts.) I have five students in my graduate class. On a recent exam (marked out of 100), the students scored

$$75,\ 63,\ 51,\ 59,\ 82.$$

The mean score is 66 (you don't to verify this). Compute the population variance.

(a) 155 (b) 112

(c) 89 (d) 66

(e) 124

Solution: $((75-66)^2+(63-66)^2+(51-66)^2+(59-66)^2+(82-66)^2)/5 = 124$

2. (5 pts.) X is a random variable with the following probability distribution:

k	0	1	2	3
$\mathbf{P}(X=k)$	.1	.2	.3	.4

Compute the standard deviation $\sigma(X)$ of X.

(a) 1.25 (b) 0.85 (c) 1

(d) 2 (e) 1.5

Solution: $E(X) = 0(.1)+1(.2)+2(.3)+3(.4) = 2$ and $E(X^2) = 0^2(.1)+1^2(.2)+2^2(.3)+3^2(.4) = 5$, so $\sigma^2(X) = E(X^2) - E(X)^2 = 5 - 2^2 = 1$ and $\sigma(X) = 1$.

3. (5 pts.) A standardized test is designed so that the scores are normally distributed with mean 140, standard deviation 20. What percentage of people who take the test get a score below 110? (Choose the closest option).

(a) 68% (b) 26.8% (c) 13.4%

(d) 6.7% (e) 93.3%

Solution: z-score of 110 is $(110 - 140)/20 = -1.5$. From table, $\mathbf{P}(Z \leq -1.5) = .0668$, so 6.7% is answer.

4. (5 pts.) I roll a dice 180 times, and I am interested in approximating the number of 6's I roll using a normal distribution. What mean μ and variance σ^2 should I choose for this normal distribution?

(a) $\mu = 30$, $\sigma^2 = 25$ (b) $\mu = 25$, $\sigma^2 = 5$ (c) $\mu = 30$, $\sigma^2 = 5$

(d) $\mu = 1/6$, $\sigma^2 = 5/36$ (e) $\mu = 90$, $\sigma^2 = 45$

Solution: Exact distribution is Binomial with $n = 180$, $p = 1/6$, so $\mu = np = 180/6 = 30$ and $\sigma^2 = npq = 180(1/6)(5/6) = 25$.

5. (5 pts.) Let Z be a standard normal random variable. Find the number x so that

$$\mathbf{P}(x \leq Z < \infty) = 0.9192.$$

(a) 1.5 (b) -2.1 (c) -1.4

(d) -0.75 (e) 1.4

Solution: From table, $\mathbf{P}(-\infty < Z \leq -1.4) = .0808$, so $\mathbf{P}(x \leq Z < \infty) = 1 - .0808 = 0.9192$, $x = -1.4$.

6. (5 pts.) In the GRE test, John's score was the 90th percentile. What does this mean?

(a) 90% of those taking the test scored 90% or better, and all of them are better than John.

(b) John's score was equal to or better than 90% of those taking the test.

(c) John scored 90% on the test.

(d) 90% of those taking the test scored more than John.

(e) 90% of the tiles belong to John.

Solution: By definition of percentiles, John's score was equal to or better than 90% of those taking the test.

7. (5 pts.) Which of the following points is in the feasible set for the system of inequalities

$$x + y \leq 5,\ x - 2y \leq 2,\ x \geq 1?$$

(a) (2,-1) (b) (3,3) (c) (0,4)

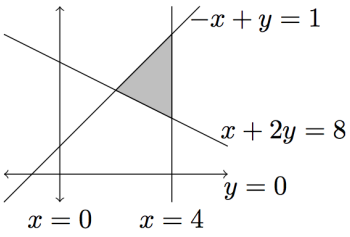
(d) (5,1) (e) (1,2)

Solution: Only $(1, 2)$ satisfies all three inequalities.

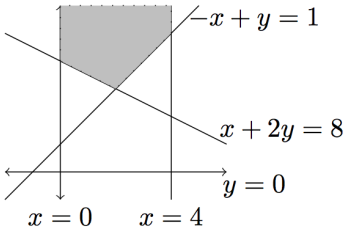
8. (5 pts.) Which of the following pictures shows the feasible set for the system of inequalities

$$-x + y \geq 1,\ x + 2y \leq 8,\ x \leq 4,\ x \geq 0,\ y \geq 0?$$

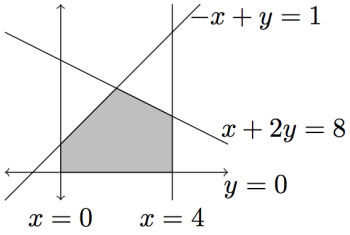
(a)



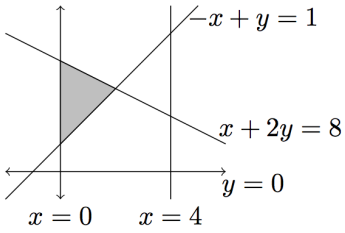
(b)



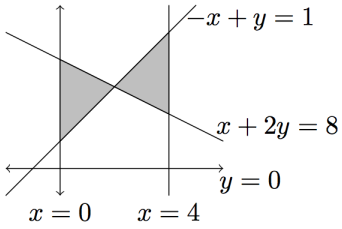
(c)



(d)



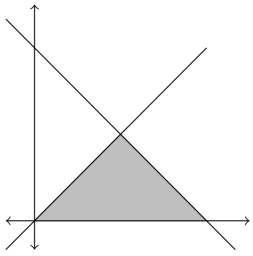
(e)



Solution: The feasible region is above the line $-x + y \geq 1$ (the opposite side to the point $(0, 0)$); below the line $x + 2y \leq 8$, (the same side as the point $(0, 0)$), to the left of the line $x \leq 4$ (the same side as the point $(0, 0)$), and in the first quadrant. Only the graph shown in (d) works.

9. (5 pts.) Consider the system of inequalities whose feasible set is sketched to the right below.

$$\begin{array}{rcrcl} -x & + & y & \leq & 0 \\ x & + & y & \leq & 6 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$



What is the maximum value of the objective function $2x + 3y$, subject to the above constraints?

(a) 15 (b) 18

(c) Objective has no maximum (d) 0

(e) 12

Solution: The corners of the feasible set are $(0, 0)$, $(3, 3)$, $(6, 0)$; the values of the objective function at these three points are 0, 15 and 12. The largest of these is 15.

10. (5 pts.) Consider the objective function $3x + 2y$, subject to the constraints $x \leq 0$ and $y \leq 0$. Which of the following statements is true? (**Hint**: sketch the feasible set; note that the constraint inequalities are $\leq$ and not $\geq$.)

(a) The objective has no maximum and no minimum (it can be made arbitrarily large and arbitrarily small).

(b) The objective has a maximum and a minimum.

(c) The objective has a maximum, but there is no minimum (it can be made arbitrarily small).

(d) The objective has a minimum, but there is no maximum (it can be made arbitrarily large).

(e) The feasible set is empty.

Solution: The objective has a maximum, 0, achieved at the corner $(0, 0)$ of the feasible region. It has no minimum — by making both x and y very small (large and negative), $3x + 2y$ can be made very small (large and negative). The correct answer is (c).

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (12 pts.) Fresh baked cookies at South dining hall are designed to each weigh 80 grammes. However, the bakers who scoop out the dough are not able to scoop out exactly 80g of dough each time. The weight of cookies is thus normally distributed with mean 85g and standard deviation 4g.

(a) What percentage of cookies weight less than 80g?

Solution: The z-score of 80 is $(80 - 85)/4 = -1.25$. From the table, $\mathbf{P}(-1.25 \leq Z) = .1056$; so around 10%.

(b) What percentage of cookies weigh between 75g and 95g?

Solution: The z-score of 95 is $(95 - 85)/4 = 2.5$. The z-score of 75 is $(75 - 85)/4 = -2.5$. From the table, $\mathbf{P}(-2.5 \leq Z \leq 2.5) = .9878$; so around 99%.

12. (12 pts.) When Justin Yoon attempts a field goal from forty yards, he is successful 75% of the time. Suppose that at an upcoming practice game, Justin plans to attempt 24 field goals from forty yards. Let X be the number of successes he has. Assume that different attempts are independent of each other.

(a) Calculate the expected value and standard deviation of X.

Solution: X is binomial with $n = 24$ and $p = .75$, so mean $\mu = np = 24 \times .75 = 18$.

(b) Write an expression involving combination numbers $C(n, k)$ (or binomial coefficients $\binom{n}{k}$) for the probability that Justin will have between 17 and 19 successes (i.e., $\mathbf{P}(17 \leq X \leq 19)$). (There is no need to evaluate the expression numerically).

Solution: $C(24, 17)(.75)^{17}(.25)^7 + C(24, 18)(.75)^{18}(.25)^6 + C(24, 19)(.75)^{19}(.25)^6$ or $\binom{24}{17}(.75)^{17}(.25)^7 + \binom{24}{18}(.75)^{18}(.25)^6 + \binom{24}{19}(.75)^{19}(.25)^6$.

(c) By using a suitable normal random variable to approximate X (one with the same mean and variance as X), estimate the probability that Justin will have between 17 and 19 successes (i.e., $\mathbf{P}(17 \leq X \leq 19)$). Don't forget to use the continuity correction.

Solution: We use a normal distribution N with mean 18 and variance $npq = 24(.75)(.25) = 4.5$ so standard deviation $\sqrt{4.5} \approx 2.12$. Keeping the continuity connection in mind, we calculate

$$\mathbf{P}(16.5 \leq N \leq 19.5) = .5208.$$

13. (12 pts.) I give \$1200 to my financial advisor. He rolls a dice.

- If the dice shows 1, he spends my \$1200 and at the end of a year reports to me that my investments didn't work out and I have no money left.
- If the dice shows 2 or 3, he puts the money in drawer, gives it back to me at the end of a year, and reports to me that my investments broke even and that I have \$1200 still.
- If the dice shows 4, 5 or 6, he invests my money wisely. He is a good investor, so in this case there is a 20% chance that he turns my \$1200 into \$1800, a 60% chance that he turns it into \$1600, and a 20% chance that he turns it into \$1400.

Let X denote the number of dollars that I have at the end of a year.

(a) Write down the probability distribution for X (a tree diagram might help).

Solution:
With probability 1/6, I end up with 0
With probability 1/3, I end up with 1200
With probability $(1/2)(1/5) = 1/10$, I end up with 1800
With probability $(1/2)(3/5) = 3/10$, I end up with 1600
With probability $(1/2)(1/5) = 1/10$, I end up with 1400

Possible value k of X	0	1200	1400	1600	1800
$\mathbf{P}(X = k)$	1/6	1/3	1/10	3/10	1/10

(b) Compute $E(X)$, the expected number of dollars I have at the end of the year.

Solution: $E(X) = 0(1/6) + 1200(1/3) + 1400(1/10) + 1600(3/10) + 1800(1/10) = 1200$.

14. (12 pts.) You are taking a Contemporary Culture class. For one of the assignments, you have to read some combination of books and magazines. Each magazine is 100 pages long and takes 3 hours to read. Each book is 200 pages long and takes 5 hours to read. The requirement is that you must read at least 800 pages, including at least two magazines and at least one book. You want to minimize the amount of time you spend reading for this assignment, while satisfying the requirements.

Let x be the number of magazines you read and let y be the number of books you read.

(a) Write down all the constraints (inequalities) that x and y must satisfy for this problem.

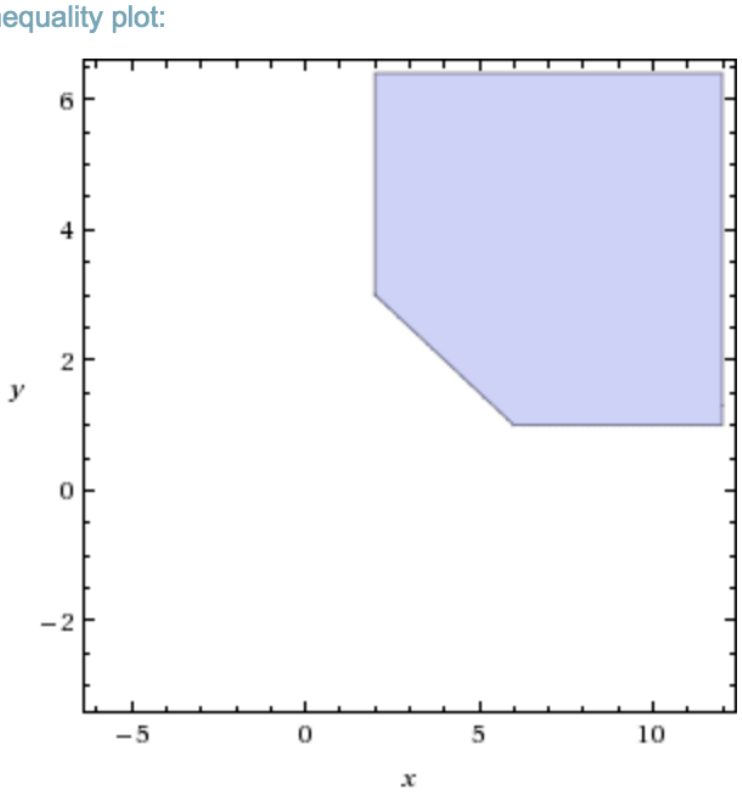
Solution: $100x + 200y \geq 800$, $x \geq 2$, $y \geq 1$ (and optionally $x \geq 0, y \geq 0$, but these are redundant).

(b) Write down the objective function for this problem.

Solution: Minimize $3x + 5y$.

(c) On the grid below, graph the inequalities, shade in the feasible set for this problem, and find the coordinates of the corners of the feasible set.

Solution: The plot is shown below. The corner points of the feasible region are $x = 2, y = 3$ and $x = 6, y = 1$.



(d) Solve the minimization problem: In order to minimize the amount of time that you spend reading for the assignment, while still satisfying the requirements, how many magazines should you read, and how many books should you read?

Solution: At $x = 2, y = 3$ the objective is $3(2) + 5(3) = 21$. At $x = 6, y = 1$ the objective is $3(6) + 5(1) = 23$. The minimum (which is achieved at a corner of the feasible set) is 21, achieved by reading 2 magazines and 3 books.

Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2016

Name:_____

Instructors: Bhattacharya/Galvin

Exam 3

April 14, 2016

This exam is in two parts on 12 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(●)
2.	(a)	(b)	(●)	(d)	(e)
3.	(a)	(b)	(c)	(●)	(e)
4.	(●)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(●)	(d)	(e)
6.	(a)	(●)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(●)
8.	(a)	(b)	(c)	(●)	(e)
9.	(●)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(●)	(d)	(e)

MC. _____
11. _____
12. _____
13. _____
14. _____

Tot. _____