Finite Mathematics (Math 10120) Sec 01, Spring 2014

Midterm Exam 1

Solutions

1. $A = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{4, 6, 8\}$, so it is **false** that A and B are disjoint. All the rest are true.

2. Use $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ to solve $n(A \cap B) = 7$

3. $A \cap (B \cup C)'$ is things which are in A, not in B, and not in C; so there are 23 of them.

4. There are 36 available symbols, so the number of different codes obtainable with k slots is 36^k. Since $36^5 = 60,466,176 < 65,000,000$ and $36^6 = 2,176,782,336 > 65,000,000$, 6 is the smallest k that will work.

5. $P(21, 5) \cdot 3! = 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 3 \cdot 2 \cdot 1 = 14,651,280$.

6. $4 times 5 \times C(8, 2) = 560$.

7. $P(10, 3) + P(10, 2) + P(10, 1) + P(10, 0) = 821$.

8. Either the three come from Carroll, ($C(7, 3)$ ways), **or** Badin ($C(6, 3)$ ways) **or** Pasquerilla East ($C(10, 3)$ ways), so $C(7, 3) + C(6, 3) + C(10, 3)$ ways in all.

9. $\binom{9}{3,3,3}$ ways to split the 9 into 3 groups, with the order of the groups mattering, but not the order within the groups; but then 3 ways to decide leader of first group, and then 3 ways to decide leader of second group, and then 3 ways to decide leader of third group, so an extra factor of 3^3 for the choice of leaders; final answer $\binom{9}{3,3,3}3^3$.

10. 26 men in all. Number of handshakes among men is number of ways of selecting 2 men from the 26, order not mattering, so $C(26, 2)$. Similarly, $C(27, 2)$ handshakes among women. So $C(27, 2) + C(26, 2)$ handshakes in all.

11. (a) $C(53, 4) = 292,825$.

    (b) $C(18, 2)$ ways to choose the juniors, then $C(35, 2)$ ways to choose the rest, so $C(18, 2)C(35, 2) = 91,035$ ways in all.

    (c) Must **either** choose 2 juniors, 1 sophomore, 1 freshman, **or** 1 junior, 2 sophomores, 1 freshman **or** 1 junior, 1 sophomore, 2 freshmen, so

    $$C(18, 2)C(14, 1)C(21, 1) + C(18, 1)C(14, 2)C(21, 1) + C(18, 1)C(14, 1)C(21, 2)$$

    Note: $C(18, 1)C(14, 1)C(21, 1)C(50, 1)$ is not correct, because it overcounts: if Joe and Jim are freshmen, Alice a sophomore and Kim a junior, it for example counts (Kim, Alice, Jim, Joe) as different from (Kim, Alice, Joe, Jim), but these are the same committee.

12. (a) 19 in the triple intersection area; 7 in the remaining football of intersection between R and C; 19 in the remaining football of intersection between E and C; 15 in the remaining football of intersection between R and E; 3 in the part of R that doesn't meet C or E; 2 in the part of C that doesn't meet R or E; 2 in the part of E that doesn't meet C or R; 1 outside the three circles.

    (b) 1

    (c) No, No, Yes

    (d) Those students would be in R, but not C; the relevant part of the Venn diagram is the crescent shaped part of R (made up of two based regions) obtained from R by deleting the football-shaped intersection between R and C.

13. Note that ACCESS CODE has six distinct letters; three vowels (A, E, O) and three consonants (C, S, D).

    (a) $P(6,5) = 720$.

    (b) The C's are indistinguishable from one another, so I get no contributing factor to the total count from choosing the two C's. I do have to decide *where* in the code the two C's go; there are $C(5,2) = 10$ ways to decide this. Then I have three remaining slots to fill, from five letters, so $P(5,3) = 60$ ways to decide. Total number of ways is $10 \times 60 = 600$.

    (c) Choosing first the first letter, then the last, then the second, third and fourth, get $3 \times 2 \times 4 \times 3 \times 2 = 144$.

    (d) Choosing first the first letter, then the last, then the second, third and fourth, get $3 \times 2 \times 3 \times 2 \times 1 = 36$.

14. (a) Must choose 4 south blocks from among the 6, so $C(6,4)$.

    (b) To get to U, must choose 4 south blocks from among the first 6, so $C(6,4)$. Then to get from there to J, must choose 1 south block from among the remaining 6, so $C(6,1)$. $C(6,4)C(6,1)$ in all.

    (c) There are $C(12,5)$ ways to go from L to U, but (from last part) $C(6,4)C(6,1)$ of these pass U. The number that *don't* pass U is $C(12,5) - C(6,4)C(6,1)$.

15. (a) We're partitioning a set of size 16 into 8 pairs, order within blocks not mattering, order of blocks not mattering, so number is $\frac{1}{8!}\binom{16}{2,2,2,2,2,2,2,2} = 2,027,025$

    (b) We take the answer from the last part and mutiply it by 2^8, because for each of 8 pairs we have to choose which is the home team; so $518,918,400$.