

Some probability problems

Math 10120, Spring 2013

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What sort of questions does probability answer?

- In Monopoly, you must roll a double-six before you get to start. How many rounds before you get to start?
- If you toss a coin 100 times, how many heads do you get?
- On *Who Wants to be a Millionaire*, I have won \$60,000. I see a 4-option multiple choice question. If I get it right, I now win \$100,000. If I get it wrong, I now win \$25,000. If I don't answer, I stay at \$60,000. If I don't know the answer, should I guess?
- 2% of the population have condition X. There's a test for X. Used on subjects who have X, it correctly detects X 98% of the time. Used on subjects who do not have X, it correctly detects the absence of X 97% of the time. I take the test, and it comes up positive. Do I have X?

Some basic elements of probability

- **Experiment:** Some well defined process that has an observable result. Usually the result can't be predicted in advance, because it involves some chance
- **Outcome:** Any one particular result of a particular running of the experiment
- **Sample space:** The set of all possible outcomes (as the experiment runs in all its possible ways)
- **Event:** A set of possible outcomes (not necessarily all possible outcomes); described either as a list of outcomes or as a rule that you can apply to test is an outcome in the event or not

Assigning probabilities

Given an experiment with finite sample space (finite set of possible outcomes)

$$S = \{s_1, s_2, s_3, \dots, s_N\}$$

we want to assign a number p_i to each outcome s_i , representing

- how likely s_i is to occur when we run the experiment
- the long-run proportion of time s_i occurs, if we repeat the experiment many times

We'll sometimes write

$$p_i = \Pr(s_i)$$

- **Fundamental rule 1:** each p_i is between 0 and 1
- **Fundamental rule 2:** probabilities add to 1:

$$p_1 + p_2 + p_3 + \dots + p_N = 1$$