The multiplication principle

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The multiplication principle

Suppose an experiment has two consecutive steps, with

- m choices for the first step, and
- n choices for the second (REGARDLESS OF WHAT CHOICE WAS MADE IN THE FIRST STEP).

Then the total number of possible outcomes for the experiment is

$$mn$$

Suppose an experiment has three consecutive steps, with

- m_1 choices for the first step,
- m_2 choices for the second (REGARDLESS OF WHAT CHOICE WAS MADE IN THE FIRST STEP), and
- m_3 choices for the third (REGARDLESS OF WHAT CHOICES WAS MADE IN THE FIRST TWO).

Then the total number of possible outcomes for the experiment is

$$m_1 m_2 m_3$$

The full multiplication principle

Suppose an experiment has t consecutive steps, with

- m_1 choices for the first step,
- m_2 choices for the second (REGARDLESS OF WHAT CHOICE WAS MADE IN THE FIRST STEP),
- m_3 choices for the third (REGARDLESS OF WHAT CHOICES WERE MADE IN THE FIRST TWO STEPS),
- $\ldots$, and
- m_t choices for the tth (REGARDLESS OF WHAT CHOICES WERE MADE IN EARLIER STEPS).

Then the total number of possible outcomes for the experiment is

$$m_1 m_2 m_3 \ldots m_t$$

Permutations of n objects taken r at a time

$$
\begin{aligned}
P(n, r) &= n(n-1)(n-2)\dots(n-(r-1)) \\
&= n(n-1)(n-2)\dots(n-r+1)
\end{aligned}
$$

This is the number of ways of taking r objects from a set of n objects, and arranging them in order

A special case:

$$
\begin{aligned}
r! &= \text{“}r\text{ factorial”} \\
&= P(r, r) \\
&= r(r-1)(r-2)\dots 3.2.1
\end{aligned}
$$

This is the number of ways arranging r objects in order

Convention: $0! = 1$

Combinations of n objects taken r at a time

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$$

This is the number of ways of taking r objects from a set of n objects, without regard to order

Alternate notation and expression:

$$\begin{aligned} C(n,r) &= \binom{n}{r} \text{ ("}n\text{ choose }r\text{")} \\ &= \frac{n(n-1)(n-2)\dots(n-r+1)\times(n-r)(n-r-1)\dots3.2.1}{r!\times(n-r)(n-r-1)\dots3.2.1} \\ &= \frac{n!}{r!(n-r)!} \end{aligned}$$