Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math.
Spring 2013

Name: SOLUTIONS

Instructor: David Galvin

Exam II

March 7, 2013

This exam is in two parts on 11 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	~~(d)~~	(e)
2.	(a)	~~(b)~~	(c)	(d)	(e)
3.	~~(a)~~	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	~~(d)~~	(e)
5.	(a)	(b)	~~(c)~~	(d)	(e)
6.	(a)	~~(b)~~	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	~~(e)~~
8.	(a)	~~(b)~~	(c)	(d)	(e)
9.	(a)	(b)	~~(c)~~	(d)	(e)
10.	(a)	(b)	(c)	(d)	~~(e)~~

MC.	______
11.	______
12.	______
13.	______
14.	______
Tot.	______

Math 10120 Spring 2013, Exam II Initials:________

Multiple Choice

1. (5 pts.) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $E = \{1, 2, 3, 4, 5\}$ and $F = \{5, 6, 7, 8, 9, 10\}$. Which of the following pairs of events are mutually exclusive?

(a) E and F (b) E' and F (c) E and F'

~~(d)~~ E' and F' (e) None of the above pairs

$E' = \{6, 7, 8, 9, 10\}$ and

$F' = \{1, 2, 3, 4\}$ have nothing in common

2. (5 pts.) An experiment has five outcomes in its sample space, s_1, s_2, s_3, s_4 and s_5. If

- $\Pr(s_1) = 0.2$,
- $\Pr(s_2) = 0.3$,
- $\Pr(s_3) = 0.1$, and
- $\Pr(s_4) = 0.2$,

then which of the following is $\Pr(s_5)$?

(a) 0.8 ~~(b) 0.2~~ (c) 0.1 (d) 0.3 (e) 0.5

$Pr(S_1) + Pr(S_2) + Pr(S_3) + Pr(S_4) + Pr(S_5) = 1$

So $Pr(S_5)$ must equal

$1 - .2 - .3 - .1 - .2 = .2$

3. (5 pts.) I choose four people from the class at random. Assuming that all birth-months are equally likely, what is the probability that at least two of the four were born in the same month?

(a) $1 - \frac{12\cdot11\cdot10\cdot9}{12^4} \approx .42708$

(b) $\frac{4\cdot3\cdot2\cdot1}{4^4} \approx .09375$

(c) $1 - \frac{4\cdot3\cdot1\cdot1}{4^4} \approx .90625$

(d) $\frac{12\cdot11\cdot10\cdot9}{12^4} \approx .57292$

(e) $1 - \frac{8\cdot7\cdot6\cdot5}{8^4} \approx .58984$

Pr (All born on different months) $= \dfrac{12\times11\times10\times9}{12^4}$,

So Pr(≥ 2 born on same month) $= 1 - \dfrac{12\times11\times10\times9}{12^4}$

4. (5 pts.) A family has four boys and five girls. Four of the children are chosen at random to shovel the sidewalk. Find the probability that exactly two boys and two girls are selected for this honor.

(a) $\frac{11}{21}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{10}{21}$ (e) $\frac{1}{5}$

\# ways of selecting four children $= C\binom{9}{4}$

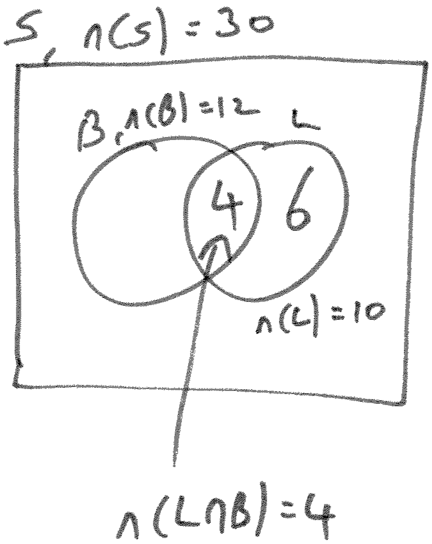
\# ways of selecting exactly two boys, two girls

$= \binom{4}{2}\times\binom{5}{2}$

$Pr = \dfrac{\binom{4}{2}\binom{5}{2}}{C\binom{9}{4}} = \dfrac{10}{21}$

5. (5 pts.) In a French Literature class of 30 students, 12 have read "Madame Bovary", 10 have read "L'Etranger", and 4 have read both. A student is selected at random from the class, and she reveals that she has read "L'Etranger". Given this information, what is the probability that she has read "Madame Bovary"?

(a) $\frac{4}{9}$ (b) $\frac{2}{9}$ ~~(c) $\frac{2}{5}$~~ (d) $\frac{5}{9}$ (e) $\frac{1}{3}$



Given that chosen student has read L'E, sample space has 10 equally likely outcomes, 4 success, so

$Pr = \frac{4}{10}$

6. (5 pts.) I toss a coin 7 times. What is the probability that I get **exactly** 5 heads and 2 tails (not necessarily in that order)?

(a) $\frac{21}{32}$ ~~(b) $\frac{21}{128}$~~ (c) $\frac{5}{7}$ (d) $\frac{5}{32}$ (e) $\frac{5}{128}$

outcomes is 2^7

with exactly 5 heads, 2 tails is $\binom{7}{5}$

So $Pr = \frac{\binom{7}{5}}{2^7} = \frac{21}{128}$

7. (5 pts.) A politician will answer a question truthfully 10% of the time, and the truthfulness of an answer is independent of whether or not he has answered truthfully on previous questions. I met my congressman a few days ago, and asked him 4 questions. What is the probability that I got a truthful response to **at least one** of my questions?

(a) $(0.9)^4 = 65.61\%$ (b) 0 (c) $1 - (0.1)^4 = 99.99\%$

(d) $(0.1)^4 = .01\%$ ~~(e)~~ $1 - (0.9)^4 = 34.39\%$

Pr (no truthful responses) = Pr (all lies) = $.9^4$

Pr (≥ 1 truthful response) = $1 - (.9)^4$

8. (5 pts.) I paint "1" on one side of a coin, and "2" on the other. I flip the coin three times, and record the **sum** of the three numbers that come up. Which of the following is the correct assignment of probabilities for this experiment? (Here "Pr(3) = 1/8" means that the probability that the sum is 3 is 1/8.)

(a) Pr(3) = 1/8, Pr(4) = 1/4, Pr(5) = 1/4, Pr(6) = 1/8

~~(b)~~ Pr(3) = 1/8, Pr(4) = 3/8, Pr(5) = 3/8, Pr(6) = 1/8

(c) Pr(3) = 1/4, Pr(4) = 1/4, Pr(5) = 1/4, Pr(6) = 1/4

(d) Pr(3) = 1/4, Pr(4) = 1/2, Pr(5) = 1/2, Pr(6) = 1/4

(e) Pr(3) = 1/6, Pr(4) = 1/3, Pr(5) = 1/3, Pr(6) = 1/6

8 equally likely outcomes:

1 1 1 → sum 3, $Pr(3) = \frac{1}{8}$

$\left.\begin{array}{l}1\ 1\ 2\\ 1\ 2\ 1\\ 2\ 1\ 1\end{array}\right\} \to$ sum 4, $Pr(4) = \frac{3}{8}$

$\left.\begin{array}{l}1\ 2\ 2\\ 2\ 1\ 2\\ 2\ 2\ 1\end{array}\right\} \to$ sum 5, $Pr(5) = \frac{3}{8}$

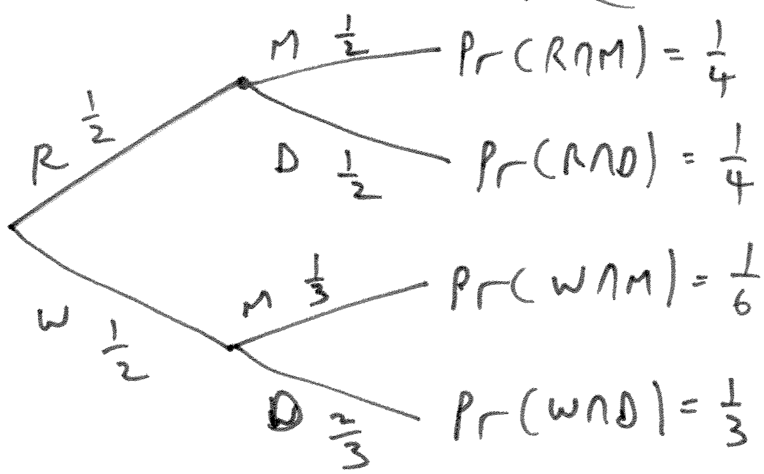
2 2 2 → sum 6, $Pr(6) = \frac{1}{8}$

9. (5 pts.) I gave my wife two boxes of chocolates for Valentines's day. One of them was a red heart box, and the other was a white square box. The compositions of the two boxes were as follows:

- **Red heart box**: 6 milk chocolates and 6 dark chocolates
- **White square box**: 8 milk chocolates and 16 dark chocolates.

She immediately picked a box at random to open, and picked a chocolate at random from that box. It was a milk chocolate. What was the probability that she had chosen the red heart box? [**Hint**: a tree diagram will help.]

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ ~~(c)~~ $\frac{3}{5}$ (d) $\frac{2}{5}$ (e) $\frac{3}{7}$



$$Pr(R|M) = \frac{Pr(R \cap M)}{Pr(M)} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{6}} = \frac{3}{5}$$

10. (5 pts.) I heard on the radio this morning that there is a 60% chance of snow overnight tonight. Assuming that this is correct, what are the odds *against* it snowing overnight? (**Note**: I want the odds *against* snow, not in favor.)

(a) 3 to 2 (b) 3 to 5 (c) 2 to 5 (d) 1 to 3 ~~(e)~~ 2 to 3

$Pr(\text{No snow}) = .4$

Odds against snow are ".4 to .6"

or "4 to 6"

or 2 to 3

Partial Credit

You must show **all of your work** on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (12.5 pts.) Let E and F be events in a sample space with

$$\Pr(E) = 0.4, \ \Pr(F) = 0.6, \ \text{and} \ \Pr(E \cap F) = 0.3.$$

(a) Are E and F independent? Give a reason for your answer.

If independent, would have $Pr(E \cap F) = Pr(E)Pr(F)$, but $.3 \neq .4 \times .6$, so not independent

(b) Calculate $\Pr(E|F)$.

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{.3}{.6} = \frac{1}{2}$$

(c) Calculate $\Pr(F|E)$.

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{.3}{.4} = \frac{3}{4}$$

(c) Calculate $\Pr(E \cup F)$.

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$
$$= .4 + .6 - .3 = .7$$

12. (12.5 pts.) In the game show *Who Wants to be a Millionaire*, a contestant is asked 10 questions, worth different dollar amounts: \$100, \$500, \$1K, \$2K, \$3K, \$5K, \$7K, \$10K, \$15K and \$25K. The questions are asked in a random order, determined before the questioning starts. (For example, it is possible that a contestant sees the questions in the order \$25K, \$100, \$500, \$2K, \$15K, \$5K, \$1K, \$3K, \$7K and \$10K.)

(a) What is the probability that the first question a contestant sees is the \$25K question?

10! ways to arrange 10 questions

$1 \times 9!$ ways to put \$25K question first

$$Pr = \frac{1 \times 9!}{10!} = \frac{1}{10}$$

(b) What is the probability that the first three questions a contestant sees are the three big-money questions (\$10K, \$15K, \$25K), in *some* order? [This happened in a show I saw late last week; my wife was not at all surprised when I paused the show to calculate the probability!]

$3! \times 7!$ ways to put \$25K, \$15K, \$10K questions in first three slots, in some order, so

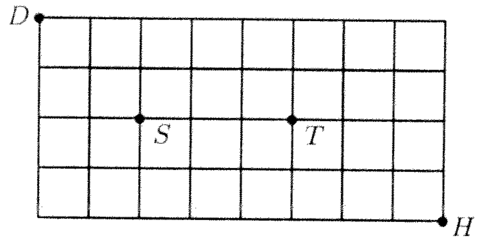
$$Pr = \frac{3! \times 7!}{10!} = \frac{1}{120}$$

(c) Use your answer from the last part to calculate the *odds in favor* of the event that the first three questions a contestant sees are the three big-money questions, in some order.

Odds in favor: " $\frac{1}{120}$ to $\frac{119}{120}$ "

or 1 to 119

13. (12.5 pts.) Both parts of this problem refer to the following city map, where I have marked the Dunkin' Donuts (D), the Hermes store (H), the statue of Sam Spade (S), and the time travel station (T). For this problem, you may leave your answer in terms of mixtures of combinations and permutations (i.e. $C(n,r)$ and $P(n,r)$ for appropriate n and r) if you choose.



(a) The shortest routes from Dunkin' Donuts to the Hermes store are all 12 blocks long. I choose one of them at random, and take it. What is the probability that my route takes me past the time travel station? (Ignore the statue of Sam Spade for this part).

routes in total : $\binom{12}{4}$ → 12 blocks, 4 must be South

routes passing T : $\binom{7}{2} \times \binom{5}{2}$ ← ways to go from T to H

$$Pr(T) = \frac{\binom{7}{2}\binom{5}{2}}{\binom{12}{4}}$$

ways to go from D to T

(b) At some point during my randomly-chosen 12 block trip from Dunkin' Donuts to the Hermes store, I am spotted by the statue of Sam Spade. Given this new information, what is now the probability that my route takes me past the time travel station?

New sample space : All routes from S to H, total # $\binom{8}{2}$

1 way to go from S to T

New # of successful routes : $1 \times \binom{5}{2}$

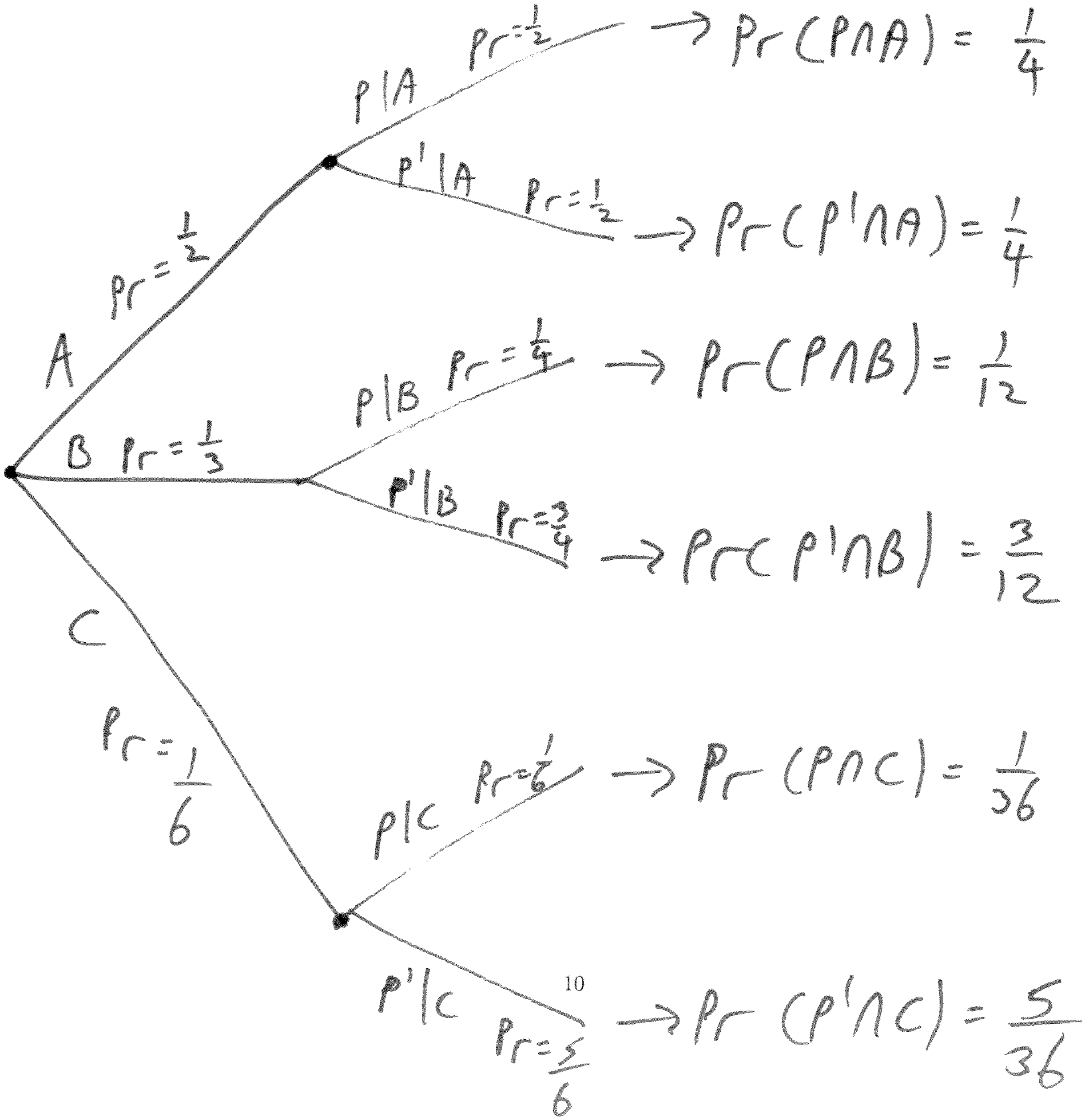
ways to go from T to H

$$Pr(T|S) = \frac{\binom{5}{2}}{\binom{8}{2}}$$

14. (12.5 pts.) Should I grade exams this evening, or go to a party? I let randomness decide. I roll an ordinary dice.

- If it comes up 1, 2 or 3 (event A), I then flip a coin. If the coin comes up heads, I go to the party (event P), and if it comes up tails, I grade (event P').
- If the dice comes up 4 or 5 (event B), I let a well-shuffled ordinary deck of decide for me: if a randomly drawn card is a club, I go to the party, and if not, I grade.
- Finally, If the dice comes up 6 (event C), I let another roll of the dice decide for me: if I roll a 6 again, I go to the party, and if not, I grade.

(a) Draw a tree diagram that represents all the possible outcomes of this two-stage experiment. Label each branch of the tree diagram with **both** the event that it corresponds to (such as A, C, $P|A$, $P'|B$), **and** the associated probability. For each of the paths through the tree, indicate to the right of the end-point of the path **both** the event that the whole path corresponds to (such as $A \cap P$, $B \cap P'$), **and** the associated probability.



(b) Using the data from the tree diagram, calculate the probability that I go to the party (i.e., calculate $\Pr(P)$).

$$\Pr(P) = \Pr(P \cap A) + \Pr(P \cap B) + \Pr(P \cap C)$$

$$= \frac{1}{4} + \frac{1}{12} + \frac{1}{36} = \frac{13}{36}$$

(c) You see me at the party. Given this information, what is the probability that when I rolled the dice initially, It came up 1, 2 or 3? (i.e., calculate $\Pr(A|P)$.)

$$\Pr(A|P) = \frac{\Pr(A \cap P)}{\Pr(P)}$$

$$= \frac{\frac{1}{4}}{\frac{13}{36}} = \frac{9}{13}$$