Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2012

Name:______________________________

Instructor: Migliore

Practice Exam I Answers

February 9, 2012

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record here your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an $\times$ through your answer to each problem.

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Multiple Choice

1. (5 pts.) The 16 members of the Hatfield family and the 18 members of the McCoy family have finally arrived at a truce. However, they still hate each other, so the Hatfields **only** shake hands with other Hatfields, and the McCoys **only** shake hands with other McCoys. If everyone shakes hands with all other members of his/her family, how many handshakes take place?

(a) $16! \cdot 18!$ (b) $C(16,2) + C(18,2)$ (c) $C(16,2) \cdot C(18,2)$

(d) $P(16,2) \cdot P(18,2)$ (e) $P(16,2) + P(18,2)$

Correct answer: (b). There are two kinds of handshakes: Hatfield handshakes and McCoy handshakes. The number of Hatfield handshakes is $C(16,2)$. The number of McCoy handshakes is $C(18,2)$. So the total number of handshakes is the sum of these.

2. (5 pts.) Which of the following is **not** equal to $C(n,r)$?

(a) $\dfrac{n!}{r! \cdot (n-r)!}$ (b) $\dfrac{(n)(n-1)\cdots(n-r+1)}{r!}$ (c) $\dfrac{P(n,r)}{r!}$

(d) $C(n, n-r)$ (e) $\dfrac{(n-r)!}{r!}$

Correct answer: (e). We've seen all the other ones in class.

3. (5 pts.) There are 11 members of a club, and they need to choose a president, a vice president and a treasurer (who are different people). In how many ways can they do this?

(a) 1331 (b) 165 (c) 3 (d) 30 (e) 990

The correct answer is (e). Order is important! There are 11 choices for president. For each of these, there are then 10 choices for VP, then 9 choices for treasurer. So the answer is

$$11 \cdot 10 \cdot 9 = P(11, 3) = 990.$$

4. (5 pts.) There are 11 members of a club, 5 of whom are Phillies fans. They need to choose three officers, of whom **at least** one has to be a Phillies fan. In how many ways can they do this?

(a) 165 (b) 75 (c) 145

(d) 150 (e) 335

Correct answer: (c).

First solution: Notice that there are 6 non-fans. They could have one Phillies fan and two non-fans, or two Phillies fans and one non-fan, or three Phillies fans.The number of ways to achieve the first is

$$C(5, 1) \cdot C(6, 2) = 5 \cdot \frac{6 \cdot 5}{2} = 5 \cdot 15 = 75.$$

The number of ways to achieve the second is

$$C(5, 2) \cdot C(6, 1) = \frac{5 \cdot 4}{2} \cdot 6 = 10 \cdot 6 = 60.$$

The number of ways to achieve the third is

$$C(5, 3) = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10.$$

The total number of ways is the sum of these.

Second solution: There are $C(11, 3)$ possible choices of three officers. Of these, the "bad" ones are the ones without a Phillies fan as a member; there are $C(6, 3)$ of these. So the answer is

$$C(11, 3) - C(6, 3) = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} - \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 165 - 20 = 145.$$

5. (5 pts.) How many different 4-letter "words" (including nonsense words) can be made from the letters

M A T H R U L E S

if one of the letters must be an M and no repetition is allowed?

(a) 1344 (b) 5376 (c) 224 (d) 126 (e) 504

Correct answer: (a). There are 4 choices for where the "M" goes (first spot, second, third or fourth). Once you have chosen that, there are 3 spots remaining, and 8 letters to choose from, so there are $P(8, 3)$ choices. So the total number of choices is

$$4 \cdot 8 \cdot 7 \cdot 6 = 1344.$$

6. (5 pts.) Let

$$\begin{aligned} U &= \{a, b, c, d, e, f, g, h\} \\ A &= \{a, e, g, h\} \\ B &= \{b, d, e, g\} \\ C &= \{c, d, e\} \end{aligned}$$

Which of the following sets is equal to $(A \cap B)' \cap C$?

(a) $\{a, b, c, d, e, f, h\}$ (b) $\emptyset$ (c) $\{e\}$

(d) $\{c, d\}$ (e) U

Correct answer: (d). We have

$$\begin{aligned} A \cap B &= \{e, g\} \\ (A \cap B)' &= \{a, b, c, d, f, h\} \\ (A \cap B)' \cap C &= \{c, d\} \end{aligned}$$

7. (5 pts.) To order a pizza, you have to first choose a sauce and then choose toppings. There are three kinds of sauces (red, white and green) and five kinds of toppings (mushroom, pepperoni, sausage, green pepper and artichoke). You must choose one of the three sauces, but you can choose any number of toppings, from zero to all five. How many different pizzas can be created?

(a) 15 (b) 96 (c) 18 (d) 48 (e) 35

Correct answer: (b). You have 3 choices for the sauce. For each of these, you have to choose the toppings. There are five toppings, and you have to choose a subset of those 5. There are 2^5 choices for a subset (including the whole set and the empty set). So combining, there are

$$3 \cdot 2^5 = 3 \cdot 32 = 96$$

possible pizzas.

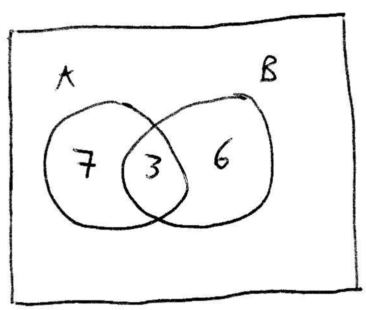
8. (5 pts.) Let A and B be sets, and assume that

$$n(A \cap B) = 3, \qquad n(A \cup B) = 16, \qquad n(A' \cap B) = 6.$$

Find $n(A \cap B')$

(a) 7 (b) 10 (c) 13 (d) 3 (e) 9

Correct answer: (a). Here is a Venn diagram:



First you fill in the $3 = n(A \cap B)$, then the $6 = n(A' \cap B)$. Then since $n(A \cup B) = 16$, it follows that the last region is 7. But this is $n(A \cap B')$.

9. (5 pts.) During the course of a season, the Trenton University football team plays 12 games. At the start of each game, a coin is tossed to see who kicks off first. At the end of the 2008 season, it was noticed that the team correctly guessed 8 of the 12 coin tosses, but no one could remember in which games that happened. How many possibilities are there?

(a) $C(12,8)\cdot C(12,4)$ (b) $P(12,8)$ (c) $C(8,4)$

(d) $P(8,4)$ (e) $C(12,8)$

Correct answer: (e). There were 12 coin tosses, of which the team correctly guessed 8. So there are $C(12,8)$ choices for the 8 correct tosses out of the 12.

10. (5 pts.) Calculate the value of

$$\frac{100!}{98!}\cdot C(5,2).$$

(a) 1,000 (b) $\dfrac{500}{49}$ (c) 99,000

(d) 59,400 (e) 990

Correct answer: (c). Notice that $C(5,2)=\frac{5\cdot 4}{2!}=\frac{20}{2}=10$, so we have

$$\begin{aligned}\frac{100!}{98!}\cdot C(5,2) &= \frac{(100)(99)(98)(97)\cdots(2)(1)}{(98)(97)\cdots(2)(1)}\cdot 10\\ &= (100)(99)(10)\\ &= 99{,}000.\end{aligned}$$

(Notice that the $(98)(97)\cdots(2)(1)$ in the numerator and in the denominator cancelled out.)

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) A club decides to hold a chess tournament. There are 8 participants.

(a) If each participant plays every other participant once, how many games will be played? For this problem I'd like a numerical answer.

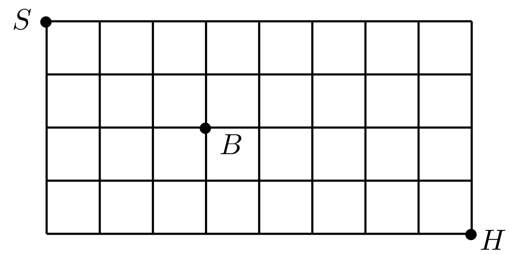
Answer: Each game represents a choice of two of the 8 participants (to be the players in the game). So the number of ways this can be done is

$$C(8,2) = \frac{8 \cdot 7}{2} = 28.$$

(b) Suppose instead that each participant is to play every other participant 3 times. In total, how many games will be played? For this problem I'd like a numerical answer.

Answer: $3 \cdot C(8,2) = 3 \cdot \frac{8 \cdot 7}{2} = 3 \cdot 28 = 84.$

12. (10 pts.) Both parts of this problem refer to the following city map. For this problem, you may leave your answer in terms of mixtures of combinations and permutations (i.e. $C(n,r)$ and $P(n,r)$ for appropriate n and r) if you like.



(a) Billy starts at school (marked S) and needs to go home (marked H). Along the way, he has to pass by the bookstore, marked B, to buy a math book. If he has to do it in as few blocks as possible (12), in how many ways can it be done?

Answer: All blocks that he walks will be East or South. To get from S to B there is a total of 5 blocks. Two of these have to be South. So there are $C(5,2) = \frac{5 \cdot 4}{2} = 10$ ways to choose the two south blocks among the 5 total blocks. Then to get from B to H has has to go 7 blocks, of which again two are South. So there is a total of $C(7,2) = \frac{7 \cdot 6}{2} = 21$ choices for the second part of the trip. All together, there are $10 \cdot 21 = 210$ possible routes.

(b) Bobby starts at school (marked S) and needs to go to the bookstore (marked B) to buy a math book. However, he has no money, so he has to go home first (marked H). If he needs to do the whole trip in as few blocks as possible (19), in how many ways can it be done? For the first part of the trip, he does **not** need to pass by B on his way to H.

Answer: For the first part of the trip he has to go 12 blocks, of which 4 are South. So the number of choices for the first part of the trip is

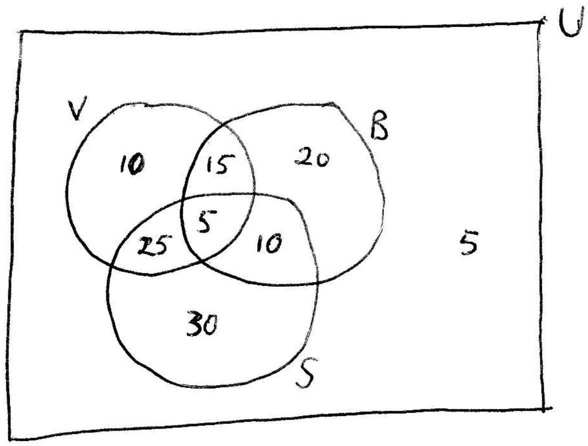
$$C(12,4) = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 495.$$

For the second part of the trip, going from H to B, he goes only North and West, but the total number of choices is the same as it was for going from B to H, namely 21 (see (a). So the total number of choices for the whole trip is $495 \cdot 21 = 10{,}395$.

13. (10 pts.) Among the 120 women varsity athletes at State University, suppose 55 play volleyball, 50 play basketball, 70 play softball, 25 play volleyball and softball but not basketball, 15 play volleyball and basketball but not softball, 10 play only volleyball, 20 play only basketball.

(a) Draw and label a Venn Diagram representing the above information.

Answer: Let V represent the set of volleyball players, B the set of basketball players and S the set of softball players. We can fill in the 25, 15, 10 and 20 first. Then from the fact that there are 55 volleyball players, we deduce the intersection $n(V \cap B \cap S) = 5$. From the fact that there are 50 basketball players we now deduce that the number of women that play basketball and softball but not volleyball is 10. We can then deduce that since there are 70 softball players, there are 30 that play only softball. Adding these numbers all up, we get 115 women play at least one of the sports. Since there are 120 women athletes all together, we are left with 5 that play none of the three. Here is the Venn diagram:



(b) How many play only softball?

Answer: From the Venn diagram we see that there are 30 (see the discussion above).

(c) How many do not play any of the three sports?

Answer: From the Venn diagram we see that there are 5 (see the discussion above).

14. (10 pts.) Seven horses will be in tomorrow's race. To place a bet on the race, you have to predict which horse will finish first, which will finish second, and which will finish third. The other four horses do not matter.

(a) How many different outcomes (predicting the first, second and third place finishers) are possible?

Answer: There are 7 choices for coming in first, then 6 for coming in second, then 5 for coming in third. So the total number is $P(7,3) = 7 \cdot 6 \cdot 5 = 210$.

(b) Bob is sure that Stewball will finish either first or second. Taking this into account, how many different outcomes (predicting the first, second and third place finishers) are possible?

Answer: There are two choices for Stewball (first or second). Once this is determined, there are 6 possibilities for the other top-two spot, and 5 possibilities for the third spot. So the total number is $2 \cdot 6 \cdot 5 = 60$.

(c) Mary is pretty sure that the horse with no legs will finish last. Taking only this into account (i.e. ignore the information from (b)), how many different outcomes (predicting the first, second and third place finishers) are possible?

Answer: Now there are 6 possibilities for first place, then 5 for second place, then 4 for third place. So the total number is $P(6,3) = 6 \cdot 5 \cdot 4 = 120$.

15. (10 pts.) Five married couples are going to be in a group picture, all lined up in a row.

(a) In how many ways can the 10 people line up? You can give your answer using either C, P or factorial notation, or you can give a numerical answer.

Answer: There are no other conditions, so it's just $P(10, 10) = 10! = 3{,}628{,}800$.

(b) In how many ways can they line up if everyone has to be standing next to their spouse? You can give your answer using either C, P or factorial notation, or you can give a numerical answer.

Answer: First we decide the order of the couples. Since there are 5 couples, the number of possible orders is $P(5, 5) = 5! = 120$. Having settled the order of the couples, for each such possible order there are two orders for the husband and wife (husband on left or husband on right). So the number of choices is $2^5 = 32$. All together, the number of possible orders keeping spouses together is

$$P(5, 5) \cdot 2^5 = 5! \cdot 2^5 = 120 \cdot 32 = 3{,}840.$$

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