# Anomaly Detection in the WIPER System using A Markov Modulated Poisson Distribution

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# Outline

- Background
- WIPER
- MMPP framework
- Application
- Experimental Results
- Conclusions and Future work

# Background

- Time Series
  - A sequence of observations measured on a continuous time period at time intervals
  - Example: economy (Stock, financial), weather, medical etc…
  - Characteristics
    - Data are not independent
    - Displays underlying trends

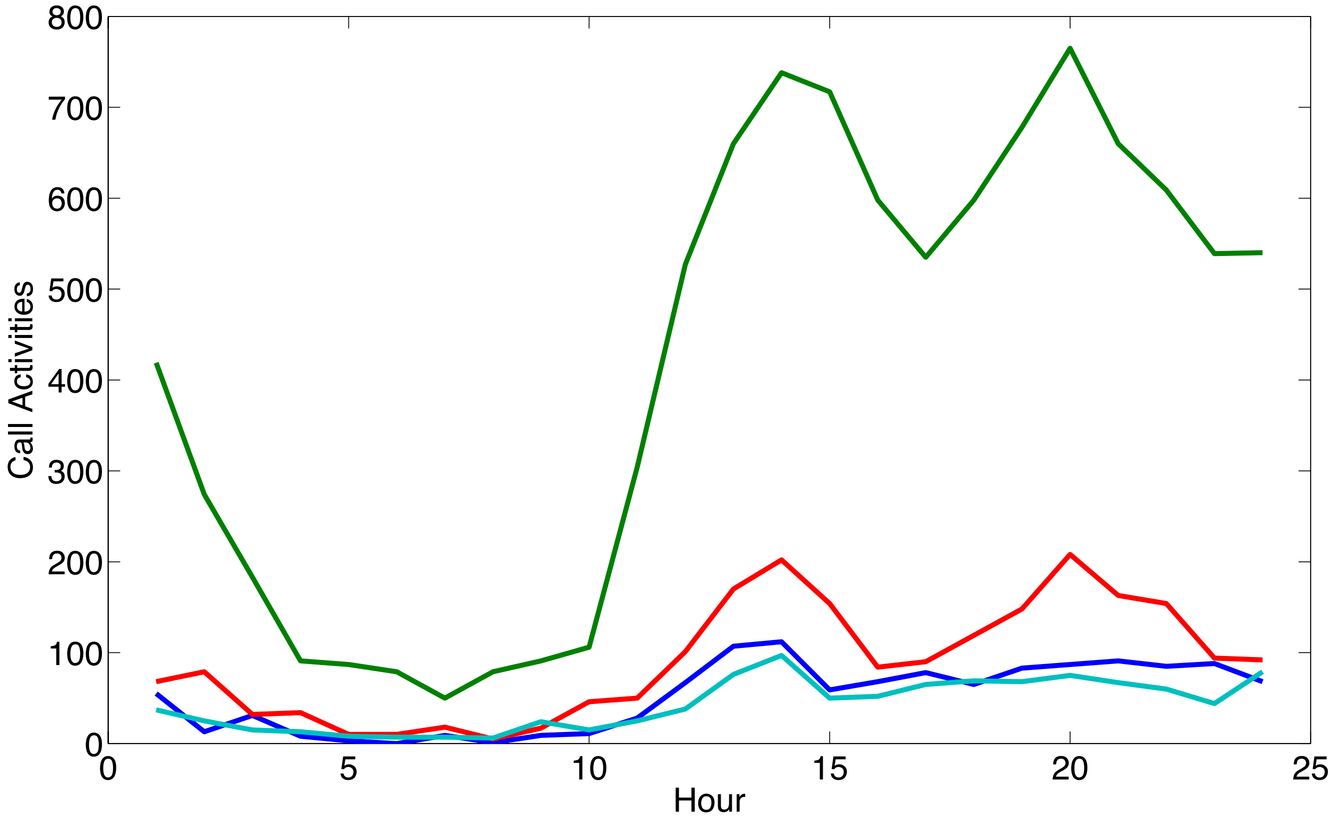
# WIPER System

- Wireless Phone-based Emergency Response System
- Functions
  - Detect possible emergencies
  - Improve situational awareness
- Cell phone call activities reflect human behavior

# Data Characteristics

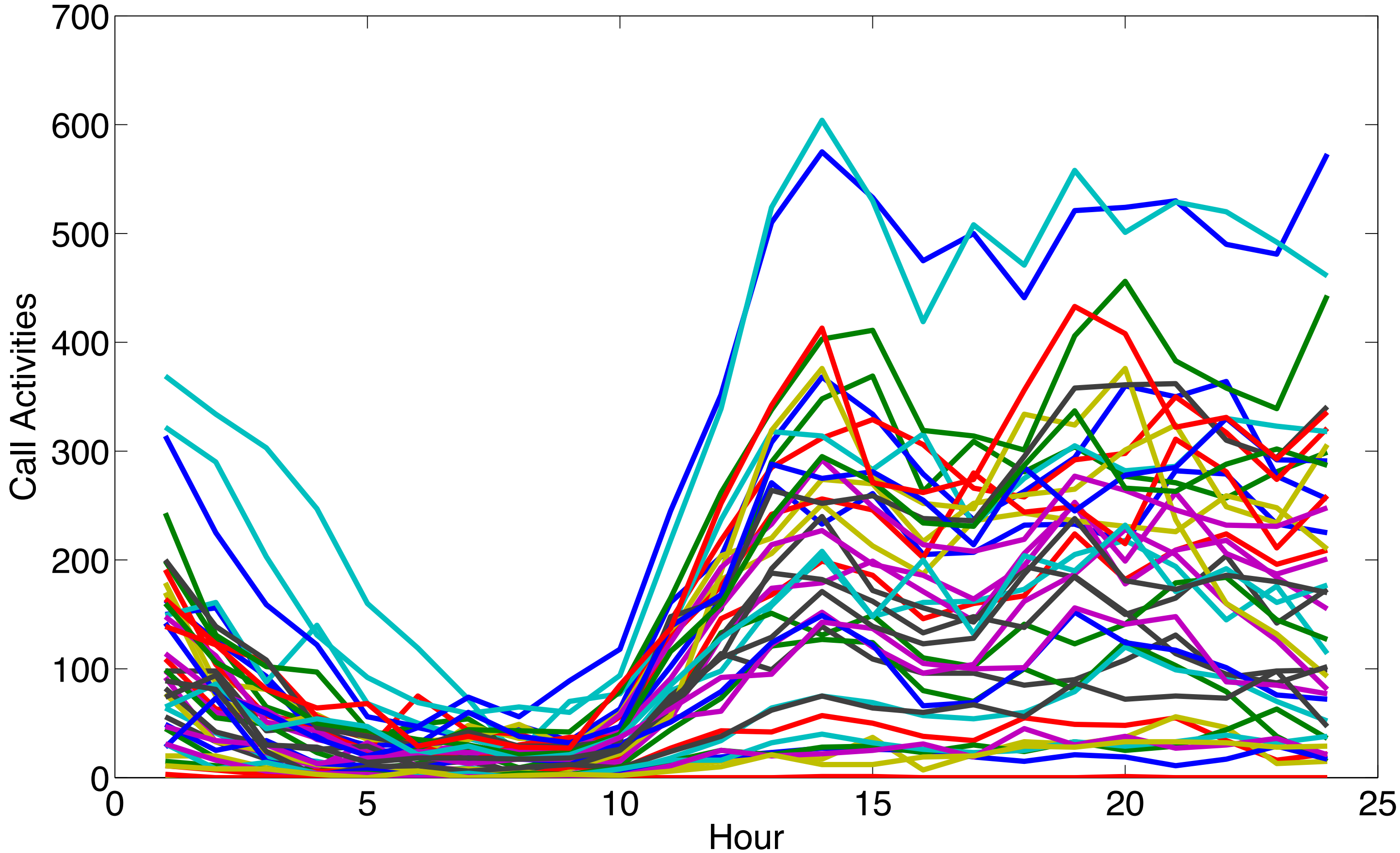
- Two cities:
  - Small city A:
    - Population – 20,000 Towers – 4
  - Large city B:
    - Population – 200,000 Towers – 31
- Time Period
  - Jan. 15 – Feb. 12, 2006

# Tower Activity



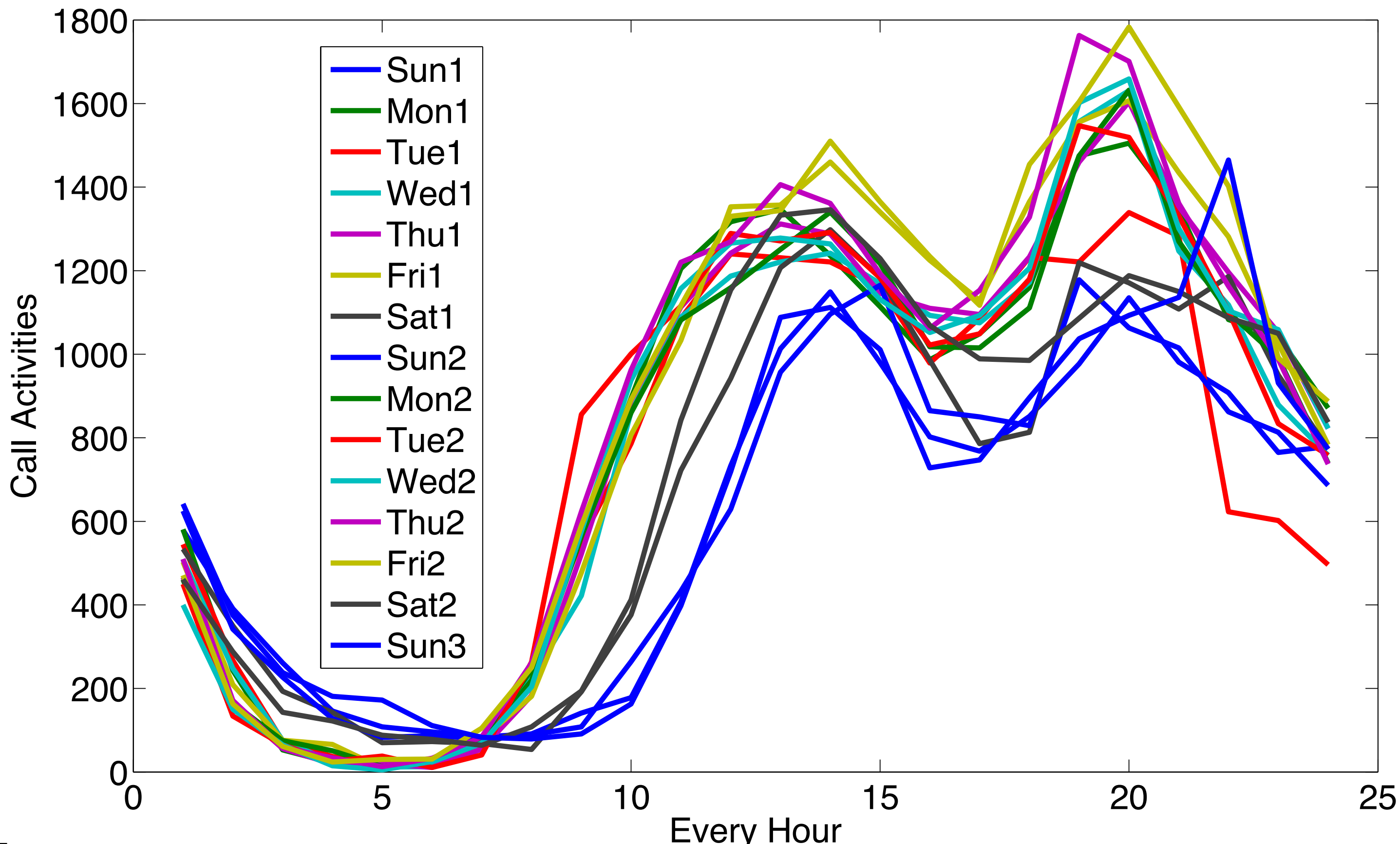

**Small City (4 Towers)**

# Tower Activity



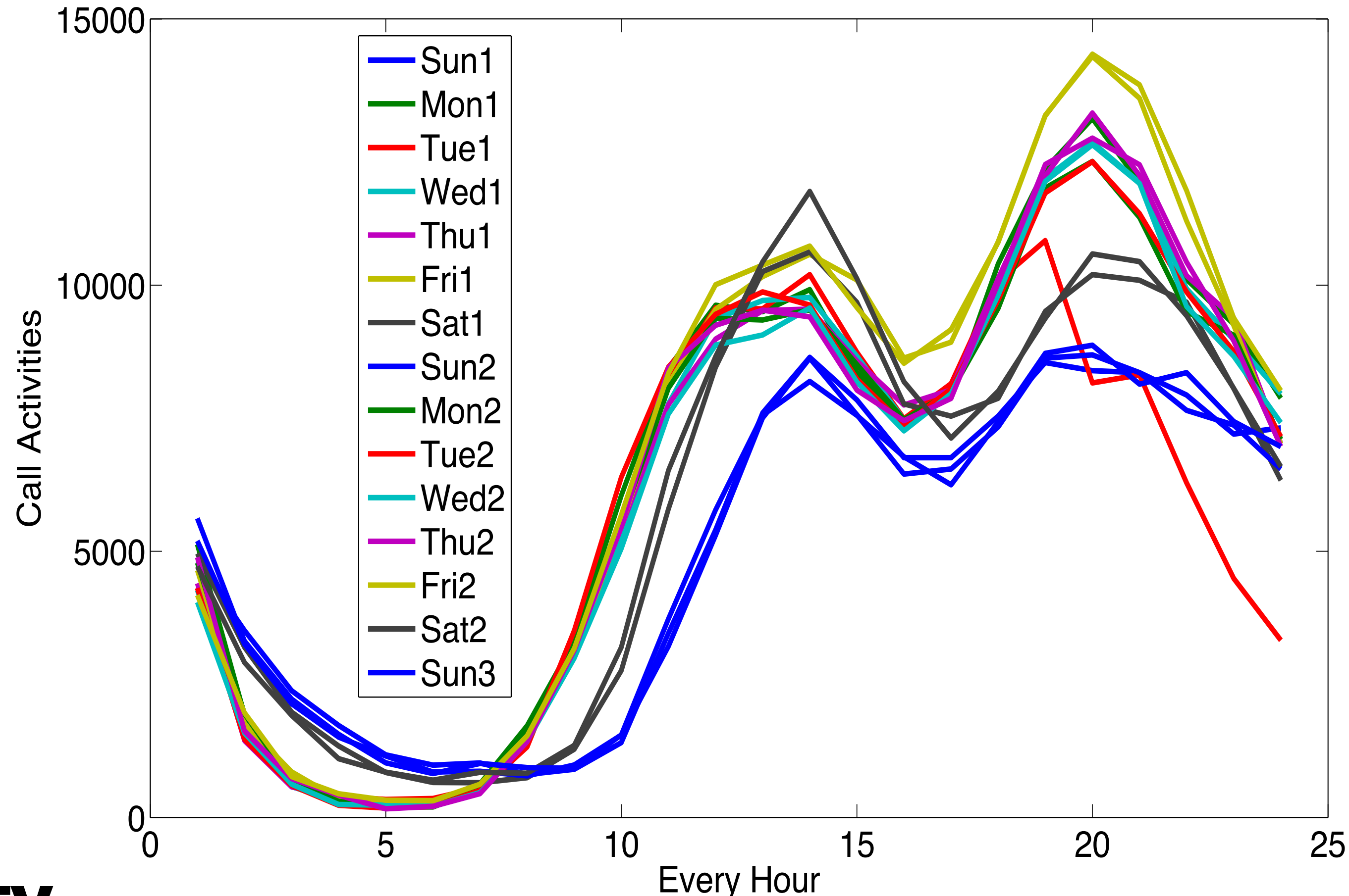

**Large City (31 Towers)**

# 15-Day Time Period Data




**Small City**

# 15-Day Time Period Data




**Large City**

# Observations

- Overall call activity of a city are more uniform than a single tower
- Call activity for each day displays similar trend
- Call activity for each day of the week shares similar behavior

# MMPP Modeling

$$N(t) = N_0(t) + N_A(t)$$

$N(t)$ : **Observed Data**

$N_0(t)$: **Unobserved Data with normal behavior**

$N_A(t)$: **Unobserved Data with abnormal behavior**

**Both $N_0(t)$ and $N_A(t)$ can be modulated as a Poisson Process.**

# Modeling Normal Data

- **Poisson distribution**

$$P(N;\lambda) = \frac{e^{-\lambda}\lambda^N}{N!} \qquad N = 0, 1, \ldots$$

- **Rate Parameter: a function of time**

$$\lambda \sim \lambda(t)$$

# Adding Day/hour effects

$$\lambda(t) = \lambda_0 \, \delta_{d(t)} \, \eta_{d(t),h(t)}$$

$$d(t) \in [1, 2, ..., 7]$$

**Associated with Monday, Tuesday … Sunday**

$h(t)$ **: Time interval, such as minute, half hour, hour etc**

$\lambda_0$ **: Average rate of the Poisson process over one week**

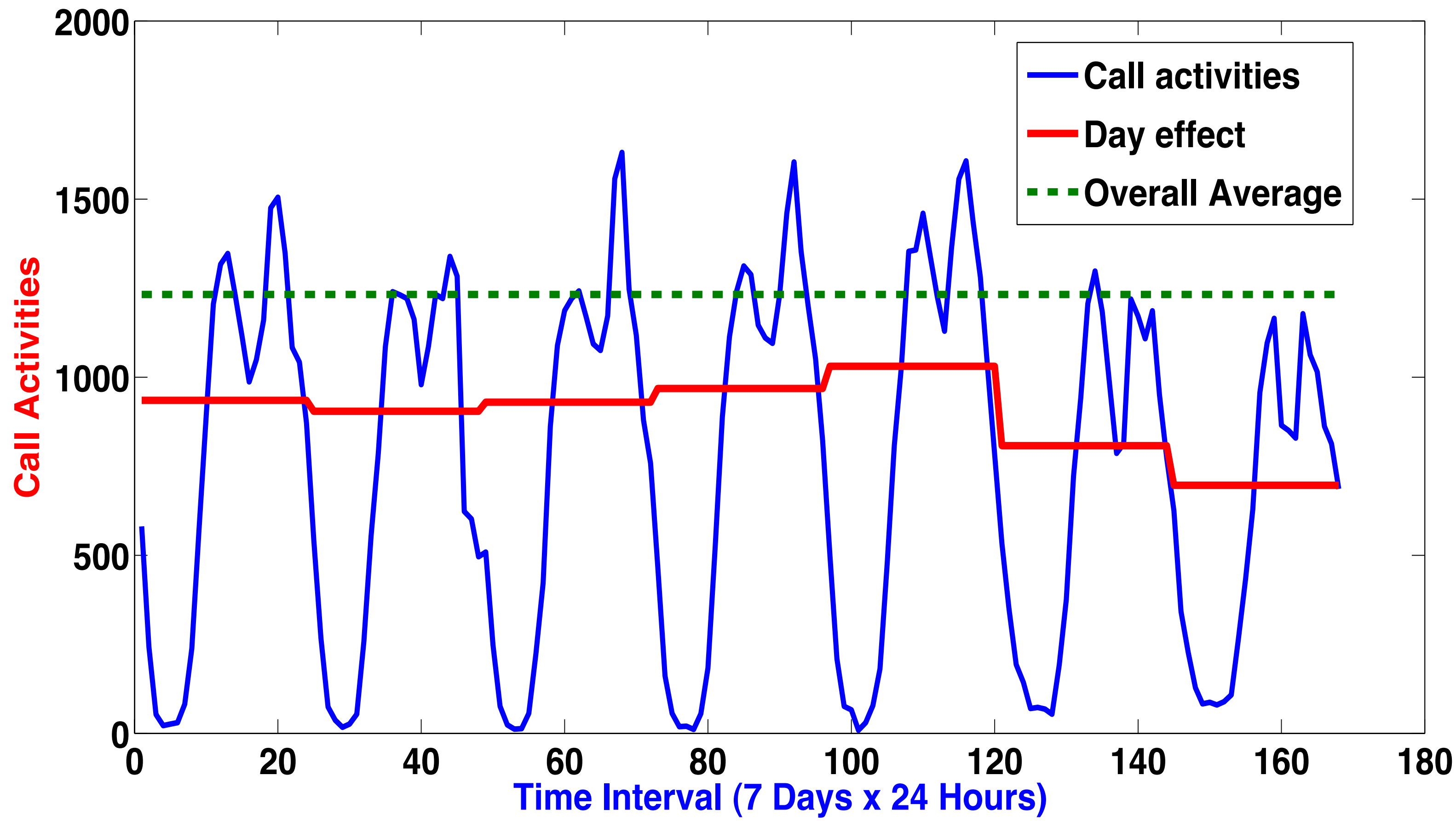
# Requirements:

$$\sum_{i=1}^{7} \delta_i = 7 \qquad \sum_{j=1}^{D} \eta_{i,j} = D, \ \ \forall i$$

$\delta_i$ **: Day effect, indicates the changes over the day of the week**

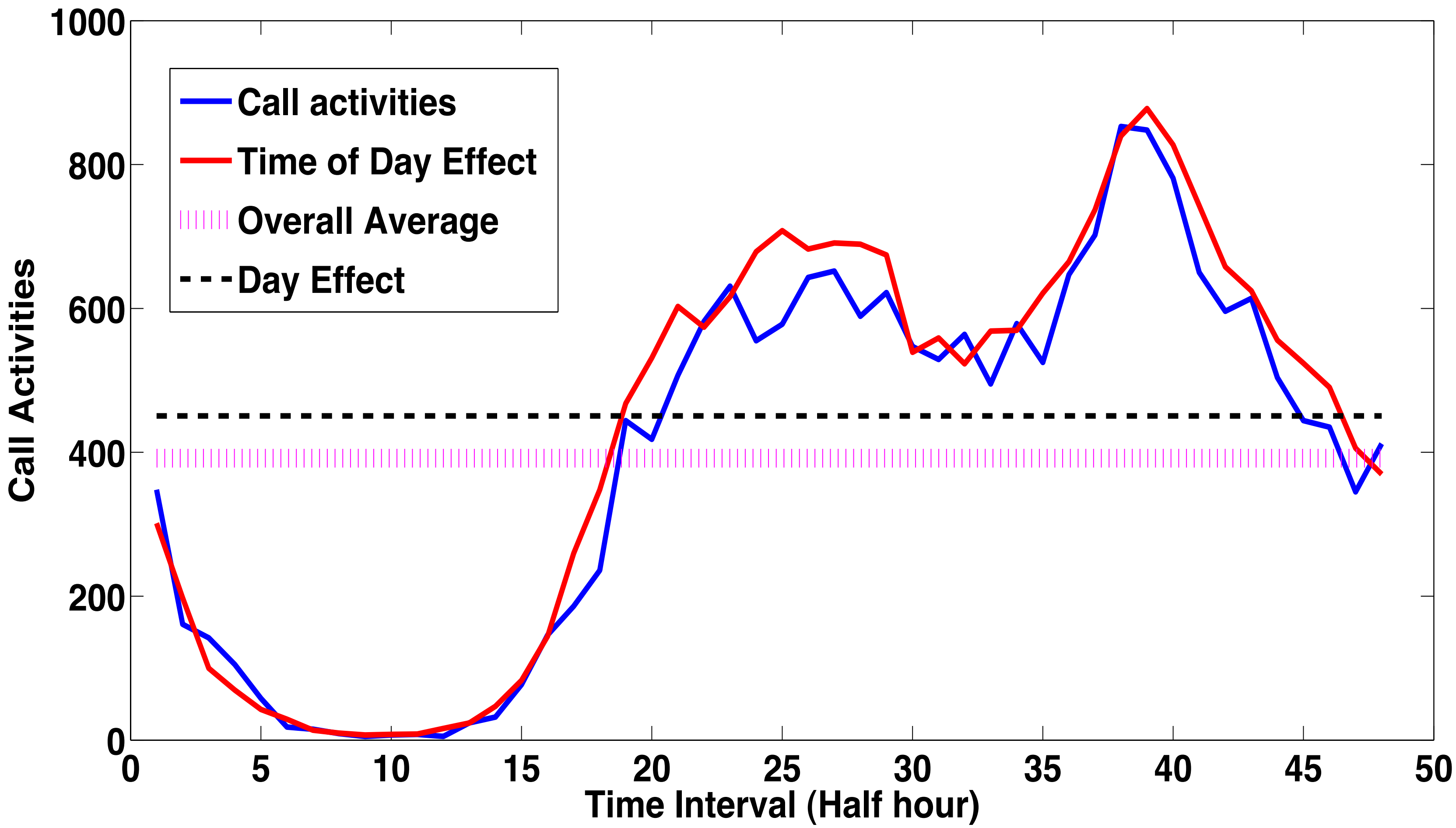
$\eta_{i,j}$ **: Time of day effect, indicates the changes over the time period j on a given day of i**

# Day Effect



Call activities
Day effect
Overall Average
Call Activities
Time Interval (7 Days x 24 Hours)
0
500
1000
1500
2000
0
20
40
60
80
100
120
140
160
180

# Time of Day Effect



# Prior Distributions for Parameters

$$\lambda_0 \sim \Gamma(\lambda; a^L, b^L)$$

$\Gamma(.)$ is the Gamma distribution

$$\frac{1}{7}[\delta_1, \delta_2, ..., \delta_7] \sim Dir(\alpha_1^d, \alpha_2^d, ..., \alpha_7^d, )$$

$$\frac{1}{D}[\eta_{i,1}, \eta_{i,2}, ..., \eta_{i,D}] \sim Dir(\alpha_1^h, \alpha_2^h, ..., \alpha_D^d)$$

$Dir(.)$ is a Dirichlet distribution

# Modeling Anomalous Data

- $N_A(t)$ **is also a Poisson process with rate** $\lambda_A(t)$

- **Markov process A(t) is used to determine the existence of anomalous events at time t**

$$A(t) = \begin{cases} 1 & \text{an event is occuring at time t} \\ 0 & \text{otherwise} \end{cases}$$

# Continued

- **Transition probabilities matrix**

$$M_A = \begin{pmatrix} 1 - A_0 & A_1 \\ A_0 & 1 - A_1 \end{pmatrix}$$

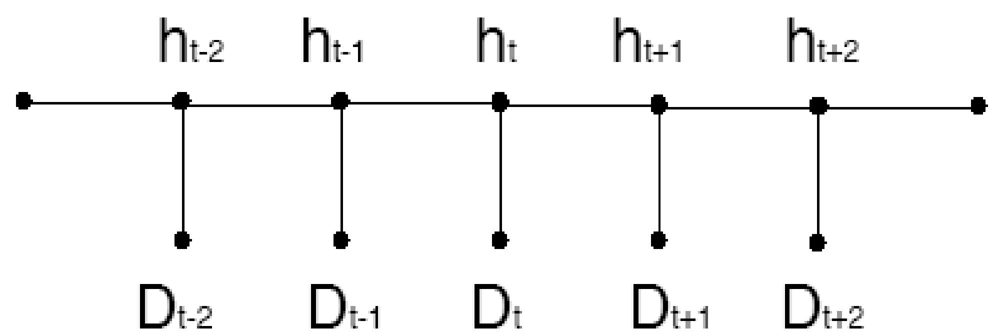
$$A_0 \sim \beta(A, a_0^A, b_0^A)$$

$$A_1 \sim \beta(A, a_1^A, b_1^A)$$

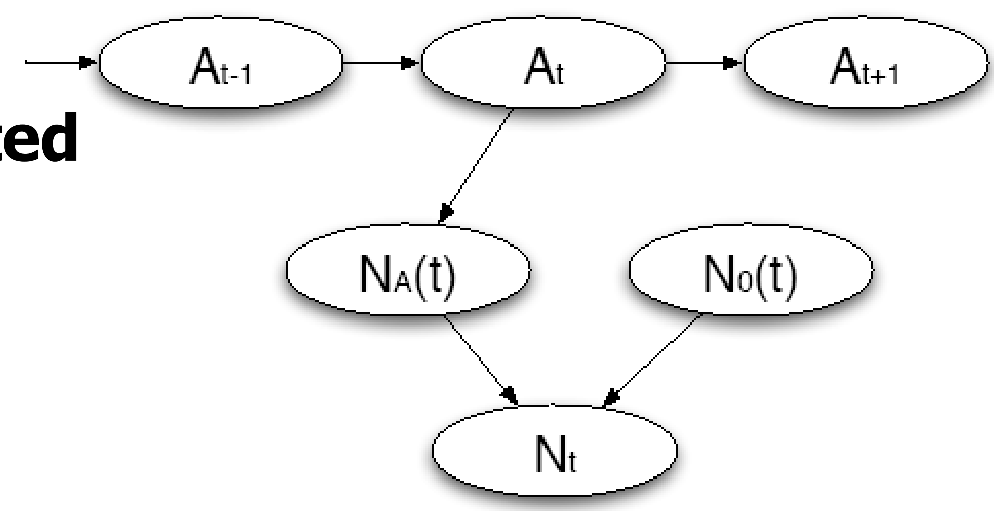
$$N_A(t) \sim \begin{cases} 0 & \mathrm{A(t) = 0} \\ P(N; \lambda_A(t)) & \mathrm{A(t) = 1} \end{cases}$$

# MMPP ~ HMM

- **Typical HMM**
  **(Hidden Markov Model)**




- **MMPP**
  **(Markov Modulated Poisson Process)**



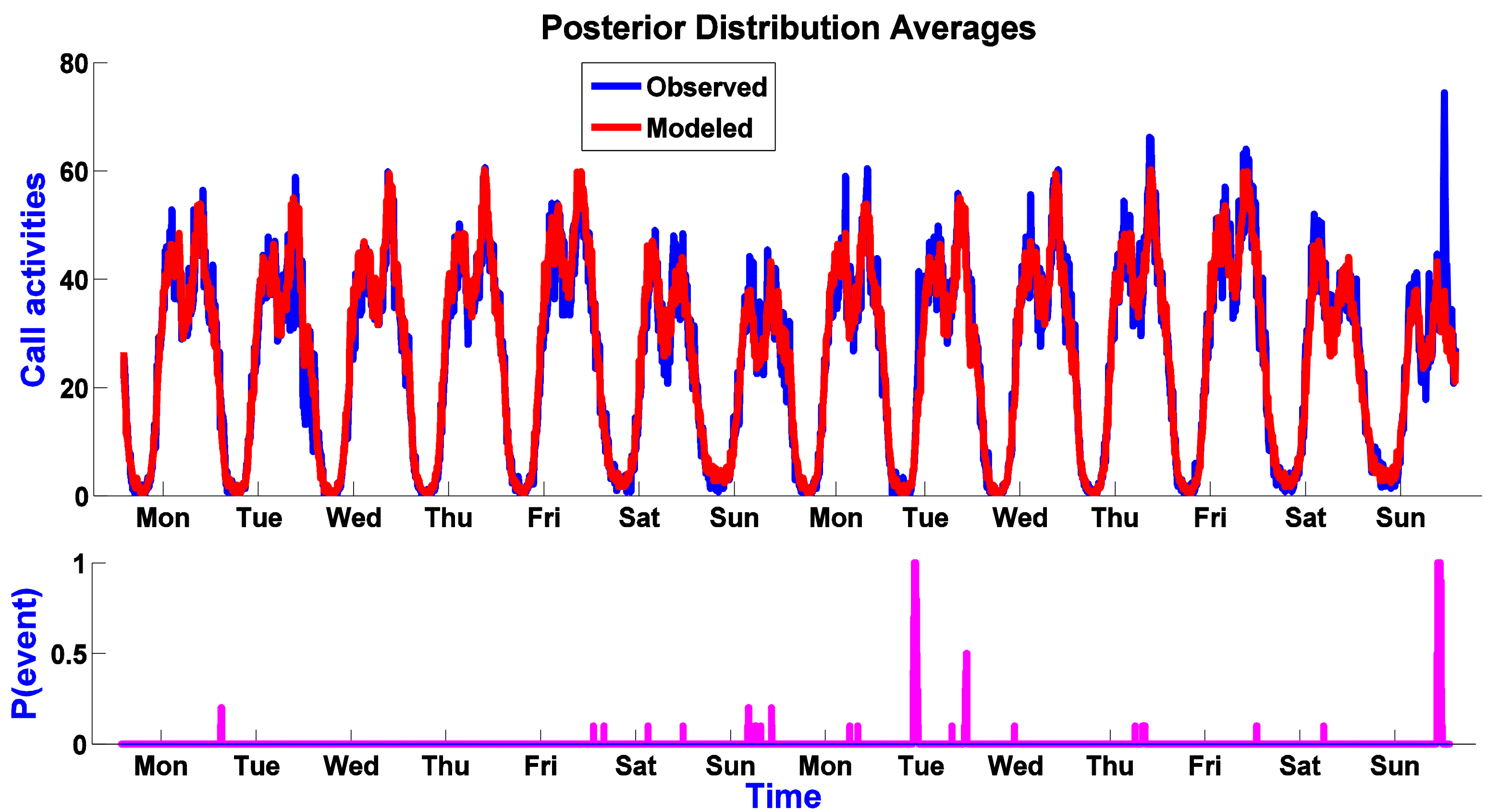
# Apply MCMC

- **Forward Recursion**
  - **Calculate conditional distribution of P( A(t) | N(t) )**
- **Backward Recursion**
  - **Draw sample of $N_A(t)$ and $N_0(t)$**
- **Draw Transition Matrix from Complete Data**

# Anomaly Detection

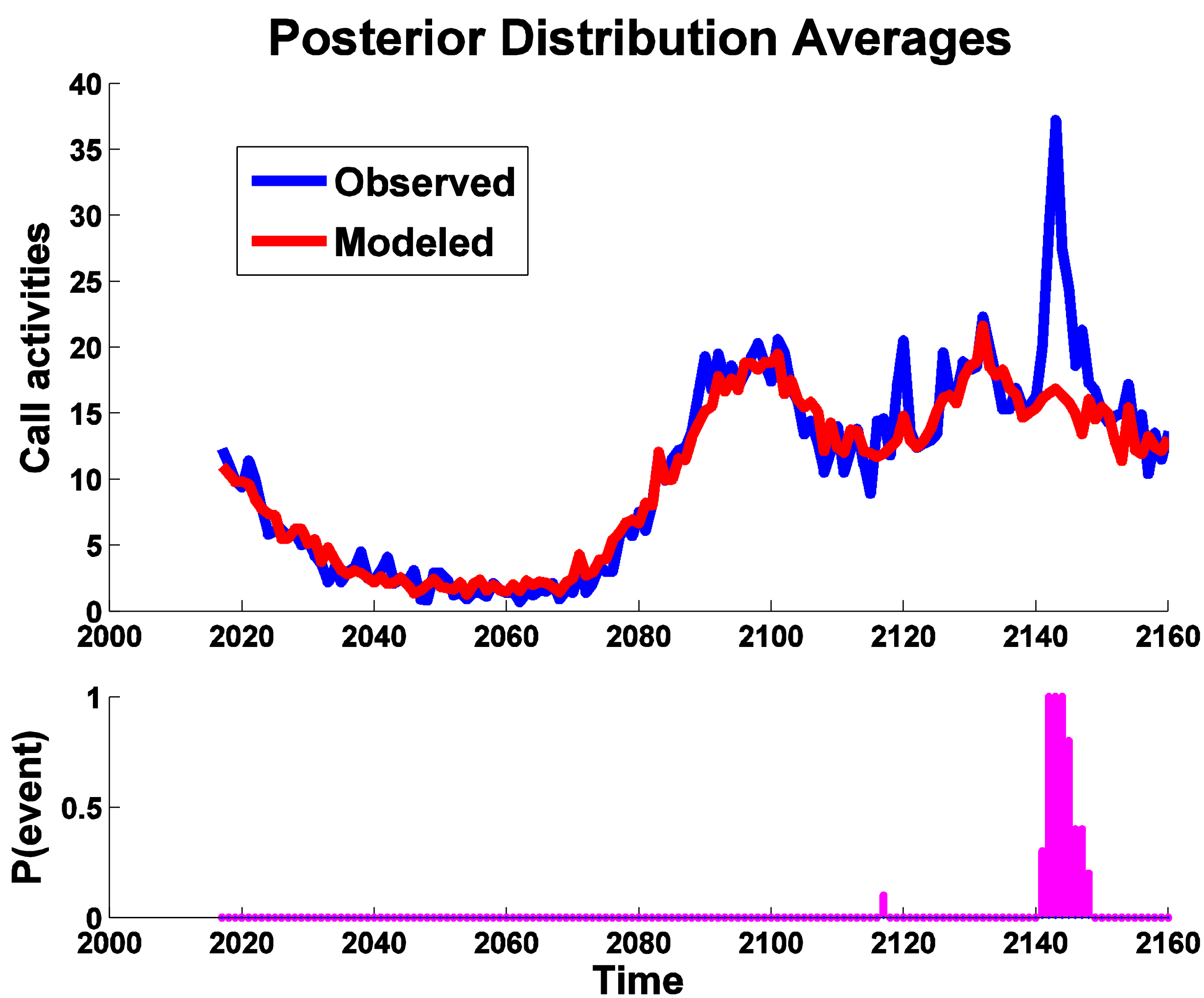
- Posterior probability of A(t) at each time t is an indicator of anomalies
- Apply MCMC algorithm:
  - 50 iterations

# Results



Posterior Distribution Averages
Observed
Modeled
Call activities
80
60
40
20
0
Mon Tue Wed Thu Fri Sat Sun Mon Tue Wed Thu Fri Sat Sun
P(event)
1
0.5
0
Mon Tue Wed Thu Fri Sat Sun Mon Tue Wed Thu Fri Sat Sun
Time

# Continued


Posterior Distribution Averages
Observed
Modeled
Call activities
40
35
30
25
20
15
10
5
0
2000
2020
2040
2060
2080
2100
2120
2140
2160
P(event)
1
0.5
0
2000
2020
2040
2060
2080
2100
2120
2140
2160
Time

# Conclusions

- Cell phone data reflects human activities on hourly, daily scale
- MMPP provides a method of modeling call activity, and detecting anomalous events

# Future Work

- Apply on longer time period, and investigate monthly and seasonal behavior
- Implement MMPP model as part of real time system on streaming data
- Incorporate into WIPER system