Name: _____   Date: April 12, 2007

(1) (a) Write down a (non-constant) sequence that converges.

(b) Can you construct a sequence that diverges but doesn't go to $+\infty$ or $-\infty$?

(2) The Fibonacci sequence is defined recursively by

$$a_{n+2} = a_n + a_{n+1} \text{ where } a_1 = 1 \text{ and } a_2 = 1.$$

(a) Write the first 9 terms of the Fibonacci sequence.

(b) Write the first 8 terms of the ratio sequence defined by

$$b_n = \frac{a_{n+1}}{a_n}, \ n \geq 1.$$

(c) Using the above definition, show that

$$b_n = 1 + \frac{1}{b_{n-1}}.$$

(d) The *golden ratio* φ is defined by

$$\lim_{n \to \infty} b_n = \varphi.$$

(i) Show that $\varphi = 1 + \frac{1}{\varphi}$. (Hint: first notice that $\lim_{n \to \infty} b_n = \lim_{n \to \infty} b_{n-1}$, and then use the properties of limits.)

(ii) Solve the equation in (i) for φ. (Hint: use the quadratic equation.)