

Practice Exam 1 — Math 10240

This questions should give an idea of how the exam will be. The actual exam will have fewer questions—probably around 20.

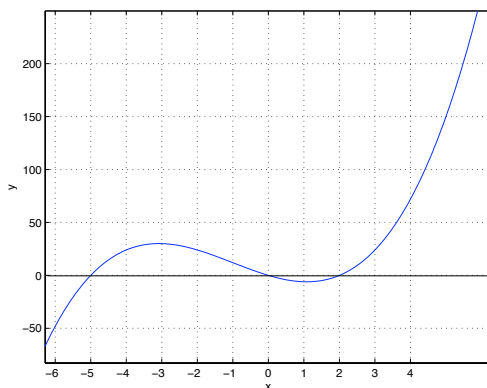
1. Suppose $f(x) = 4x^2 - 3$. Find the following values: $f(2)$, $f(-2)$, $f(\frac{1}{2})$, $f(b)$, $f(a + h)$.
2. Suppose f is given by the following table. Evaluate: $f(9)$, $f(f(2))$, $f(2) + f(3)$. If this table completely defines f , what is its domain and range?

x	0	2	3	4	7	9	10
$f(x)$	1	7	$\frac{1}{2}$	-6	0	π	-1

3. Find the following sum: $\frac{1}{x+h} + \frac{1}{x}$.

4. Find the following: $\frac{\frac{3}{4}}{\frac{5}{6}}$.

5. Consider the following curve:



- (a) Is this curve the graph of a function? Why or why not?
 - (b) If your answer to (a) was yes, does the function represented by the graph appear to be even, odd, or neither? Justify this answer. If your answer to (a) was no, why is the curve not the graph of a function?
 - (c) What are the x- and y-intercept(s) of the curve above, rounded to the nearest integer?
6. When the price of gelato was \$3.50, Cafe Amalia sold about 400 per week. When the price was reduced to \$2, sales rose to 550 per week.
 - (a) Find the equation of the demand curve for gelato.
 - (b) About how many gelatos should the cafe expect to sell, per week, if they charged \$ 3?

Find the natural domain of the following functions:

- (a) $f(x) = \frac{x}{1-x}$
- (b) $f(x) = \frac{15}{x+5}$
- (c) $f(x) = \sqrt{x-5}$
- (d) $f(x) = \sqrt{x+4}$

(e) $f(x) = x^2 + 2x + 7$

(f) $f(x) = 8x$

(g) $f(x) = 25$

(h) $f(x) = \frac{2}{\sqrt{x-3}}$

(i) $f(x) = \frac{1}{6x-1}$

7. Find the range of the following functions:

(a) $g(x) = 45$

(b) $h(x) = 2x + 1$

(c) $f(x) = x^2$

(d) $f(x) = \sqrt{x}$

(e) $f(x) = x^2 + 6$

(f) $f(x) = \sqrt{x} - 2$

8. Find the y -intercept of the following functions:

(a) $f(x) = 7x - 4$

(b) $f(x) = x^5 + 9x^3 - 8x^2$

(c) $f(x) = (x - 4)^2 + 7$

(d) The line given by the equation $y - 4 = 6(x - 3)$

9. I was considering going into the business of selling videogames. I figured that I would have a fixed monthly expense of \$750, (for rent, advertising, etc.). In addition, I found out that I can buy videogames wholesale for \$30 each. I plan to sell the games for \$50 each.

(a) Find my cost function $C(x)$ of producing x items.

(b) Find my revenue function $R(x)$ of earned income from the sale of x items.

(c) Write the profit function $P(x)$.

(d) I figured I could sell about 100 videogames in a month. Should I go into business (i.e. will I make a profit?) Show all your work.

(e) How many videogames would I have to sell to break-even?

10. Find the equation of the line connecting the points $(4, 4)$ and $(-1, 0)$.

11. Find the equation of the line through the point $(5, 9)$ with slope $\frac{3}{2}$.

12. Find the equation of the line through the points $(3, 2)$ and $(-1, 2)$.

13. Find the equation of the line connecting the points $(-2, -4)$ and $(-3, -1)$.

14. What is the slope of the function given by $-3x + 3y = -6$?

15. What are the slope and the intercept of the function $y - 2 = 5x - 3$? Graph this function.

16. Factor $f(x) = x^2 - 5x + 6$.

17. Factor $f(x) = x^2 + 5x - 6$.

18. Factor $g(x) = x^2 + 2x - 3$. Determine where $g(x)$ is positive and where it is negative.

19. Graph $g(x) = x^2 - 4x + 3$ by hand. Show all your work. Figure out if the graph opens up or down, show where the vertex goes, etc.

20. Factor the following functions:

(a) $x^2 - 4x - 5$

(b) $x^2 - 7x + 12$

(c) $x^2 - x - 1$

(d) $x^2 - 25$

(e) $x^2 - 8x + 12$

(f) $x^2 - x - 6$

21. Find the x -intercepts of $f(x) = 3x^2 - 15x - 18$.

22. Find $\lim_{x \rightarrow 0^+} \frac{x-1}{\sqrt{x}-1}$

23. Find $\lim_{x \rightarrow 7} \frac{x^2-5x-14}{x-7}$

24. Find $\lim_{x \rightarrow 5} \frac{x^2+10x+25}{(x-5)^2}$

25. Find $\lim_{x \rightarrow 20} 15$

26. Find $\lim_{x \rightarrow \infty} 35$

27. Let $f(x) = -6x^8 + 5x^2 - 3x + 10$. What happens to $f(x)$ as $x \rightarrow \infty$?

28. What is $\lim_{x \rightarrow \infty} -6x^8 + 5x^2 - 3x + 10$?

29. Determine what happens to $g(x)$ as $x \rightarrow \infty$ and $x \rightarrow -\infty$ for the function $g(x) = 250x^{29} - x^{13} + 76x^5 - 8x^4 + 7x^2 - 5x + 210$. In other words calculate $\lim_{x \rightarrow \infty} g(x)$ and $\lim_{x \rightarrow -\infty} g(x)$.

30. Find all the holes and vertical asymptotes of the following functions:

(a) $\frac{1}{(x-1)^3}$

(b) $\frac{x+1}{x^2-7x-8}$

(c) $\frac{x^2+12x+20}{x+10}$

(d) $\frac{x^2-4}{x^2+5x+6}$

(e) $\frac{x+1}{x-5}$

31. Calculate the following:

(a) $(64)^{-\frac{2}{3}}$

(b) $(32)^{\frac{1}{5}}$

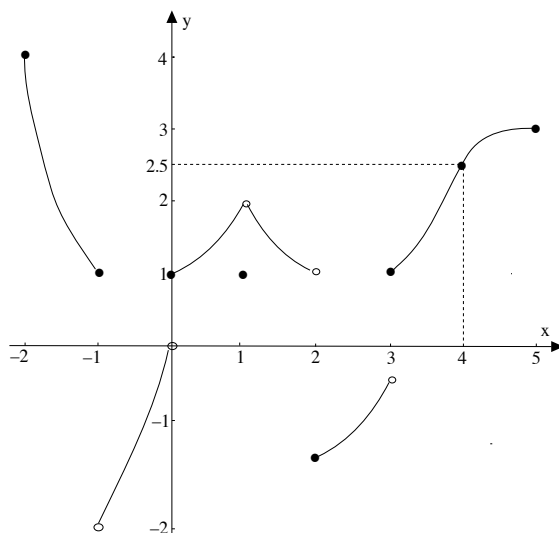
(c) $(32)^{\frac{4}{5}}$

(d) $(\frac{1}{4})^{-\frac{3}{2}}$

(e) $(-8)^{\frac{1}{3}}$

(f) Can you calculate $(-4)^{\frac{3}{4}}$? Why or why not?

32. Consider the following graph of a function, $f(x)$:



- (a) Find $\lim_{x \rightarrow 1^-} f(x)$
- (b) Find $\lim_{x \rightarrow 1^+} f(x)$
- (c) Find $\lim_{x \rightarrow 1} f(x)$
- (d) Find $\lim_{x \rightarrow 4^+} f(x)$
- (e) Find $f(1)$
- (f) Find $f(4)$
- (g) Find $\lim_{x \rightarrow 1} f(x)$
- (h) Find $\lim_{x \rightarrow 3^-} f(x)$
- (i) Does it make sense to ask for $\lim_{x \rightarrow 2} f(x)$?
- (j) Specify the x -values for which $f(x)$ has no limit. Explain your answers briefly.
- (k) Is this function continuous at $x = 4$? Why or why not?
- (l) Is this function continuous at $x = 1$? Why or why not?
- (m) Specify where $f(x)$ is discontinuous. Explain why.
33. Find $\lim_{x \rightarrow \infty} \frac{10x^3 - 9x^2 + 8x - 2}{4x^2 + 7x + 54}$
34. Find $\lim_{x \rightarrow -\infty} \frac{4x^2 + 7x + 54}{10x^3 - 9x^2 + 8x - 2}$
35. Find $\lim_{x \rightarrow -\infty} \frac{4x^3 + 7x + 54}{10x^3 - 9x^2 + 8x - 2}$
36. Consider the function $g(x) = \frac{3x-1}{2x-4}$.
- (a) Find $\lim_{x \rightarrow 2^-} g(x)$.
- (b) Does $g(x)$ have any horizontal asymptotes? If so, what are they? If not, why not? Justify your answer either way.
37. Find $\lim_{x \rightarrow 2} \frac{x^2 + 7x - 4}{x + 2}$
38. Find $\lim_{x \rightarrow 3} x^3 + 6x^2 + 4x - 10$
39. Find $\lim_{x \rightarrow 2} \frac{1}{x-2}$
40. Explain what $\lim_{x \rightarrow 3} x^3$ means in your own words. (Or more generally, what $\lim_{x \rightarrow a} f(x)$ means).

41. What does $x^{\frac{4}{5}}$ mean?
42. What does x^{-5} mean?
43. Determine the natural domains of the following functions:
- $x^{\frac{2}{3}}$
 - $x^{-\frac{5}{2}}$
 - $x^{\frac{7}{2}}$
 - $x^{-\frac{11}{6}}$
 - $x^{-\frac{1}{7}}$
44. Find $\lim_{h \rightarrow 0} \frac{(h-3)^2 - 9}{h}$.
45. Is there a difference between $f(a)$ and $\lim_{x \rightarrow a} f(x)$ for a general function $f(x)$. Explain.
46. Let $g(x) = \frac{x+1}{x+1}$. What is $g(-1)$? What is $\lim_{x \rightarrow -1} g(x)$?
47. Let $h(x) = \frac{x+1}{x-2}$. What is $h(2)$? What is $\lim_{x \rightarrow 2} h(x)$?
48. What is $\lim_{x \rightarrow 2} \frac{x+1}{(x-2)^2}$?
49. Does a function have to have a limit that exists at every point? Explain (you can draw a picture if you want).
50. Write the definition of continuity. Then, in your own words, say what this means.
51. If $f(x) = x^2 - 2x - 5$ have to cross the x axis between the x -values $x = 1$ and $x = 5$? If so, why?
52. Consider the following table of values for a continuous function $g(x)$. At least how many roots can we be positive that $g(x)$ has between $x = -4$ and $x = 4$? Which intervals are they located in? What theorem lets us determine these answers?

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-6.1	-3	-5	2	1	-1	1	-3	-7