Homework Solutions – M10240 Summer 2007

Since the answers to the odd numbered exercises are in the back of the book, this mostly contains answers to everything else.

Homework 1

(§0.2) 10 Natural domain is $[-1, 1]$. We need $1 - x^2 \geq 0$ which means $1 \geq x^2$ so $1 \geq |x|$.

18 Range is $[0, \infty)$. One way to see this: $x^4 = xxxx = x^2x^2 \geq 0$ since $x^2 \geq 0$ for all x.

Homework 2

(§0.4) 4 slope is -2, y-intercept is 0.

10 $y = \frac{1}{2}x - 1$

22 Parallel lines have the same slope. The slope of the given line is $-\frac{1}{2}$ so the desired equation is $y = -\frac{1}{2}(x - 0) + 4$. (or $y = -\frac{1}{2}x + 4$)

Homework 3

(§0.4) 25 $I(x) = 400 + 0.08x$. As revenue increases by \$1, income increases by \$0.08.

26 (a) $C(x) = 2100 + 450x$, $R(x) = 1050x$, $P(x) = R(x) - C(x) = 600x - 2100$. (b) Want to find x where $P(x) = 0$. So $600x - 2100 = 0$ and solve: $x = \frac{7}{2}$. (c) Calculate $P(9) = 3300$. (d) New functions $R(x) = 950x$ and $P(x) = 500x - 2100$. Find x where new $P(x) = 0$ to be $x = \frac{21}{5} = 4\frac{1}{5}$.

(§0.5) 9 Have $x^2 - 5x - 14 = (x + 2)(x - 7)$ so roots are at $x = -2$ and $x = 7$. Since this quadratic is pointing up it is positive on intervals $(-\infty, -2)$ and $(7, \infty)$; negative on $(-2, 7)$.

12 $4x^2 - 9 = 4(x + \frac{3}{2})(x - \frac{3}{2})$. The quadratic is pointing up so it is positive on $(-\infty, -\frac{3}{2})$ and $(\frac{3}{2}, \infty)$ and negative on $(-\frac{3}{2}, \frac{3}{2})$.

26 Function is $h(t) = -16t^2 + 48t + 56$ for $t \geq 0$ (a) Find zeros by quadratic formula to be $\frac{3 \pm \sqrt{23}}{2}$. One root is positive, the other negative. Since domain is $t \geq 0$ discard the negative root, so the rock hits the ground at $t = \frac{3+\sqrt{23}}{2}$. (b) Since the quadratic points down, the vertex gives the maximum. The vertex has t-coordinate $\frac{-b}{2a} = \frac{3}{2}$. So rock is at maximum height $t = \frac{3}{2}$ seconds after being thrown. (c) The maximum height is $h(\frac{3}{2}) = 92$ feet.

Homework 4

(§0.5) 2 $3x^2 - 6x + 7 = 3(x - 1)^2 + 4$. The parabola opens up and has vertex at $(1, 4)$.

4 $-2x^2 + x + 1 = -2(x - \frac{1}{4})^2 - \frac{9}{8}$. It opens down; vertex is at $(\frac{1}{4}, \frac{9}{8})$.

10 $2x^2 - x - 1 = 2(x^2 - \frac{1}{2}x - \frac{1}{2}) = 2(x + \frac{1}{2})(x - 1)$. The parabola opens up so the function is positive on $(-\infty, -\frac{1}{2})$ and $(1, \infty)$. It is negative $(-\frac{1}{2}, 1)$.

14 $3x^2 + 5x - 2 = 2(x - \frac{1}{3})(x + 2)$. It is positive on $(-\infty, -2)$ and $(\frac{1}{3}, \infty)$. It is negative on $(-2, \frac{1}{3})$.

(§0.6) 2 As x goes to the right the function decreases. As x goes to the left, it also decreases.

4 As $x \to \infty$ the function increases. As $x \to -\infty$ the function increases.

Homework 5

(§0.6) 6 n is even and a_n is positive.

8 n is even and a_n is negative.

10 Asymptote at $x = 1$. Function is positive on right, negative on left.

Homework 6

(§1.1) 2 (a) -2, (b) 3, (c) 2

4 At $x = 1$ and $x = 3$ since at these values the one sided limits are unequal.

10 (a) omitted, (b) $\lim_{t\to 8^-} r(t) = 5$, $\lim_{t\to 8^+} r(t) = 8$. (c) All $x \neq 8, 16$ ($16 = 4$ p.m. in 24 hour time). Include $x \neq 0, 24$ if the domain of the function is $[0, 24]$ (this is unclear from the statement).

12 $\lim_{x\to 2}(x^2 + x - 3) = 4 + 2 - 3 = 3$.

16 $\lim_{x\to 1} \frac{x+1}{x^2-1} = \lim_{x\to 1} \frac{x+1}{(x+1)(x-1)} = \lim_{x\to 1} \frac{1}{x-1}$. This limit does not exist.

30 $\lim_{h\to 0} \frac{(h-3)^2-9}{h} = \lim_{h\to 0} \frac{h^2-6h}{h} = \lim_{h\to 0} h - 6 = -6$.

Homework 7

(§1.1) 6 (a) 3, (b) -1, (c) 0, (d) $g(2) = 3$ and $g(3) = 0$.

8 At $x = 2$ and $x = 3$ since the one-sided limits are different at these two places.

40 0

46 Since $f(x) = \frac{x}{x}$ when $x > 0$, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$.

(§1.2) 2 (a) -1, (b) 0, (c) $+\infty$, (d) -1, (e) 0.

32 How does the function $W(y)$ behave as the years pass? This is asking for $\lim_{y\to\infty} W(y)$. Calculate: $\lim_{y\to\infty} W(y) = \lim_{y\to\infty}[200 + \frac{1000}{y+1}] = \lim_{y\to\infty} 200 + \lim_{y\to\infty} \frac{1000}{y+1} = 200 + 1000 \lim_{y\to\infty} \frac{1}{y+1} = 200 + 1000 \cdot 0 = 200$.

Homework 8

(§1.3) 1 f is discontinuous at $x = -1$ (jump), $x = 1$ (hole), $x = 4$ (asymptote), $x = 6$ (hole), and $x = 8$ (jump).

6 At $x = 1$ and $x = 6$.

10 omitted

22 (a) $f(1) = 1^2 = 1$, $f(2) = 2^2 = 4$, $f(3) = 2 \cdot 3 = 6$, $f(4) = \sqrt{4} = 2$, $f(5) = \sqrt{5}$. (b) Each piece of f is continuous on where they are defined so f is continuous on $(-\infty, 2)$, $(2, 4)$ and $(4, \infty)$. Since $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = 4$ and $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 2x = 4$ and $f(2) = 4$, f is continuous at $x = 2$. Only one point left: $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 2x = 8$ and $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \sqrt{x} = 2$. The two one-sided limits are unequal so we can stop right here; f is discontinuous at $x = 4$.

Homework 9

(§2.1) 15 $b^{2u} = (b^u)^2 = 3^2 = 9$.

16 $b^{-v} = \frac{1}{b^v} = \frac{1}{4}$.

17 $b^{v-u} = \frac{b^v}{b^u} = \frac{4}{3}$.

18 $b^{u+2v} = b^u b^{2v} = b^u (b^v)^2 = 3 \cdot 4^2 = 48$.

19 $b^0 = 1$.

20 $b^{v/2} = \sqrt{b^v} = \sqrt{4} = 2$.

23 (a) $500(1+0.07)^t$, (b) Plug in $t = 10$: $500(1.07)^{10} \approx 983.58$.

26 Let P be the starting amount. Then we have $5000 = P(1.05)^8$. Now solve for $P = 5000/(1.05)^8 \approx 3384.20$.

34 After 5 days we will have $y_0(0.93)^5$ grams. We need to solve for y_0. To do that look at the third day data. On the third day we have $200 = y_0(0.93)^3$. Solve for $y_0 = \frac{200}{(0.93^3)} \approx 248.65$. Now we can calculate the fifth day: $(248.65)(0.93)^5 \approx 172.98$ grams.

Homework 10

(§2.2) 3 (a) Compounded daily: $100(1+\frac{0.07}{365})^{20\cdot 365} \approx 405.47$. (b) Compounded continuously: $100e^{0.07\cdot 20} \approx 405.52$.

4 (a) Compounded annually: $5000(1+0.045)^21 \approx 12601.21$. (b) Compounded quarterly: $5000(1+\frac{0.045}{4})^{4\cdot 21} \approx 12796.37$. (c) Compounded continuously: $5000e^{0.045\cdot 21} \approx 12864.07$.

14 $\lim_{n\to\infty} \left(1+\frac{1}{4n}\right) = \lim_{n\to\infty} \left(1+\frac{1/4}{n}\right) = e^{1/4}$.

16 $\lim_{n\to\infty} 100\left(1-\frac{1}{2n}\right) = 100 \lim_{n\to\infty} \left(1+\frac{-1/2}{n}\right) = 100e^{-1/2}$.

(§2.3) 2 $\log_2 \frac{1}{4} = -2$ since $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$.

4 $\log_9 3 = \frac{1}{2}$ since $9^{1/2} = \sqrt{9} = 3$.

6 $\log_8 4 = \frac{2}{3}$ since $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$.

Homework 11

(§2.3) 8 $\log_{(0.1)} 100 = -2$ since $(0.1)^{-2} = (\frac{1}{10})^{-2} = \frac{1}{(1/10)^2} = \frac{1}{1/100} = 100$. This is a hard one since the base is less than 1. I would first rewrite the base as $1/10 = 10^{-1}$, find the base 10 logarithm and then negate it to get the base 1/10 logarithm.

14 Let $\log_{1/3} 3\sqrt{3} = n$. Then $\frac{1}{3}^n = 3\sqrt{3} = 3^{1+1/2} = 3^{3/2}$. Since $\frac{1}{3}^n = \frac{1}{3^n} = 3^{-n}$ equating the two expressions gives $3^{-n} = 3^{3/2}$, so $n = -3/2$. Thus $\log_{1/3} 3\sqrt{3} = -\frac{3}{2}$.

17 (a) $\log_{10} 40 = \log_{10} 4 \cdot 10 = \log_{10} 4 + \log_{10} 10 = \log_{10} 4 + 1 \approx 1.602$.
(b) $\log_{10} .4 = \log_{10} \frac{4}{10} = \log_{10} 4 - \log_{10} 10 = \log_{10} 4 - 1 \approx -0.398$.
(c) $\log_{10} .25 = \log_{10} \frac{1}{4} = \log_{10} 1 - \log_{10} 4 = 0 - \log_{10} 4 \approx -0.602$.
(d) $\log_{10} 2 = \log_{10} \sqrt{4} = \frac{1}{2} \log_{10} 4 \approx 0.301$.

(§2.4) 2 $5^x = e^{\ln 5^x} = e^{x \ln 5}$.

10 $\ln(e^{-2}) = -2 \ln e = -2$.

14 $e^{-\ln 2} = e^{\ln 2^{-1}} = e^{\ln \frac{1}{2}} = \frac{1}{2}$.

36 $e^{x/2} = 7$. Take the logarithm of both sides: $\ln e^{x/2} = \ln 7$. Simplify: $\frac{x}{2} = \ln 7$. Solve for $x = 2\ln 7 = \ln 49$.

38 We have $e^{x^2} = 10$. Take logarithm of both sides and simplify: $x^2 = \ln 10$. So $x = \pm\sqrt{\ln 10}$.

Homework 12

(§2.4) 39 $\ln(x+1) = 1$. Raise e to the power of both sides $e^{\ln(x+1)} = x + 1 = e^1$. Solve for x: $x = e - 1$.

40 $\ln(2x) = 3$ gives $2x = e^3$. Solve for x: $x = \frac{1}{2}e^3$.

46 $\ln(\ln x) = 0$. Exponentiate by e: $\ln x = 1$, and again: $x = e^1 = e$.

70 Put in the constants to get $78 = 72 + (98.6 - 72)(0.6)^h$. Simplify to get $\frac{6}{26.6} = (0.6)^h$. Take logarithms: $\log_{0.6} \frac{6}{26.6} = h$. So $h = \frac{\ln(6/26.6)}{\ln 0.6} \approx 2.92$ hours.

(p. 157) 16 (a)

17 (b)

18 (d)

19 (b) (This case is identical to problem 17 since $2^{-x} = (\frac{1}{2})^x$).

Homework 13

(§3.1) 1 Eyeball the slope to be $\frac{1/2}{1/4} = 2$.

2 A, B.

3 C, D.

4 A, B, D, C.

9 Using the fact that the for $f(x) = x^2$ the slope of the tangent line at (a, a^2) is $2a$: (a) the slope is 1, line is $y = (x - \frac{1}{2}) - \frac{1}{4}$, (b) slope 0, (c) slope is -6.

16 Using the work from the activity, the slope of $f(x) = x^3$ at (a, a^3) is $3a^2$. The question is then for which a is $-1 = 3a^2$. Solve for a: $a = \sqrt{-\frac{1}{3}}$. There are no such a.

25 (a) Calculate $f(1+h) - f(1) = 1 + 2h + h^2 + 1 + h - 1 - 1 = h^2 + 3h$. So slope is $\frac{h^2+3h}{h}$. (b) We have $\lim_{h\to 0} \frac{h^2+3h}{h} = 3$. (c) In general $f(x+h) - f(x) = x^2 + 2hx + h^2 + x + h - x^2 - x = 2hx + h^2 + h$. Take the limit: $\lim_{h\to 0} \frac{2hx+h^2+h}{h} = 2x + 1$. (d) At $(1, 2)$ have $y = 3(x - 1) + 2$. At $(-2, 2)$ have $y = -3(x + 2) + 2$. And at $(-1, 0)$ have $y = -(x + 1)$.

26 (a) Calculate $f(x + h) - f(x) = 3x^2 + 6xh + 3h^2 - 3x^2 = 6xh + 3h^2$. So the slope is $\frac{6xh+3h^2}{h}$. (b) The limit is then $6x$. (c) at $(1, 3)$ have $y = 6(x - 1) + 3$; at $(-1, 3)$ have $y = -6(x + 1) + 3$; and at $(\frac{1}{2}, \frac{3}{4})$ have $y = 3(x - \frac{1}{2}) + \frac{3}{4}$.

34 (a) graph omitted. Since $f(t+h) - f(t) = -t^2 - 2th - h^2 + 100 - (-t^2 + 100) = -2th - h^2$. Then $\lim_{h\to 0} \frac{-2th-h^2}{h} = -2t$. (b) At $t = 4$ the slope is -8. So the sales are falling. (c) the line is $y = -8(t - 4) + 84$. The x intercept is then $t = 14.5$ weeks.