Handout 1 — Fall 2010 — ACMS 20340

Working With Conditional Probability

I thought about asking some questions about this as homework, but on reflection it seems it might be more helpful to show an example of working with conditional probability.

The following is from the South Bend Tribune.

> **History lesson**
>
> Though last year's Irish team did survive at Purdue (24-21) and went on to a coach-purging 6-6 mark, ND's early-season matchups with the Boilermakers have been almost scary in their accuracy in intuiting how Notre Dame's seasons will end up.
>
> Of the 63 meetings since World War II and up until last season, 41 Irish squads have passed the Purdue test and have gone on to win at an .824 clip for the season. The 22 ND teams that lost to the Boilermakers averaged a modest winning percentage of .536.[1]

In this article two conditional probabilities are given: the probability of wining a game, provided ND won against Purdue that season, and the probability of losing a game, provided ND lost against Purdue. Lets name the relevant events.

$$Q : \text{ND won against Purdue in the season}$$
$$W : \text{ND wins any given game}$$

I use Q instead of P since there are already enough P's.

The article then states $P(W|Q) = 0.824$ and $P(W|Q^c) = 0.536$. Of course, conveniently forget that these numbers were calculated by looking at previous games and may not provide a good future prediction.

Here are some questions:

1. So, what is the overall probability of winning a given game?

2. Are these two events independent?

Before going farther, try to answer these questions.

[1] From "Recruit visits few on opening weekend" by Eric Hansen, South Bend Tribune, September 4, 2010, page W2.

1. For the first question we need to find $P(W)$, the probability of winning any given game. If we think of a tree diagram then we see that

$$P(W) = P(Q) \cdot P(W|Q) + P(Q^c) \cdot P(W|Q^c)$$

(I wish I could include a picture. Try drawing one and check the above equality.) All we need now is $P(Q)$ and $P(Q^c)$. But, these are given in the problem statement. Over 63 meetings, ND won against Purdue 41 times, making $P(Q) = \frac{41}{63}$. Conversely, ND lost to Purdue 22 times, so $P(Q^c) = \frac{22}{63}$. Combine everything:

$$\begin{aligned} P(W) &= P(Q) \cdot P(W|Q) + P(Q^c) \cdot P(W|Q^c) \\ &= \tfrac{41}{63} \cdot 0.824 + \tfrac{22}{63} \cdot 0.536 \\ &= 0.723 \end{aligned}$$

Looking up the all time winning percentage online[2], I find it is 0.736, which is pretty close to what we calculated since the numbers in the article did not include the 2009 season.

2. Are the events independent?

   It is enough to check whether $P(W|Q)$ is equal to $P(W)$. But $P(W|Q) = 0.824$ and $P(W) = 0.723$, so these events are dependent.

[2]For the years 1887–2009.
From http://www.nationalchamps.net/NCAA/database/notredame_database.htm