Risk Aversion in Stochastic Bargaining Games[1]

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1 Introduction

This paper address the role of risk aversion in a non-cooperative model of bargaining similar to the classic Ståhl/Rubinstein model. The original model, and most variants of it, typically involve no exogenous uncertainty and have subgame perfect equilibria in pure strategies. Even when risk is introduced in the models, such as when the discount factor is seen as including a probability of the game ending, the risk tends to be symmetric (Roth, 1985). We consider a model in which the bargainers are identical in every respect, except in the uncertainty they face in the bargaining environment. In the model, rejection of an offer leads to a random move by nature that determines future order of play. When considering the offers that a player would be willing to accept, the player's risk aversion consequently enters the calculation. We also demonstrate that under risk aversion, our model has a unique subgame perfect equilibrium. We show that as players become infinitely patient, the equilibrium offer converges to the generalized Nash bargaining solution(Harsanyi and Selten, 1972).

The remainder of this paper is organized as follows. Section 2 introduces the bargaining model. Properties of the subgame perfect equilibrium to this bargaining game are investigated in Section 3. Section 4 presents analysis of equilibrium agreements for various specifications of the players' utility functions.

2 The Model

We consider a standard pie partitioning bargaining problem involving two agents, quite similar to Rubinstein's (1982) alternating offer model. Just as in Rubinstein's model, the pie is only divided once a player has proposed a partition, and the other player consents to this partition. If the second player does not consent, one of the players makes another proposal. In the interest of studying the effect of risk aversion, we introduce uncertainty by selecting the proposer randomly after each rejected offer according to a stationary Markov process. Let p_{ij} be the probability that player j will

be selected as the proposer in the next round given that player i is the proposer in the current round.[1] Merlo and Wilson (1995) investigate the uniqueness and efficiency properties of the equilibrium offers for a similar class of bargaining models. The current paper concentrates more on the distributional characteristics of the equilibrium.

The agents are assumed to have identical, concave, strictly increasing utility functions for wealth, so $u'(w) > 0$ and $u''(w) < 0$.[2] Without loss of generality, we adopt the normalization that $u(0) = 0$ and $u(1) = 1$. The agents are also taken to have a fixed common discount factor $\delta \in (0, 1)$. This framework was established to make the players identical in all respects except for their chances of being selected as the proposer.

The game starts with the selection of the first proposer, player one will be selected with probability p and player two will be selected with probability $1 - p$. The selection process of the initial proposer is not defined by the Markov process. The equilibrium proposals will not depend on this initialization procedure, it would only affect the *ex-ante* expected payoffs of the players. One possible specification for p is the steady state of the Markov process, ie. $\pi = p_{21}/(1 - p_{11} + p_{21})$. If player one (two) is selected, he proposes a partition $x_1 \in [0, 1]$ ($y_1 \in [0, 1]$). Player two (one) observes this, then can either accept it or reject it. If player two (one) accepts, the payoffs for player one and two are $u(x_1)$ and $u(1-x_1)$ ($u(y_1)$ and $u(1-y_1)$). If player two (one) rejects the offer, period one ends and period two starts with a random determination of the first mover in which player one is selected with probability p_{11} (p_{21}) . If player one is the proposer in the second period, he selects an $x_2 \in [0, 1]$. If this is accepted, the discounted payoffs are $\delta u(x_2)$ and $\delta u(1-x_2)$. The game continues indefinitely until a partition is accepted. Figure 1 depicts the extensive form of the bargaining process.

[1] These transition probabilities must satisfy $p_{i1} + p_{i2} = 1$ for $i = 1, 2$. Henceforth, we shall specify this transition matrix with the probability that player one is selected to be the proposer next period given the identity of the current proposer, ie. p_{11} and p_{21}.

[2] All of the results of Section 3 are trivially extended to the case where the players may have different utility functions, u_1 and u_2. This focus of this paper is mainly on the effect of asymmetries in the selection process of proposers, so we take the utility functions to be identical to avoid introducing an additional asymmetry.

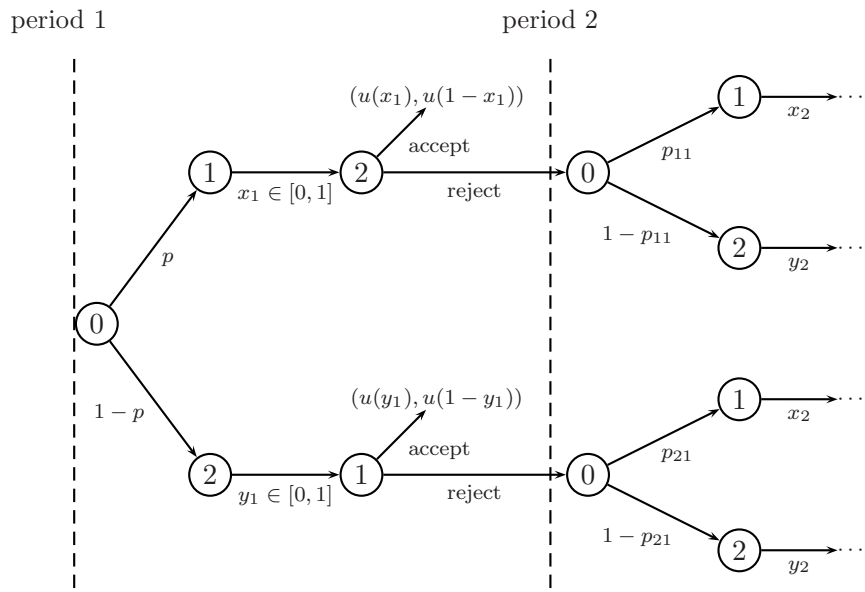

Figure 1: Stochastic Order Bargaining Game

The inspiration for this model was a board appointment model studied by Waller (1997), in which candidates would be appointed to a board by a party that was currently in power. The opposing party must confirm the appointee in order for the appointee to be placed on the board. The uncertainty in the model is that political parties are unsure if they will be in control following the next election. The model allows for any number of expectations about future elections. It might be that party one simply wins sixty percent of the elections, regardless of who won the previous election, which would correspond to $p_{11} = p_{21} = 0.6$. It might be that incumbents have an advantage (or disadvantage) but there is no other difference between the two parties, which would correspond to $p_{11} = 1 - p_{21}$. The current model differs from Waller's original model in that Waller cast this appointment process in an OLG framework in which individuals had finite time horizons.

3 Properties of Subgame Perfect Equilibria

Characterization of the set of subgame perfect equilibria is achieved through techniques similar to those used by Rubinstein. Let $\mathbb{X}$ be the set of values of x_1 for which there is a subgame perfect equilibrium in which player two accepts this first offer. We will need to consider the players' best and worst equilibria, so let $\bar{x} = \sup \mathbb{X}$ and $\underline{x} = \inf \mathbb{X}$. Similarly, $\mathbb{Y}$ is the set of values of y_1 that are accepted in some subgame perfect equilibrium, and $\bar{y} = \sup \mathbb{Y}$ and $\underline{y} = \inf \mathbb{Y}$. If player one is selected to propose in any period, regardless of the history of rejected offers up to that point, the proper subgame starting from player one's selection of a partition is isomorphic to the proper subgame in which player one proposes in the first round, so the set of subgame perfect equilibria must be identical. Thus, $\mathbb{X}$ is the set of values of x_t that can be accepted in a subgame perfect equilibrium for any $t = 1, 2, \ldots$. This isomorphism also holds for the subgames in which player two proposes, so all values of y_t that are accepted in equilibrium belong to $\mathbb{Y}$.

Suppose player one is selected to propose in the first period. Player two will only accept if the offer provides at least as much utility as her worst expected equilibrium payoff in round two,

$$u(1 - \bar{x}) \geq \delta p_{11} u(1 - \bar{x}) + \delta(1 - p_{11}) u(1 - \bar{y}). \tag{1}$$

Similarly, player one realizes that player two would certainly accept any offer in which she received a payoff higher than her best expected equilibrium payoff in round two. To maximize his own payoff, player one cannot offer player two more than this amount,

$$u(1 - \underline{x}) \leq \delta p_{11} u(1 - \underline{x}) + \delta(1 - p_{11}) u(1 - \underline{y}). \tag{2}$$

Parallel arguments hold for the case when player two is the proposer in the first round and these lead to the two following conditions:

$$u(\underline{y}) \geq \delta p_{21} u(\underline{x}) + \delta(1 - p_{21}) u(\underline{y}), \text{ and} \tag{3}$$

$$u(\bar{y}) \leq \delta p_{21} u(\bar{x}) + \delta(1 - p_{21}) u(\bar{y}). \tag{4}$$

Together, (1) and (4) can be shown to imply that

$$u^{-1}\Big(\frac{\delta p_{21}}{1 - \delta(1 - p_{21})} u(\bar{x})\Big) + u^{-1}\Big(\frac{1 - \delta p_{11}}{\delta(1 - p_{11})} u(1 - \bar{x})\Big) \geq 1. \tag{5}$$

Note that the strict monotonicity of the utility function guarantees that there is an monotonic inverse utility function $u^{-1}()$. Also, (1) and (4) together imply that

$$u^{-1}\Big(\frac{\delta p_{21}}{1 - \delta(1 - p_{21})} u(\underline{x})\Big) + u^{-1}\Big(\frac{1 - \delta p_{11}}{\delta(1 - p_{11})} u(1 - \underline{x})\Big) \leq 1. \tag{6}$$

Theorem 3.1. *For strictly monotonic, strictly concave, differentiable utility functions, the set of partitions proposed by player one that get accepted in equilibrium is a singleton.*

Proof. We will first show that $\bar{x} = \underline{x}$ if (5) and (6) are jointly satisfied. Second, a fixed point argument is used to show existence of a proposal that jointly satisfies these conditions.

We start by defining a function

$$f(x) = u^{-1}\Big(\frac{\delta p_{21}}{1 - \delta(1 - p_{21})} u(x)\Big) + u^{-1}\Big(\frac{1 - \delta p_{11}}{\delta(1 - p_{11})} u(1 - x)\Big),$$

so that (5) and (6) can be rewritten as $f(\bar{x}) \geq 1$ and $f(\underline{x}) \leq 1$, which together imply the following condition:

$$f(\bar{x}) \geq f(\underline{x}). \tag{7}$$

We now show that $f(x)$ is strictly decreasing in x for $\delta \in (0, 1)$ by signing the derivative

$$f'(x) = (u^{-1})'\Big(\frac{\delta p_{21}}{1 - \delta + \delta p_{21}} u(x)\Big) u'(x) - (u^{-1})'\Big(\frac{1 - \delta p_{11}}{\delta - \delta p_{11}} u(1 - x)\Big) u'(1 - x). \tag{8}$$

From the definition of the inverse utility function, $u^{-1}[u(x)] \equiv x$, we can derive that $(u^{-1})'[u(x)]u'(x) = 1$. The strict concavity of the utility function implies that the inverse utility function is strictly convex, from which it follows that $(u^{-1})'[\lambda u(x)]u'(x) < 1$ for $\lambda < 1$, and the inequality is reversed for $\lambda > 1$. Given that $\frac{\delta p_{21}}{1-\delta+\delta p_{21}} < 1$ and $\frac{1-\delta p_{11}}{\delta-\delta p_{11}} > 1$ for $\delta \in (0,1)$, it follows that $f'(x) < 0$.

Since $f(x)$ is strictly decreasing in x and by definition, $\bar{x} \geq \underline{x}$, (7) can only hold at $x^* = \bar{x} = \underline{x}$.

Finally, using the continuity of $f(x)$ along with the observations that $f(0) = u^{-1}(\frac{1-\delta p_{11}}{\delta-\delta p_{11}}) > 1$ and $f(1) = u^{-1}(\frac{\delta p_{21}}{1-\delta+\delta p_{21}}) < 1$, the Intermediate Value Theorem guarantees that $f(x) = 1$ for some $x \in (0,1)$. □

Player one's offer, denoted as x^*, must satisfy a condition similar to (5) which must hold with equality,

$$u^{-1}\Big(\frac{\delta p_{21}}{1-\delta(1-p_{21})}u(x^*)\Big) + u^{-1}\Big(\frac{1-\delta p_{11}}{\delta(1-p_{11})}u(1-x^*)\Big) = 1. \qquad (9)$$

An identical argument shows that player two also has a unique equilibrium offer y^* that must satisfy an equation similar to (9). Furthermore, as δ tends to one, (1) and (4) together imply that $y^* \to x^*$. Therefore, as the players become infinitely patient, they will both suggest the same partition in equilibrium. However, Theorem 3.2 shows that this limiting partition is not an equal split as it is in the standard Rubinstein model, even though the players have identical utility functions. Instead, equilibrium offers are shown to converge to the asymmetric Nash bargaining solution (Harsanyi and Selten (1972)) where the players' bargaining powers are given by their long run probabilities of being selected as the proposer.

The asymmetric Nash bargaining solution calculated by solving the following maximization problem:

$$\max_{x_N \in [0,1]} u(x_N)^{\pi} u(1-x_N)^{1-\pi}.$$

Given the normalization that $u(0) = 0$ and the concavity of u, the unique

solution to this problem must satisfy the first order condition:

$$\pi u(1-x_N)u'(x_N) - (1-\pi)u(x_N)u'(1-x_N) = 0 \tag{10}$$

Theorem 3.2. *As players become infinitely patient, $\delta \to 1$, the subgame perfect equilibrium offer that satisfies equation (9) converges to the asymmetric Nash bargaining solution.*

Proof. Let $x(\delta)$[3] be the solution to (9), so that

$$u^{-1}\Big(\frac{\delta p_{21}}{1-\delta(1-p_{21})}u(x(\delta))\Big) + u^{-1}\Big(\frac{1-\delta p_{11}}{\delta(1-p_{11})}u(1-x(\delta))\Big) \equiv 1. \tag{11}$$

Because this is an identity, the equation obtained from differentiating this with respect to δ must also hold as an identity:

$$\begin{aligned}(u^{-1})'\Big(\frac{\delta p_{21}}{1-\delta(1-p_{21})}u(x(\delta))\Big)\Big(&\frac{\delta p_{21}}{[1-\delta(1-p_{21})]^2}u(x(\delta))\\ &\frac{\delta p_{21}}{1-\delta(1-p_{21})}u'(x(\delta))x_\delta(\delta)\Big)\\ &-(u^{-1})'\Big(\frac{1-\delta p_{11}}{\delta(1-p_{11})}u(1-x(\delta))\Big)\\ \Big(\frac{1}{\delta^2(1-p_{11})}u(1-x(\delta)) &+ \frac{1-\delta p_{11}}{\delta(1-p_{11})}u'(1-x(\delta))x_\delta(\delta)\Big) \equiv 0.\end{aligned} \tag{12}$$

Let $\bar{x} \equiv \lim_{\delta\to 1} x(\delta, p_{11}, p_{21})$ and $\bar{x}_\delta \equiv \lim_{\delta\to 1} x_\delta(\delta, p_{11}, p_{21})$. Taking the limit of this (12) as $\delta \to 1$ yields

$$\begin{aligned}(u^{-1})'[u(\bar{x})]\Big(\frac{u(\bar{x})}{p_{21}} &+ u'(\bar{x})\bar{x}_\delta\Big)\\ &- u^{-1'}[u(1-\bar{x})]\Big(\frac{u(1-\bar{x})}{1-p_{11}} + u'(1-\bar{x})\bar{x}_\delta\Big) \equiv 0.\end{aligned} \tag{13}$$

Once again, we can employ the identity: $u^{-1'}[u(y)] \equiv 1/u'(y)$, to reduce this

[3] More accurately, this should also be a function of p_{11} and p_{21}, but the probabilities have been omitted to simplify the notation.

to:

$$\frac{u(\bar{x})}{p_{21}u'(\bar{x})} + \bar{x}_\delta - \frac{u(1-\bar{x})}{(1-p_{11})u'(1-\bar{x})} - \bar{x}_\delta \equiv 0. \tag{14}$$

After cancelling the $\bar{x}_\delta$ terms, multiplying by the product of the denominators, and dividing by $1 - p_{11} + p_{21}$, this equation becomes identical to the first order condition used to solve for the Nash solution, ie. (10). Theorem 3.1 shows that there is a unique subgame perfect equilibrium offer to the stochastic bargaining game. It has just been shown that this offer must satisfy equation (10). The concavity of u guarantees that the unique solution of (10) is the asymmetric Nash bargaining solution. Thus, as players become infinitely patient, the subgame perfect offers in the stochastic bargaining game converge to the asymmetric Nash bargaining solution. □

The requirement that the limiting equilibrium offer must satisfy (10) suggests a simple criterion for determining how this offer compares to the long-run probability of being the proposer based on the elasticity of utility. The elasticity of utility evaluated at some level of wealth w is defined as $E(w) \equiv wu'(w)/u(w)$.

Theorem 3.3. *If the long-run probability of player one being selected as the proposer is less than* $1/2$ *and the elasticity of utility is increasing (decreasing) in wealth, the limiting subgame perfect equilibrium offer as players become extremely patient is less (greater) than* π*. If* π *is greater than* $1/2$*, the relations between* $\bar{x}$ *and* π *are reversed. If* $\pi = 1/2$*,* $\bar{x}$ *must equal* $1/2$*.*

Proof. The proof consists of two steps. First, we show that if $\pi < 1/2$ then $\bar{x} < 1/2$. Second, we show that if $E(w)$ is increasing in w, then the $\bar{x} < \pi$.
1) Equation (10) can be rewritten as

$$\frac{\pi u'(\bar{x})}{u(\bar{x})} = \frac{(1-\pi)u'(1-\bar{x})}{u(1-\bar{x})} \tag{15}$$

By the concavity of u, $u'(x)$ is decreasing in x and u is increasing in x, so $u'(x)/u(x)$ must be decreasing in x. If $\pi < 1/2$ then $\pi < 1-\pi$, so in order to satisfy (15) it must be that $u'(\bar{x})/u(\bar{x}) > u'(1-\bar{x})/u(1-\bar{x})$, which requires

that $\bar{x} < 1-\bar{x}$ or $\bar{x} < 1/2$. This inequality is obviously reversed for $\pi > 1/2$, and holds with equality when $\pi = 1/2$.
2) For $\pi < 1/2$, if the elasticity of utility is increasing in wealth, it must be that $E(\bar{x}) < E(1-\bar{x})$ since $\bar{x} < 1/2$. Dividing the left and right sides of this inequality by the appropriate sides of (15) yields the following sequence of inequalities:

$$\begin{aligned} \frac{E(\bar{x})u(\bar{x})}{\pi u'(\bar{x})} &< \frac{E(1-\bar{x})u(1-\bar{x})}{(1-\pi)u'(1-\bar{x})}, \\ \frac{\bar{x}}{\pi} &< \frac{1-\bar{x}}{1-\pi}, \text{ or} \\ \bar{x} &< \pi. \end{aligned}$$

This condition would clearly hold with equality if the elasticity of utility was constant, and would reverse if the elasticity of utility was decreasing in w. □

The economic intuition for why the elasticity of utility plays an important role in determining the equilibrium split is that if player one has a lower elasticity of utility than player two, player one values an incremental increase in wealth less than player two, so he would be less willing to delay agreement in order to get it. Table 1 summarizes the results of Theorem 3.3.

	$E'(w) < 0$	$E'(w) = 0$	$E'(w) > 0$
$\pi < 1/2$	$\bar{x} > \pi$	$\bar{x} = \pi$	$\bar{x} < \pi$
$\pi = 1/2$	$\bar{x} = \pi$	$\bar{x} = \pi$	$\bar{x} = \pi$
$\pi > 1/2$	$\bar{x} < \pi$	$\bar{x} = \pi$	$\bar{x} > \pi$

Table 1: Summary of Results from Theorem 3.3

4 Specific Utility Functions

The previous section established that for well behaved risk averse utility specifications, there is a unique subgame perfect equilibrium proposal for

each player. Below, we obtain closed form solutions for a few utility functions. Theorem 3.3 showed that properties of the elasticity of utility played an important role in determining how the equilibrium offer compared to the parameter π. Specifically, the derivative of $E(w)$ needs to be signed,

$$\begin{aligned} E'(w) &= \frac{u'(w)}{u(w)} + \frac{wu''(w)}{u(w)} - \frac{wu'(w)^2}{u(w)^2}, && (16) \\ &= \frac{u'(w)}{u(w)}\Big(1 + \frac{wu''(w)}{u'(w)} - \frac{wu'(w)}{u(w)}\Big), && (17) \\ &= \frac{u'(w)}{u(w)}\Big(1 - R(w) - E(w)\Big), && (18) \end{aligned}$$

where $R(w)$ is the Arrow-Pratt measure of relative risk aversion. We expect risk aversion to play an even more important role in determining the equilibrium offers as players become impatient. When players are extremely patient, the risk of not being selected as the proposer next period is not terribly important to them because they know they will eventually get to propose. Intuitively, if the bargaining process is set up so that one player is at a disadvantage, that player would expect to get less than half the pie. If relative risk aversion is decreasing, it means that this player's opponent will have the additional advantage of being relatively less risk averse when considering likely agreements that favor this opponent. Results related to the quantification of this effect for arbitrary utility functions have been elusive. For specific utility functions, we can solve for the closed form solution of the equilibrium offers. We observe that the Arrow–Pratt measure of relative risk aversion coincides with certain regularities in these solutions. This regularity is based on the difference between a player's expected equilibrium offer and that player's long run steady state probability of being selected by the Markov process. We shall call this difference the player's "risk bonus."

4.1 Constant Relative Risk Aversion

We start by considering the constant relative risk aversion (CRRA) utility specification. Solving equation 9 given that $u(w) = w^a$ for $a \in (0,1)$ yields:

$$x^* = \underline{x} = \bar{x} = \frac{1 - \left(\frac{1-\delta p_{11}}{\delta - \delta p_{11}}\right)^{\frac{1}{a}}}{\left(\frac{\delta p_{21}}{1-\delta+\delta p_{21}}\right)^{\frac{1}{a}} - \left(\frac{1-\delta p_{11}}{\delta - \delta p_{11}}\right)^{\frac{1}{a}}}.$$

For this utility specification, $E(w) = a$, so Theorem 3.3 implies that this equilibrium offer should converge to π as players become patient. It is easy to verify that this solution does indeed converge to π,

$$\lim_{\delta \to 1} x^* = \frac{p_{21}}{1 - p_{11} + p_{21}} = \pi.$$

When considering impatient players, it is no longer appropriate to only consider player one's equilibrium offer, x^*, because it will now differ from player two's equilibrium offer, y^*. In order to compare the player's payoff to π, we will now consider the *ex-ante* expected payoffs of player one, which we denote as $z^* \equiv \pi x^* + (1-\pi) y^*$. Using the closed form solutions for the CRRA utility function, we have been able to show algebraically that for $\delta < 1$, if $\pi < 1/2$ then $z^* < \pi$, and $z^* > \pi$ if $\pi > 1/2$.[4] Figure 2 plots $x^* - y^*$ as a function of p_{11} and p_{22} for $\delta = 0.8$ Light areas of the surface represent regions where there is a positive risk bonus, ie. $z^* > \pi$. Notice that the dividing line between the light and dark regions is along the line $p_{11} = 1 - p_{21}$, which is where $\pi = 1/2$. This is not a coincidence, when $\pi = 1/2$ it must be that $p_{21} = 1 - p_{11}$, which means that the Markov proposer selection process, and thus the game, is symmetric for any specification of utilities. The uniqueness of the subgame perfect equilibrium then implies that the subgame perfect equilibrium offers must be symmetric, ie. $y^* = 1 - x^*$ for any discount factor, thus $z^* = \pi = 1/2$. To the southwest of this border is the region over which $\pi < 1/2$, and it is dark, indicating that $z^* < \pi$. Figure 3 presents

[4] This will likely be included in the next version of this paper, unless a more general result is obtained.

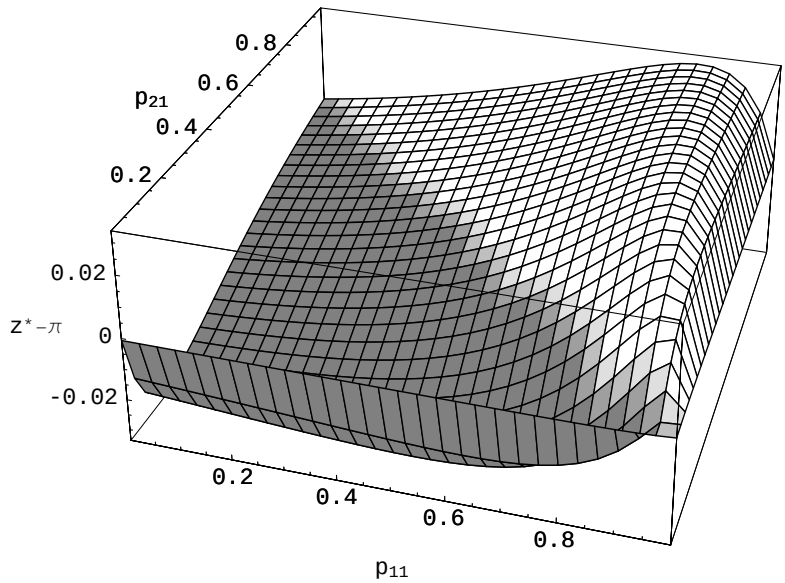

Figure 2: CRRA: Risk Bonus for $\delta = 0.8$

a similar graph for a discount factor of 0.99. The main difference between these figures is the scale on the $z^* - \pi$ axis, which is larger for $\delta = 0.8$, indicating larger risk bonuses when players are impatient.

4.2 Increasing Relative Risk Aversion

We have solved for equilibrium offers under a number of specifications of utility that exhibit increasing relative risk aversion (IRRA), we only report the results of one such specification here. The qualitative results of all IRRA utility functions were identical to those presented here.

For $u(w) = \frac{2w}{1+w}$, the closed form solution is given by:

$$x^* = \frac{\sqrt{\delta^2 (1 - p_{11} + 3p_{21})^2 + 8 (1 - \delta + \delta p_{21}) (2 - \delta - \delta p_{11} - \delta p_{21})} - \delta + \delta p_{11} - 3\delta p_{21}}{4 - 2\delta - 2\delta p_{11} - 2\delta p_{21}}$$

For this utility function, the elasticity of utility is $E(w) = \frac{1}{1+w}$ and the Arrow–Pratt measure of relative risk aversion is $R(w) = \frac{2w}{1+w}$, which are

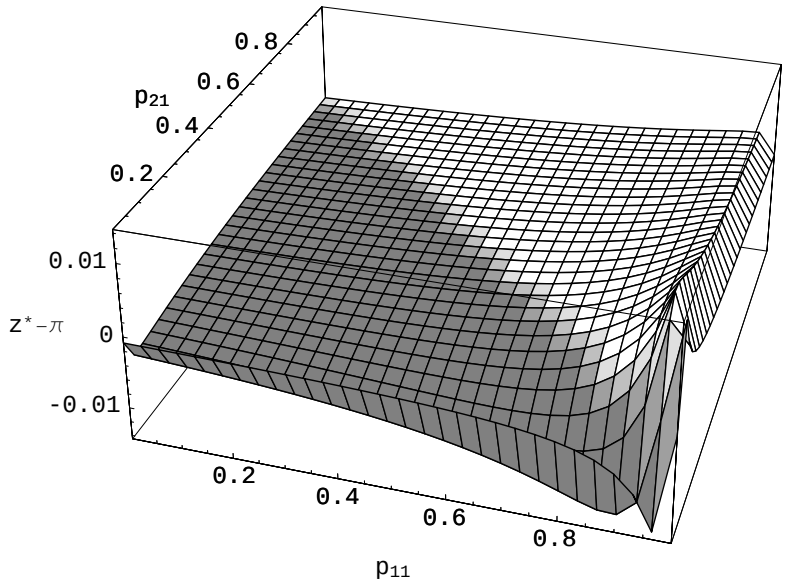

Figure 3: CRRA: Risk Bonus for $\delta = 0.99$

both increasing in w.

Again, we investigate equilibrium behavior when players are somewhat impatient. Figures 4 and 5 depict the risk bonus for $\delta = 0.8$ and $\delta = 0.99$. The graphic is shaded so that the surface is white when risk bonus is positive, and gray otherwise. Notice that in both figures, the risk bonus is now positive for $p_{11} < 1 - p_{21}$, which implies that $\pi < \frac{1}{2}$, the exact opposite of the CRRA case.

5 Discussion of other utility functions

We have also worked through an example with decreasing relative risk aversion, and the risk bonuses look very similar to the CRRA case, so we omit the details. We would like to untangle the effects of elasticity of utility from risk aversion, but for every valid utility function we tried, the derivative of the Arrow-Pratt measure of relative risk aversion always has the same sign as the derivative of the elasticity of utility. For instance, all CRRA utility

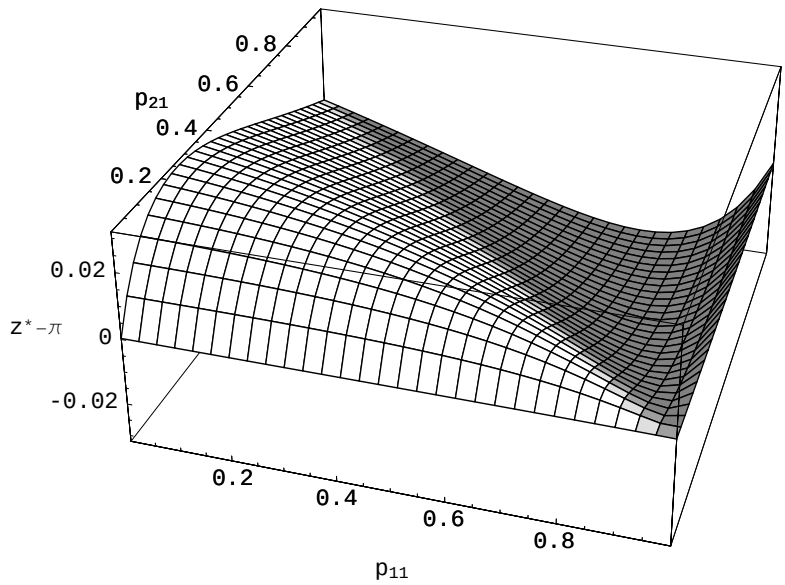

Figure 4: IRRA: Risk Bonus for $\delta = 0.8$

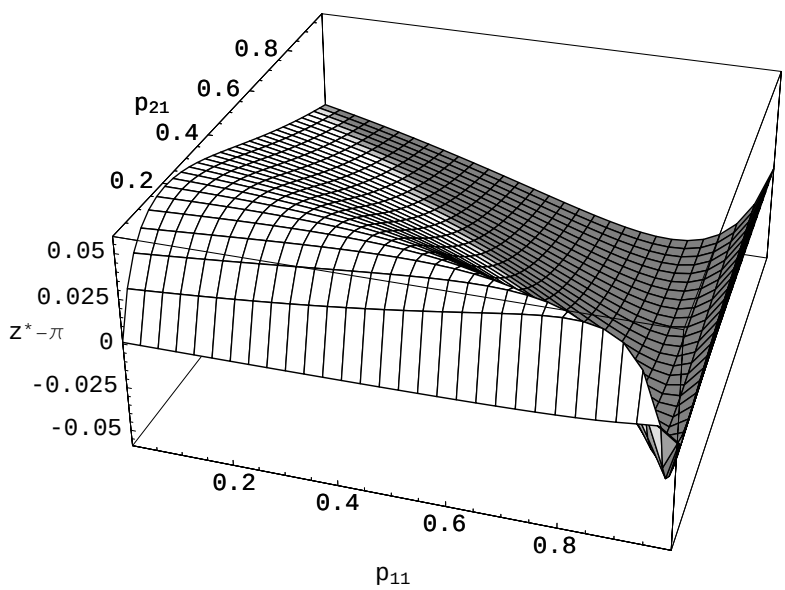

Figure 5: IRRA: Risk Bonus for $\delta = 0.99$

functions also have a constant elasticity of utility, and the converse is also true. One item yet to be resolved in this paper is to either: find an example of a utility function in which these measures are moving in opposite directions; or prove that this cannot happen for the class of utility functions we are considering.

6 Conclusion

Obviously, the highest priority right now is to analytically demonstrate this relationship between the risk bonus and the relative risk aversion properties of the utility function when players are impatient. Other avenues of research might include an attempt to tie these results on risk aversion to the experimental literature on ultimatum bargaining, in which subjects often make offers that are generous relative to the subgame perfect equilibrium of the one-shot game. However, if subjects view the experiment in the context of a repeated setting, in which the roles of the players may be switched in the future, modeling techniques similar to those presented here may provide some insight.

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