Money, Credit and Banking

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Abstract

In monetary models in which agents are subject to trading shocks there is typically an ex-post inefficiency in that some agents are holding idle balances while others are cash constrained. This problem creates a role for financial intermediaries, such as banks, who accept nominal deposits and make nominal loans. In general, financial intermediation improves the allocation. The gains in welfare come from the payment of interest on deposits and not from relaxing borrowers' liquidity constraints. We also demonstrate that when credit rationing occurs increasing the rate of inflation can be welfare improving .

Keywords : Money, Credit, Rationing, Banking.

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1 Introduction

In monetary models in which agents are subject to trading shocks there is typically an ex-post inefficiency in that some agents are holding idle balances while others are cash constrained.[1] Given this inefficiency it seems natural that a credit market reduces or eliminates this welfare loss. This seems obvious at first glance but it overlooks a fundamental tension between money and credit. A standard result in monetary theory is that for money to be essential in exchange there must be an absence of record keeping. In contrast, by definition credit requires record keeping. This tension has made it inherently difficult to introduce credit in a model where money is essential.[2] Furthermore, the existence of credit raises the issues of repayment and enforcement.

This paper constructs a framework in which money and credit both arise from the same trading frictions. The objective is to answer the following questions. First, how can money and credit coexist in a model where money is essential? Second, can financial intermediation improve the allocation? Third, what is the optimal monetary policy in an environment where all trades are voluntary, i.e., when there is no enforcement?

To answer these questions we construct a monetary model with financial intermediation. We call these financial intermediaries banks because they accept nominal deposits and make nominal loans. Banks have a record-keeping technology that allows them to keep track of financial histories but agents still trade with each other in anonymous goods markets. Hence, there is no record keeping of good market trades. Consequently, the existence of financial record keeping does not eliminate the need for money as a medium of exchange. We characterize the monetary equilibria in two cases: with and without enforcement. By enforcement we mean that banks can force repayment at no cost, which prevents any default, and the monetary authority can impose lump-sum taxes. In an environment with no enforcement, the only penalty for default is

[1] Models with this property include Bewley (1980), Levine (1991), Molico (1999), Camera and Corbae (1999), Berentsen (2002), Green and Zhou (2005), and Berentsen, Camera and Waller (2004).

[2] By essential we mean that the use of money expands the set of allocations (Kocherlakota (1998) and Wallace (2001)).

exclusion from the financial system.

With regard to the second question, we show that the equilibrium with credit improves the allocation. The gain in welfare comes from payment of interest to agents holding idle balances and not from relaxing borrowers' liquidity constraints. With respect to the third question, the answer depends on whether enforcement is feasible or not. If enforcement is feasible, the Friedman rule attains the first-best allocation. The intuition is that under the Friedman rule agents can perfectly self-insure against consumption risk by holding money at no cost. Consequently, there is no need for financial intermediation – the allocation is the same with or without credit. In contrast, without enforcement, deflation cannot be implemented nor can banks force agents to repay loans. In this situation, we show that price stability is not the optimal policy since some inflation can be welfare improving. The reason is that inflation makes holding money more costly, which increases the severity of punishment from being excluded from the financial system and increases the incentives to repay loans.[3]

How does our approach differ from the existing literature? Other mechanisms have been proposed to address the inefficiencies that arise when some agents are holding idle balances while others are cash constrained. These mechanisms involve either trading cash against some other illiquid asset (Kocherlakota 2003), collateralized trade credit (Shi 1996) or inside money (Cavalcanti and Wallace 1999a,b, Cavalcanti et al. 1999, and He, Huang and Wright, 2005).

The banks in our model have a similar function as Kocherlakota's (2003) 'illiquid' bonds – they transfer money at a price from those with low marginal value of money to those with a high valuation. The key difference is that, in Kocherlakota's model, agents adjust their portfolios by trading assets while in our model agents acquire one asset, namely money, by issuing liabilities. Although both approaches have the same implications for the allocation, in general, the presence of illiquid bonds does not eliminate the role of credit. The reason is that some agents may hold so little money and bonds that they

[3]This confirms the intuition of Aiyagari and Williamson (2000) for the numerical results in their model for why the optimal inflation rate is positive when enforcement is not feasible.

would like to borrow additional money to acquire goods. Finally, Kocherlakota never addresses why the interest-bearing asset is illiquid; it is simply assumed to be illiquid. This issue is not a problem for us because in our environment the holders of interest-bearing debt instruments (depositors) do not want to consume so they are 'naturally' illiquid.

The mechanism in Shi (1996) and Cavalcanti and Wallace (1999a,b) are related in the sense that the allocation is improved because in each period some buyers are able to relax their cash constraint by issuing personal liabilities directly to sellers. However, the inefficiency created by idle cash balances is not eliminated. In contrast, in our model agents can either borrow to relax their cash constraints or lend their idle cash balances to earn interest. Contrary to these other models we find that the welfare gain is not due to relaxing buyers' cash constraints but comes from generating positive rates of returns on idle cash balances.[4]

A further key difference of our analysis from the existing literature is that since money is divisible we can study how changes in the growth rate of the money supply affects the allocation.[5] We also have competitive markets as opposed to bilateral matching and bargaining.[6] Unlike Cavalcanti and Wallace (1999a,b) and related models we do not have bank claims circulating as medium of exchange and goods market trading histories are unobservable for all agents. Finally, in contrast to He, Huang and Wright (2005), there is no security motive for depositing cash in the bank.

The paper proceeds as follows. Section 2 describes the environment and Section 3 the agents' decision problems. In Section 4 we derive the equilibrium

[4]This shows that being constrained is not per se a source of inefficiency. In any general equilibrium model all agents face a budget constraint. Nevertheless, the equilibrium is efficient because all gains from trade are exploited.

[5]Recently, Faig (2004) has also developed a model of banking in which money and goods are divisible but his banks serve a very different purpose than modeled here. Furthermore, his banks have records of goods markets trades between individuals, hence it is doubtful that money is essential in his model.

[6]Competitive pricing in the Lagos-Wright framework has been introduced by Rocheteau and Wright (2004) and further investigated in Berentsen, Camera and Waller (2005), Lagos and Rocheteau (2005), and Aruoba, Waller and Wright (2005).

when banks can force repayment at no cost and in Section 5 when punishment for a defaulter is permanent exclusion from the banking system. The last section concludes.

2 The Environment

The basic framework we use is the divisible money model developed in Lagos and Wright (2005). This model is useful because it allows us to introduce heterogenous preferences for consumption and production while still keeping the distribution of money balances analytical tractable.[7] Time is discrete and in each period there are two perfectly competitive markets that open sequentially. There is a $[0, 1]$ continuum of infinitely-lived agents and one perishable good produced and consumed by all agents.

At the beginning of the first market agents get a preference shock such that they can either consume or produce. With probability $1-n$ an agent can consume but cannot produce while with probability n the agent can produce but cannot consume. We refer to consumers as buyers and producers as sellers. Agents get utility $u(q)$ from q consumption in the first market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$. Furthermore, we impose that the elasticity of utility $e(q) = \frac{qu'(q)}{u(q)}$ is bounded. Producers incur utility cost $c(q) = q$ from producing q units of output. To motivate a role for fiat money, we assume that all goods trades are anonymous which means that agents cannot identify their trading partners. Consequently, trading histories of agents are private information and sellers require immediate compensation so buyers must pay with money.[8]

In the second market all agents consume and produce, getting utility $U(x)$ from x consumption, with $U'(x) > 0$, $U'(0) = \infty$, $U'(+\infty) = 0$ and $U''(x) \leq 0$. The difference in preferences over the good sold in the last market allows us

[7] An alternative framework would be Shi (1997) which we could amend with preference and technology shocks to generate the same results.

[8] There is no contradiction between assuming Walrasian markets and anonymity. To calculate the market clearing price, a Walrasian auctioneer only needs to know the aggregate excess demand function and not the identity of the individual traders.

to impose technical conditions such that the distribution of money holdings is degenerate at the beginning of a period. Agents can produce one unit of the consumption good with one unit of labor which generates one unit of disutility. The discount factor across dates is $\beta \in (0,1)$.

We assume a central bank exists that controls the supply of fiat currency. The growth rate of the money stock is given by $M_t = \gamma M_{t-1}$ where $\gamma > 0$ and M_t denotes the per capita money stock in t. Agents receive lump sum transfers $\tau M_{t-1} = (\gamma - 1) M_{t-1}$ over the period. Some of the transfer is received at the beginning of market 1 and some during market 2. Let τ_1 and τ_2 denote the transfers in market 1 and 2 respectively with $(\tau_1 + \tau_2) M_{t-1} = \tau M_{t-1}$. Moreover, $\tau_1 = (1-n)\tau_b + n\tau_s$ since the government might wish to treat buyers and sellers differently. This transfer scheme is merely an analytical device to see whether or not a government policy of differential lump sum transfers based on an individual's relative need for cash can replicate the same allocation that occurs with banking. For notational ease variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1.

If there is enforcement, the central bank can levy nominal taxes to extract cash from the economy, then $\tau < 0$ and hence $\gamma < 1$. Implicitly this means that the central bank can force agents to trade with it involuntarily. However, this does not mean that it can force agents to produce or consume certain quantities in the good markets nor does it mean that it knows the identity of the agents. If the central bank does not have this power, a lump-sum tax is simply not feasible and so $\gamma \geq 1$. We will derive the equilibrium for both environments.

Banks and record keeping. We model credit as financial intermediation done by perfectly competitive firms who accept nominal deposits and make nominal loans. For this process to work we assume that there is a technology for record keeping on financial histories but not trading histories in the goods market amongst the agents themselves. We call the firms that operate this record keeping technology banks and they can do so at zero cost. We call them banks because the financial intermediaries who perform these activities -

taking deposits, making loans, keeping track of credit histories - are classified as 'banks' by regulators around the world. Since record keeping can only be done for financial transactions but not good market transactions trade credit is not feasible.[9] Record keeping does not imply that banks can issue tangible objects as inside money. Hence, we assume that there are no bank notes in circulation. This ensures that outside fiat currency is still used as a medium of exchange in the goods market.[10] Finally, we assume that loans and deposits are not rolled over. Consequently, all financial contracts are one-period contracts. One-period debt contracts are optimal in these environments because of the quasi-linear preferences. Unlike standard dynamic contracting models, with linear disutility of production in market 2, there is no gain from spreading out repayment of loans or redemption of deposits across periods in order to smooth the disutility of production.[11]

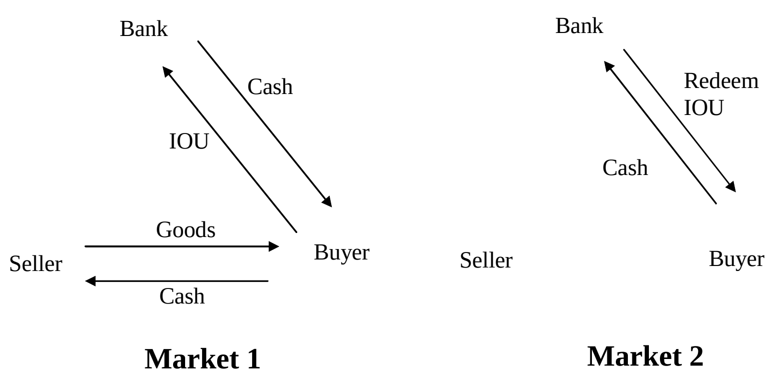

Figure 1. Cash and Credit.

Although all goods transactions require money, buyers do not face a standard cash-in-advance constraint. Before trading they can borrow cash from the

[9]There is also no collateral in our model so bilateral trade credit cannot be supported as in Shi (1996).

[10]Alternatively, we could assume that our banks issue their own currencies but there is a 100 percent reserve requirement in place. In this case the financial system would be similar to narrow banking (Wallace 1996).

[11]In steady state, the LW framework turns the economy into a sequence of repeated static problems. Hence, a one period contract is sufficient to deal with any trading frictions occurring within the period.

bank to supplement their money holdings but do so at the cost of the nominal interest rate as illustrated in Figure 1 which describes the flow of goods, credit and money in our model in markets 1 and 2. Note the absence of links between the seller and the bank. The missing link is consistent with the assumption that there is no record-keeping in the goods market. For example, it rules out the following mechanism. At the end of each period, every agent reports to the bank the identity of the trading partners and the quantities of trade. If the report of an agent does not match the report of his trading partner, then the bank punishes both agents by excluding them from the banking system. This mechanism requires that the agents in the good market can accurately identify their trading partners which violates our assumption of anonymity.

Default. In any model of credit, default is a serious issue. We first assume that banks can force repayment at no cost. In such an environment, default is not possible so agents face no borrowing constraints. This allows us to focus on how the provision of liquidity via borrowing and lending affects the allocation. In this case, banks are nothing more than cash machines that post interest rates for deposits and loans. In equilibrium these posted interest rates clear the market. We then consider an environment where banks cannot enforce repayment. The only punishment available is that a borrower who fails to repay his loan is excluded from the banking sector in all future periods. Given this punishment, we derive conditions to ensure voluntary repayment and show that this may involve imposing binding borrowing constraints, i.e. credit rationing.

3 Symmetric equilibrium

The timing in our model is as follows. At the beginning of the first market agents observe their production and consumption shocks and they receive the lump-sum transfers τ_1. Then, the banking sector opens and agents can borrow and deposit money. Finally, the banking sector closes and agents trade goods. In the second market agents trade goods and settle financial claims.

In period t, let ϕ be the real price of money in the second market. We study equilibria where end-of-period real money balances are time-invariant

$$\phi M = \phi_{-1} M_{-1} = \Omega \tag{1}$$

which implies that $\frac{\phi_{-1}}{\phi} = \gamma$. We refer to it as a stationary equilibrium.

Consider a stationary equilibrium. Let $V(m_1)$ denote the expected value from trading in market 1 with m_1 money balances conditional on the aggregate shock. Let $W(m_2, l, d)$ denote the expected value from entering the second market with m_2 units of money, l loans, and d deposits. In what follows, we look at a representative period t and work backwards from the second to the first market.

3.1 The second market

In the second market agents produce h goods and consume x, repay loans, redeem deposits and adjust their money balances. If an agent has borrowed l units of money, then he pays $(1+i)\,l$ units of money, where i is the nominal loan rate. If he has deposited d units of money, he receives $(1+i_d)\,d$, where i_d is the nominal deposit rate. The representative agent's program is

$$W(m_2, l, d) = \max_{x,h,m_{1,+1}} \left[U(x) - h + \beta V(m_{1,+1})\right] \tag{2}$$

$$\text{s.t.} \quad x + \phi m_{1,+1} = h + \phi\,(m_2 + \tau_2 M_{-1}) + \phi\,(1+i_d)\,d - \phi\,(1+i)\,l$$

where $m_{1,+1}$ is the money taken into period $t+1$ and ϕ is the real price of money. Rewriting the budget constraint in terms of h and substituting into (2) yields

$$\begin{aligned} W(m_2, l, d) &= \phi\left[m_2 + \tau_2 M_{-1} - (1+i)\,l + (1+i_d)\,d\right] \\ &+ \max_{x,m_{1,+1}} \left[U(x) - x - \phi m_{1,+1} + \beta V(m_{1,+1})\right]. \end{aligned}$$

The first-order conditions are $U'(x) = 1$ and

$$\phi = \beta V'(m_{1,+1}) \tag{3}$$

where $V'(m_{1,+1})$ is the marginal value of an additional unit of money taken into period $t+1$.

Notice that the optimal choice of x is the same across time for all agents and the $m_{1,+1}$ is independent of m_2. As a result, the distribution of money holdings is degenerate at the beginning of the following period.

The envelope conditions are

$$W_m = \phi \tag{4}$$

$$W_l = -\phi\,(1+i) \tag{5}$$

$$W_d = \phi\,(1+i_d)\,. \tag{6}$$

Finally, to ensure that all loans can be feasibly repaid in market two, let v be the ratio of aggregate nominal loan repayments, $(1-n)(1+i)\,l$, to the money stock, M. If $v \leq 1$, then borrowers can repay their loans with one trip to the bank only since the nominal demand for cash by borrowers for repayment of loans is less than M. If $v > 1$, borrowers cannot acquire sufficient balances in the aggregate to repay loans at once. This implies that they repay part of their loans which is then used to settle deposit claims and the cash reenters the goods market as depositors use the cash to acquire more goods. This recycling of cash occurs until all claims are settled.

3.2 The first market

Let q_b and q_s respectively denote the quantities consumed by a buyer and produced by a seller trading in market 1. Let p be the nominal price of goods in market 1. It is straightforward to show that agents who are buyers will never deposit funds in the bank and sellers will never take out loans. Thus, $l_s = d_b = 0$. In what follows we let l denote loans taken out by buyers and d deposits of sellers. We also drop these arguments in $W(m, l, d)$ where relevant for notational simplicity.

An agent who has m_1 money at the opening of the first market has expected lifetime utility

$$\begin{aligned} V\,(m_1) = \quad & (1-n)\left[u\,(q_b) + W\,(m_1 + \tau_b M_{-1} + l - pq_b, l)\right] \\ & + n\left[-c\,(q_s) + W\,(m_1 + \tau_s M_{-1} - d + pq_s, d)\right] \end{aligned} \tag{7}$$

where pq_b is the amount of money spent as a buyer, and pq_s the money received as a seller. Once the preference shock occurs, agents become either a buyer or a seller.

Sellers' decisions. If an agent is a seller in the first market, his problem is

$$\max_{q_s,d} \left[-c\left(q_s\right) + W\left(m_1 + \tau_s M_{-1} - d + pq_s, d\right)\right]$$
$$s.t.\;\; d \leq m_1 + \tau_s M_{-1}$$

The first-order conditions are

$$-c'\left(q_s\right) + pW_m = 0$$
$$-W_m + W_d - \lambda_d = 0.$$

where λ_d is the Lagrangian multiplier on the deposit constraint. Using (4), the first condition reduces to

$$c'\left(q_s\right) = p\phi. \tag{8}$$

Sellers produce such that the ratio of marginal costs across markets $\left(c'\left(q_s\right)/1\right)$ is equal to the relative price $(p\phi)$ of goods across markets. Due to the linearity of the envelope conditions, q_s is independent of m_1 and d. Consequently, sellers produce the same amount no matter how much money they hold or what financial decisions they make. Finally, it is straightforward to show that for any $i_d > 0$ the deposit constraint is binding and so sellers deposit all their money balances.

Buyers' decisions. If an agent is a buyer in the first market, his problem is:

$$\max_{q_b,l} \left[u\left(q_b\right) + W\left(m_1 + \tau_b M_{-1} + l - pq_b, l\right)\right]$$
$$\text{s.t.}\quad pq_b \leq m_1 + \tau_b M_{-1} + l$$
$$l \leq \bar{l}$$

Notice that buyers cannot spend more cash than what they bring into the first market, m_1, plus their borrowing, l, and the transfer $\tau_b M_{-1}$. He also faces the constraint that the loan size is bounded above by $\bar{l}$. For now we assume that this constraint is arbitrary. However, it is intended to capture the fact that there is a threshold $\bar{l}$ above which an agent would choose to default.

Using (4), (5) and (8) the buyer's first-order conditions reduce to

$$u'(q_b) = c'(q_s)(1 + \lambda/\phi) \tag{9}$$

$$\phi i = \lambda - \lambda_l \tag{10}$$

where λ is the multiplier on the buyer's cash constraint and λ_l on the borrowing constraint. If $\lambda = 0$, then (9) reduces to $u'(q_b) = c'(q_s)$ implying trades are efficient.[12]

For $\lambda > 0$, these first-order conditions yield

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i + \lambda_l/\phi$$

If $\lambda_l = 0$, then

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i \tag{11}$$

In this case the buyer borrows up to the point where the marginal benefit of borrowing equals the marginal cost. He spends all his money and consumes $q_b = (m_1 + \tau_b M_{-1} + l)/p$. Note that for $i > 0$ trades are inefficient. In effect, a positive nominal interest rate acts as tax on consumption.

Finally, if $\lambda_l > 0$

$$\frac{u'(q_b)}{c'(q_s)} > 1 + i \tag{12}$$

In this case the marginal value of an extra unit of a loan exceeds the marginal cost. Hence, a borrower would be willing to pay more than the prevailing loan rate. However, if banks are worried about default, then the interest rate may not rise to clear the market and credit rationing occurs. Consequently, the buyer borrows $\bar{l}$, spends all of his money and consumes $q_b = \left(m_1 + \tau_b M_{-1} + \bar{l}\right)/p$.

Since all buyers enter the period with the same amount of money and face the same problem q_b is the same for all of them. The same is true for the sellers. Market clearing implies

$$q_s = \frac{1-n}{n} q_b. \tag{13}$$

so that efficiency is achieved at the quantity q^* solving $u'(q^*) = c'\left(\frac{1-n}{n} q^*\right)$.

[12] With $1-n$ buyers and n sellers, the planner maximizes $(1-n)\, u(q_b) - nc(q_s)$ s.t. $(1-n)\, q_b = nq_s$. Use the constraint to replace q_s in the maximand. The first-order condition for q_b is $u'(q_b) = c'(q_s)$.

Banks. Banks accept nominal deposits, paying the nominal interest rate i_d, and make nominal loans l at nominal rate i. The banking sector is perfectly competitive with free entry, so banks take these rates as given. There is no strategic interaction among banks or between banks and agents. In particular, there is no bargaining over terms of the loan contract. Finally, we assume that there are no operating costs or reserve requirements.

The representative bank solves the following problem per borrower

$$\max_{l} \ (i - i_d)\, l$$
$$\text{s.t. } l \leq \bar{l}$$
$$u(q_b) - (1+i)\, l\phi \geq \Gamma$$

where Γ is the reservation value of the borrower. The reservation value is the borrower's surplus from receiving a loan at another bank. We investigate two assumptions about the borrowing constraint $\bar{l}$. In the first case, banks can force repayment at no cost. In this case the borrowing constraint is exogenous and we set it to $\bar{l} = \infty$. In the second case, we assume that a borrower who fails to repay his loan will be shut out of the banking sector in all future periods. Given this punishment, the borrowing constraint is endogenous and we need to derive conditions to ensure voluntary repayment.

The first-order condition is

$$i - i_d - \lambda_L + \lambda_\Gamma \left[u'(q_b) \frac{dq_b}{dl} - (1+i)\,\phi \right] = 0$$

where $\lambda_L \geq 0$ is the Lagrange multiplier on the lending constraint and λ_Γ on the participation constraint of the borrower. Profit maximization implies $\lambda_\Gamma > 0$.

With free entry banks make zero profits so $i = i_d$. Since, $\frac{dq_b}{dl} = \phi / c'(q_s)$ we have

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i + \frac{\lambda_L}{\lambda_\Gamma \phi}.$$

If repayment is not an issue, $\lambda_L = 0$ and so the loan offered by the bank satisfies (11). If the constraint on the loan size is binding, i.e. $\lambda_L > 0$, it satisfies (12).

In a symmetric equilibrium all buyers borrow the same amount, l, and sellers deposit the same amount, d, where profit maximization implies

$$(1-n)\,l = nd. \tag{14}$$

Marginal value of money. Using (7) the marginal value of money is

$$V'(m_1) = (1-n)\frac{u'(q_b)}{p} + n\phi(1+i_d).$$

In the appendix we show that the value function is concave in m so the solution to (3) is well defined.

Using (8) $V'(m_1)$ reduces to

$$V'(m_1) = \phi\left[(1-n)\frac{u'(q_b)}{c'(q_s)} + n(1+i_d)\right]. \tag{15}$$

Note that banks increase the marginal value of money because sellers can deposit idle cash and earn interest. This is captured by the second term on the right-hand side. If there are no banks, this term is just n.

4 Equilibrium with enforcement

In this section as a benchmark we assume that the monetary authority can impose lump-sum taxes and that banks can force repayment of loans at no cost. This does not imply that the banks or the monetary authority can dictate the terms of trade between private agents in the goods market.

In any stationary monetary equilibrium use (3) lagged one period to eliminate $V'(m_1)$ from (15). Then use (1) and (13) to get

$$\frac{\gamma-\beta}{\beta} = (1-n)\left[\frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} - 1\right] + ni_d. \tag{16}$$

The right-hand side measures the value of bringing one extra unit of money into the first market. The first term reflects the net benefit (marginal utility minus marginal cost) of spending the unit of money on goods when a buyer and the second term is the value of depositing an extra unit of idle balances when a seller.

Since banks can force agents to repay their loans, agents are unconstrained and so $\bar{l} = \infty$. This implies that (11) holds. Using it in (16) yields[13]

$$\frac{\gamma - \beta}{\beta} = (1 - n)\, i + n i_d. \tag{17}$$

Now, the first term on the right-hand side reflects the interest saving from borrowing one less unit of money when a buyer.

Zero profit implies $i = i_d$ and so

$$\frac{\gamma - \beta}{\beta} = i. \tag{18}$$

We can rewrite this in terms of q_b using (11) to get

$$\frac{\gamma - \beta}{\beta} = \frac{u'(q_b)}{c'\left(\frac{1-n}{n} q_b\right)} - 1. \tag{19}$$

Definition 1 *When repayment of loans can be enforced, a monetary equilibrium with credit is an interest rate i satisfying (18) and a quantity q_b satisfying (19).*

Proposition 1 *Assume repayment of loans can be enforced. Then if $\gamma > \beta$, a unique monetary equilibrium with credit exists. Equilibrium consumption is decreasing in γ, and satisfies $q_b < q^*$ with $q_b \to q^*$ as $\gamma \to \beta$.*

It is clear from (19) that money is neutral, but not super-neutral. Increasing its stock has no effect on q_b, while changing the growth rate γ does. Moreover, the Friedman rule ($\gamma = \beta$) generates the first-best allocation.

How does this allocation differ from the allocation in an economy without credit? Let q_b^n denote the quantity consumed in such an economy. It is straightforward to show that q_b^n solves (16) with $i_d = 0$, i.e.,

$$\frac{\gamma - \beta}{\beta} = (1 - n) \left[\frac{u'(q_b^n)}{c'\left(\frac{1-n}{n} q_b^n\right)} - 1 \right]. \tag{20}$$

Comparing (20) to (19), it is clear that $q_b^n < q_b$ for any $\gamma > \beta$.

[13] This equation implies that if nominal bonds could be traded in market 2, then their nominal rate of return i_b would be $(1 - n)\, i + n i_d$. For $\mu = 0$ the return would be $i = i_d$. Thus agents would be indifferent between holding a nominal bond or holding a bank deposit.

Corollary 1 *For $\gamma > \beta$ financial intermediation improves the allocation and welfare.*

The key result of this section is that financial intermediation improves the allocation away from the Friedman rule. The greatest impact on welfare is for moderate values of inflation. The reason is that near the Friedman rule there is little gain from redistributing idle cash balances while for high inflation rates money is of little value anyway. At the Friedman rule agents can perfectly self-insure against consumption risk because the cost of holding money is zero. Consequently, there is no welfare gain from financial intermediation.

Given that banks improve the allocation away from the Friedman rule, is it because they relax borrowers' liquidity constraints or because they allow payment of interest to depositors? The following Proposition answers this question.

Proposition 2 *The gain in welfare from financial intermediation is due to the fact that it allows payment of interest to depositors and not from relaxing borrowers' liquidity constraints*

According to Proposition 2 the gain in welfare comes from payment of interest to agents holding idle balances. To prove this claim we show in the proof of Proposition 2 that in equilibrium, agents are indifferent between borrowing to finance equilibrium consumption or holding the amount of cash that allows them to acquire the same amount without borrowing. The only importance of borrowing is to sustain payment of interest to depositors. That is, even though each individual agent is indifferent between borrowing and not borrowing, agents taking out loans are needed to finance the interest received by the depositors.

As a final proof of this argument, we consider a systematic government policy that redistributes cash in market 1 by imposing lump-sum taxes on sellers and giving the cash as lump-sum transfers to buyers. This clearly relaxes the liquidity constraints of the buyers while paying no interest to depositors. However, inspection of (20) reveals that neither τ_1 nor τ_b appear in this equation. Hence, varying the transfer across the two markets or by redistributing cash from sellers to buyers in a lump-sum fashion has no effect on q_b^n. It only affects the equilibrium price of money in the last market. Agents simply change the amount of money they bring into market 1 and so the demand for money changes in market 2 which alters the price of money. Note also that this implies that the allocation with credit cannot be replicated by government policies using lump-sum transfers or taxes.

Finally, we have assumed Walrasian prices in market 1 but one can also assume bilateral matching and bargaining. In an earlier version of this paper we showed that the results are qualitatively the same - financial intermediation improves the allocation and the Friedman rule is the best monetary policy. Thus the existence of a credit market increases output and welfare regardless of the pricing mechanism.[14]

5 Equilibrium without enforcement

In the previous section enforcement occurred in two occasions. First, the monetary authority could impose lump-sum taxes. Second, banks could force repayment of loans. Here, we assume away any enforcement.

The implications are that the monetary authority cannot run a deflation. Consequently, $\gamma \geq 1$. Second, since production is costly, those who borrow in market 1 have an incentive to default in market 2. To offset this short-run benefit we assume that if an agent defaults on his loan then the only punishment is permanent exclusion from the banking system. This is consistent with the requirement that all trades are voluntary since banks can refuse to trade with private agents. Furthermore, it is in the banks' best interest to share information about agents' repayment histories.

For credit to exist, it must be the case that borrowers prefer repaying loans to being banished from the banking system. Given this punishment, the borrowing constraint $\bar{l}$ is endogenous and we need to derive conditions to ensure voluntary repayment. In what follows, since the transfers only affect

[14] A similar result is found by Faig and Huangfu (2004) in a model of competitive search in which market-makers can charge differential entry fees for buyers and sellers.

prices, we set $\tau_b = \tau_s = \tau_1 > 0$.

For buyers entering the second market with no money and who repay their loans, the expected discounted utility in a steady state is

$$\mathcal{U} = U(x^*) - h_b + \beta V(m_{1,+1})$$

where h_b is a buyer's production in the second market if he repays his loan.

Consider the case of a buyer who reneges on his loan. The benefit of reneging is that he has more leisure in the second market because he does not work to repay the loan. The cost is that he is out of the banking system, meaning that he cannot borrow or deposit funds for the rest of his life. He cannot lend because the bank would confiscate his deposits to settle his loan arrears. Thus, a deviating buyer's expected discounted utility is

$$\widehat{\mathcal{U}} = U(\widehat{x}) - \widehat{h}_b + \beta \widehat{V}(\hat{m}_{1,+1})$$

where the hat indicates the optimal choice by a deviator. The value of being in the banking system $\mathcal{U}$ as well as the expected discounted utility of defection $\widehat{\mathcal{U}}$ depend on the growth rate of the money supply γ. This puts constraints on γ that the monetary authority can impose without destroying financial intermediation.

Existence of a monetary equilibrium with credit requires that $\mathcal{U} \geq \widehat{\mathcal{U}}$, where the borrowing constraint $\bar{l}$ satisfies

$$\mathcal{U} = \widehat{\mathcal{U}}. \tag{21}$$

In the proof of Proposition 3 we show that $\bar{l}$ satisfies

$$\bar{l} = \frac{\beta}{\phi(1+i)(1-\beta)} \left\{ (1-n)\Psi + c'(q_s)\left(\frac{\gamma - \beta}{\beta}\right)[\widehat{q}_b - (1-n)q_b] \right\} \tag{22}$$

where $\Psi = u(q_b) - u(\widehat{q}_b) - c'(q_s)(q_b - \widehat{q}_b) \geq 0$. In an unconstrained equilibrium with credit we have $l \leq \bar{l}$ and in a constrained equilibrium $l = \bar{l}$.

In order to determine the sign of $\bar{l}$, we need to determine whether a deviator carries more money than a non-deviator since in (22) $\widehat{q}_b - (1-n)q_b = (\hat{m}_1 - m_1)/p$. This in turn depends on his degree of risk aversion. It is reasonable to assume that the deviator is sufficiently risk averse that he holds more real balances to compensate for the loss of consumption insurance provided by the banking system. In the appendix we show that a *sufficient* condition for this to be true is that the degree of relative risk aversion is greater than one. If this condition holds, then $\widehat{q}_b - (1-n) q_b > 0$ for any $\gamma \geq 1$ so $\bar{l}$ is non-negative.

In any equilibrium, profit maximization implies that banks lend out all of their deposits. This implies that (14) holds also in a constrained equilibrium, i.e.,

$$\bar{l} = \frac{n}{1-n} M.$$

To guarantee repayment in a constrained equilibrium banks charge a nominal loan rate, $\bar{\imath}$, that is below the market clearing rate.[15]

Definition 2 *A monetary equilibrium with unconstrained credit is a triple* $(q_b, \widehat{q}_b, i)$ *satisfying*

$$\frac{\gamma - \beta}{\beta} = (1-n) \left[\frac{u'(q_b)}{c'(q_s)} - 1 \right] + ni \tag{23}$$

$$\frac{\gamma - \beta}{\beta} = (1-n) \left[\frac{u'(\widehat{q}_b)}{c'(q_s)} - 1 \right] \tag{24}$$

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i \tag{25}$$

such that $0 < \phi l = nc'(q_s) q_b < \phi \bar{l}$, *where* $q_s = \frac{1-n}{n} q_b$.

Definition 3 *A monetary equilibrium with constrained credit is a triple* $(\bar{q}_b, \widehat{q}_b, \bar{\imath})$ *satisfying (23), (24) and (22) where* $nc'(\bar{q}_s) \bar{q}_b = \phi \bar{l}$ *and* $\bar{q}_s = \frac{1-n}{n} \bar{q}_b$.

Proposition 3 *Let the degree of risk aversion be greater than one. Then there exists a critical value* $\tilde{\beta}$ *such that if* $\beta \geq \tilde{\beta}$ *there is a* $\widetilde{\gamma} > 1$ *such that the*

[15] This may seem counter-intuitive since one would think that banks would reduce $\bar{l}$ to induce repayment. However, this cannot be an equilibrium since it would imply that banks are not lending out all of their deposits. If banks are not lending out all of their deposits then zero profits would require $i_d = (1-\mu) i$ where μ is the fraction of deposits held idle by the bank. If all banks were to choose a triple (i, i_d, μ) with $\mu > 0$ such that they earned zero profits, then a bank could capture the entire market and become a monopolist by raising i_d by an infinitesimal amount and lowering μ and i by an infinitesimal amount. Since all banks can do this, in a constrained equilibrium, the only feasible solution is $\mu = 0$ and $i = i_d = \bar{\imath}$.

following is true:
(i) If $\gamma > \widetilde{\gamma}$, a unique monetary equilibrium with unconstrained credit exists.
(ii) If $1 < \gamma \leq \widetilde{\gamma}$, a monetary equilibrium with constrained credit ***may*** *exist.*
(iii) If $\gamma = 1$, no monetary equilibrium with credit exists.

According to Proposition 3, existence of a monetary equilibrium with credit requires that there is some inflation. The reason for this is quite intuitive. If a borrower works to repay his loan in market 2, he is strictly worse off than when he defaults since the outside option (trading with money only) yields almost the efficient consumption q^* in all future periods. With zero inflation, agents are able to self-insure at low cost, thus having access to financial markets is of little value. As a result, borrowers will not repay their loans and so financial intermediation is impossible. This result is related to Aiyagari and Williamson (2000) who also report a break-down of financial intermediation in a dynamic contracting model with private information close to the Friedman rule.

For low rates of inflation credit rationing occurs. Again, in this case the cost of using money to self-insure is low. To induce repayment banks charge a below market-clearing interest rate since this reduces the total repayment the borrowers have to make. In short, with an endogenous borrowing constraint, the interest rate is lower than would occur in an economy where banks can force repayment.[16]

One aspect that is puzzling about this result is that the incentive to default is *higher* for low nominal interest rates and *lower* for high nominal interest rates. This seems counter-intuitive at first glance since standard credit-rationing models, such as Stiglitz and Weiss (1981), suggest that the likelihood of default increases as interest rates rise. The reason for the difference in results is that standard credit rationing models focus on *real* interest rates, while our model is concerned with *nominal* interest rates. In our model, nominal rates rise because of perfectly anticipated inflation, which acts as a tax on deviators' wealth since they carry more money for transactions purposes. This reduces the incentive to default thereby alleviating the need to ration credit. Conse-

[16]This result is similar to results reported in Kehoe and Levine (1993), Alvarez and Jermann (2000) or Hellwig and Lorenzoni (2004).

quently, a key contribution of our analysis is to show how credit rationing can arise from changes in *nominal* interest rates.

Corollary 2 *If a monetary equilibrium with constrained credit exists, consumption and welfare can be increasing in the growth rate of the money supply* γ.

When banks have to worry about default, we obtain an interesting welfare result. For $\gamma > \widetilde{\gamma}$ the monetary equilibrium with unconstrained credit displays the standard property that consumption and welfare are decreasing in γ. In contrast, consumption and welfare are increasing in γ for the constrained equilibrium. The reason is that at low rates of inflation an increase of the inflation rate reduces the incentive to default. This reduces credit rationing and allows the credit market to perform more efficiently thereby expanding consumption.

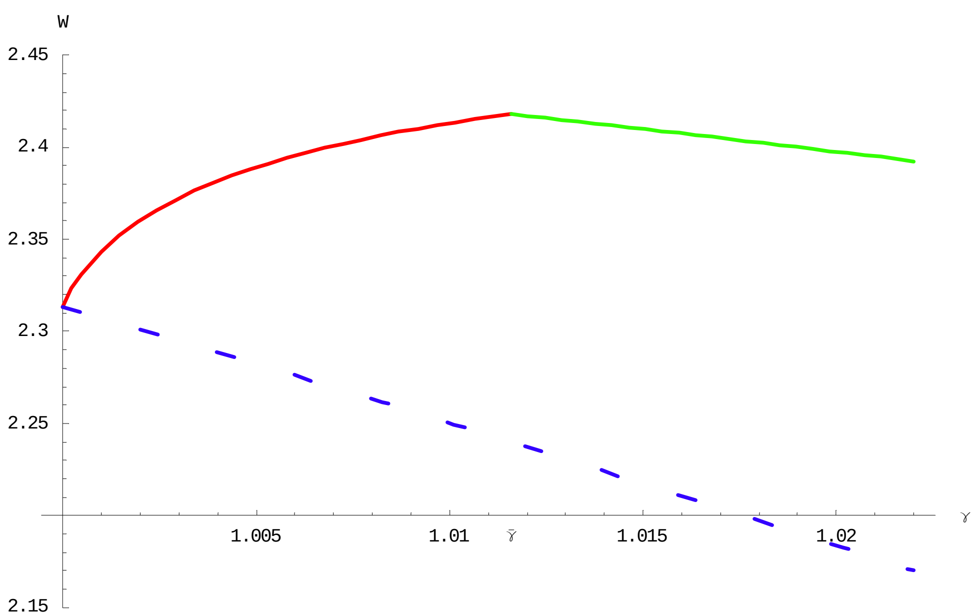

Figure 2: Welfare with endogenous borrowing constraint.

Figure 2 is a numerical example of our economy in which the constrained credit equilibrium exists and is unique. As before, the existence of credit leads to higher welfare than a regime without credit (displayed by the dotted blue line) even when credit is rationed. It also illustrates that with credit welfare rises with inflation for the constrained equilibrium while it falls in the unconstrained equilibrium.

6 Conclusion

In this paper we have shown how money and credit can coexist in an essential model of money. Our main findings are that reallocating idle cash via a banking system can expand output and improve welfare away from the Friedman rule but not at the Friedman rule. Furthermore, such an improvement cannot be achieved through a government policy of lump-sum taxes and transfers. Interestingly, banks have there greatest impact on welfare for moderate rates of inflation. Also, when voluntary repayment is a problem, credit rationing may arise and in this situation, inflation can improve welfare.

Our framework is open to many extensions such as private bank note issue, financing of investment instead of consumption, and longer term financial contracts. We could also extend the model to investigate the role of banks in transmitting other type of shocks such as productivity shocks or preference shocks. Finally, the interaction of government regulation and stabilization policies would allow analysis of different monetary arrangements such as expressed by the real-bill doctrine or the quantity theory as studied in Sargent and Wallace (1982).

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Appendix

Proof that $V(m_1)$ **is concave** $\forall m$**.** Differentiating (7) with respect to m_1

$$\begin{aligned} V'(m_1) = (1-n)\left[u'(q_b)\frac{\partial q_b}{\partial m_1} + W_m\left(1 - p\frac{\partial q_b}{\partial m_1} + \frac{\partial l}{\partial m_1}\right) + W_l\frac{\partial l}{\partial m_1}\right] \\ +n\left[-c'(q_s)\frac{\partial q_s}{\partial m_1} + W_m\left(1 + p\frac{\partial q_s}{\partial m_1} - \frac{\partial d}{\partial m_1}\right) + W_d\frac{\partial d}{\partial m_1}\right] \end{aligned}$$

Recall from (4), (5), and (6) that $W_m = \phi$, $W_l = -\phi(1+i)$ and $W_d = \phi(1+i_d)$ $\forall m_2$. Furthermore, $\frac{\partial q_s}{\partial m_1} = 0$ because the quantity a seller produces is independent of his money holdings. We also know that $\frac{\partial d}{\partial m_1} = 1$ since a seller deposits all his cash when $i > 0$. Hence,

$$\begin{aligned} V'(m_1) = \quad & (1-n)\left[u'(q_b)\frac{\partial q_b}{\partial m_1} + \phi\left(1 - p\frac{\partial q_b}{\partial m_1} + \frac{\partial l}{\partial m_1}\right) - \phi(1+i)\frac{\partial l}{\partial m_1}\right] \\ & +n\phi(1+i_d) \end{aligned}$$

Since $i > 0$ implies $pq_b = m_1 + \tau_b M_{-1} + l$ we have $1 - p\frac{\partial q_b}{\partial m_1} + \frac{\partial l}{\partial m_1} = 0$. Hence[17]

$$V'(m_1) = (1-n)\frac{u'(q_b)}{p} + n\phi(1+i_d)$$

In a symmetric equilibrium $q_s = \frac{1-n}{n}q_b$. Define $m^* = pq^*$. Then if $m_1 < m^*$, $0 < q_b < q^*$, implying $\frac{\partial q_b}{\partial m_1} > 0$ so that $V''(m_1) < 0$. If $m_1 \geq m^*$, $q_b = q^*$ implying $\frac{\partial q_b}{\partial m_1} = 0$, so that $V''(m_1) = 0$. Thus, $V(m_1)$ is concave $\forall m$. ■

Proof of Proposition 1. Because $u(q)$ is strictly concave there is a unique value q that solves (11), and for $\gamma > \beta$, $q < q^*$ where q^* is the efficient quantity solving $u'(q^*) = c'\left(\frac{1-n}{n}q^*\right)$. As $\gamma \to \beta$, $u'(q) \to c'\left(\frac{1-n}{n}q\right)$, $q \to q^*$, and from (18) $i \to 0$. In this equilibrium, the Friedman rule sustains efficient trades in the first market. Since $V(m_1)$ is concave, then for $\gamma > \beta$, the choice m_1 is maximal.

We now derive equilibrium consumption and production in the second market. Recall that, due to idiosyncratic trade shocks and financial transactions, money holdings are heterogeneous after the first market closes. Therefore, if we set $m_1 = M_{-1}$, the money holdings of agents at the opening of the second market are $m_2 = 0$ for buyers and $m_2 = \frac{1}{n}(1+\tau_1)M_{-1}$ for sellers.

[17] Note that $u'(q_b)\frac{\partial q_b}{\partial m_1} - \phi(1+i)\frac{\partial l}{\partial m_1} = u'(q_b)\frac{\partial q_b}{\partial m_1} - \phi(1+i)\left[p\frac{\partial q_b}{\partial m_1} - 1\right]$
$= \frac{\partial q_b}{\partial m_1}(u'(q_b) - \phi(1+i)p) + \phi(1+i) = \phi(1+i) = \frac{u'(q_b)}{p}$.

Equation (3) gives us $x^* = U'^{-1}(1)$. The buyer's production in the second market can be derived as follows

$$h_b = x^* + \phi\{m_{1,+1} + (1+i)l - \tau_2 M_{-1}\} = x^* + c'(q_s)q_b + \phi il$$

since in equilibrium

$$m_{1,+1} = M = M_{-1} + \tau_1 M_{-1} + \tau_2 M_{-1} \text{ and } c'(q_s)q_b = \phi[(1+\tau_1)M_{-1} + l].$$

Thus, an agent who was a buyer in market 1 has to work to recover the production cost of his consumption and the interest on his loan. The seller's production is

$$\begin{aligned} h_s &= x^* + \phi\{m_{1,+1} - [pq_s + (1+\tau_1)M_{-1} + i_d d + \tau_2 M_{-1}]\} \\ &= x^* - c'(q_s)q_s - \phi i_d d \end{aligned}$$

The expected hours worked h satisfies

$$h = (1-n)h_b + nh_s = x^* \tag{26}$$

since in equilibrium $q_b = \frac{n}{1-n}q_s$ and $i(1-n)l = i_d nd$. Finally, hours in market 2 can be also expressed in terms of q as in the following table

Trading history:	Production in the last market:
Buy	$h_b = x^* + c'(q_s)(1-n)q_b + ne(q_b)u(q_b)$
Sell	$h_s = x^* - \frac{(1-n)}{n}[c'(q_s)(1-n)q_b + ne(q_b)u(q_b)]$

Since we assumed that the elasticity of utility $e(q_b)$ is bounded, we can scale $U(x)$ such that there is a value $x^* = U'^{-1}(1)$ greater than the last term for all $q_b \in [0, q^*]$. Hence, h_s is positive for for all $q_b \in [0, q^*]$ ensuring that the equilibrium exists. ■

Proof of Proposition 2. Assume that at some point in time t an agent at the beginning of market 2 chooses never to borrow again but continues to deposit. The first thing to note is that it is optimal for him to buy the same quantity q_b since his optimal choice still satisfies (19). This implies that his money balances are $\bar{m}_{1,+1} = m_{1,+1} + l_{+1}$. An agent who decides to opt out from borrowing has to carry more money but he saves the interest on loans in the future. In particular, consumption and production in the market 1 are not affected. The difference in lifetime payoffs come from difference in hours worked.

If he enters market 2 having been a buyer, the hours worked are

$$\begin{aligned}\bar{h}_b &= x^* + \phi\left\{m_{1,+1} + l_{+1} + (1+i)\,l - \tau_2 M_{-1}\right\} \\ &= x^* + c'(q_s)\,q_b + \phi l_{+1} + \phi i l\end{aligned}$$

while if he sold in market 1 he works

$$\begin{aligned}\bar{h}_s &= x^* + \phi\left\{m_{1,+1} + l_{+1} - [pq_s + (1+i)\,d + \tau_2 M_{-1}]\right\} \\ &= x^* - c'(q_s)\,q_s + \phi l_{+1} - \phi i_d d\end{aligned}$$

The expected hours worked satisfy

$$\bar{h} = (1-n)\,\bar{h}_b + n\bar{h}_s = x^* + \phi l_{+1}$$

Consequently, from (26) the additional hours worked are

$$\bar{h} - h = \phi l_{+1} = \gamma n c'(q_s)\,q_b > 0 \tag{27}$$

since $l_{+1} = \gamma l$ in a steady state.

Let us next consider the hours worked in market 2 in some future period. Since he has no loan to repay the hours worked are

$$\begin{aligned}\breve{h}_b &= x^* + \phi\left\{m_{1,+1} + l_{+1} - \tau_2 M_{-1}\right\} \\ &= x^* + \phi\left\{M_{-1} + \tau_1 M_{-1} + l + l_{+1} - l\right\} \\ &= x^* + c'(q_s)\,q_b + \phi l\,(\gamma - 1) \\ &= x^* + c'(q_s)\,q_b + nc'(q_s)\,q_b\,(\gamma - 1)\end{aligned}$$

if he was a buyer while if he sold in market 1 he works

$$\begin{aligned}\breve{h}_s &= x^* + \phi\left\{m_{1,+1} + l_{+1} - \left[pq_s + (1+i)\,\bar{d} + \tau_2 M_{-1}\right]\right\} \\ &= x^* + \phi\left\{M_{-1} + \tau_1 M_{-1} + l + l_{+1} - l - \left[pq_s + (1+i)\,\bar{d}\right]\right\} \\ &= x^* + c'(q_s)\,q_b + \phi l\,(\gamma - 1) - \phi\left[pq_s + (1+i)\,\bar{d}\right] \\ &= x^* + c'(q_s)\,q_b + \phi l\,(\gamma - 1) - c'(q_s)\,q_s - \phi\,(1+i)\,pq_b \\ &= x^* + nc'(q_s)\,q_b\,(\gamma - 1) - c'(q_s)\,q_s - ic'(q_s)\,q_b \\ &= x^* + nc'(q_s)\,q_b\,(\gamma - 1) - c'(q_s)\,q_s - (\gamma - \beta)\,c'(q_s)\,q_b/\beta\end{aligned}$$

The expected hours worked h satisfies

$$\breve{h} = (1-n)\,\breve{h}_b + n\breve{h}_s = x^* - nc'\left(q_s\right)q_b\gamma\left(1-\beta\right)/\beta$$

The expected gain from this strategy in any future period is

$$\breve{h} - h = -nc'\left(q_s\right)q_b\gamma\left(1-\beta\right)/\beta < 0 \tag{28}$$

Then, from (27) and (28), the total expected gain from this deviation is

$$\bar{h} - h + \frac{\beta\left(\breve{h} - h\right)}{1-\beta} = 0.$$

So agents are indifferent to borrow at the current rate of interest or taking in the equivalent amount of money themselves. ■

Proof of Corollary 1. Neither τ_1 nor τ_b appear in (19). Therefore, (τ_1, τ_2) can only affect the equilibrium ϕ. Of course, by changing τ_1 we change τ_2, for a given rate of growth of money. To see how the transfers affect ϕ note that $\phi l = c'(q_s)nq_b$. Since $l = \frac{n}{1-n}M_{-1}(1+\tau_s) = \frac{n}{1-n}M(\frac{1+\tau_s}{1+\tau})$ and $q_s = \frac{1-n}{n}q_b$ then we have

$$\phi = \frac{c'(q_s)nq_b}{l} = \frac{(1-n)c'(\frac{1-n}{n}q_b)\,(1+\tau)}{M(1+\tau_s)}$$

which implies the price of money in the second market, ϕ, is affected by the timing and size of lump-sum transfers. ■

Proof of Proposition 3. Since the transfers do not affect quantities set $\tau_b = \tau_s = \tau_1$. We now derive the endogenous borrowing constraint $\bar{l}$. This quantity is the maximal loan that a borrower is willing to repay in the second market at given market prices. For buyers entering the second market with no money, who repay their loans, the expected discounted utility in a steady state is

$$\mathcal{U} = U\left(x^*\right) - h_b + \beta V\left(m_{1,+1}\right)$$

where h_b is a buyer's production in the second market if he repays his loan.

A deviating buyer's expected discounted utility is

$$\widehat{\mathcal{U}} = U\left(\widehat{x}\right) - \widehat{h}_b + \beta\widehat{V}\left(\hat{m}_{1,+1}\right)$$

where the hat indicates the optimal choice by a deviator.

We now derive $\widehat{x}$, $\widehat{h}_b$, $\widehat{q}_b$, and $\widehat{q}_s$. In the last market the deviating buyer's program is

$$\widehat{W}(\hat{m}_2) = \max_{\widehat{x},\widehat{h},\hat{m}_{1,+1}} \left[U(\widehat{x}) - \widehat{h}_b + \beta \widehat{V}(\hat{m}_{1,+1})\right]$$
$$\text{s.t.} \quad \widehat{x} + \phi \hat{m}_{1,+1} = \widehat{h}_b + \phi(\hat{m}_2 + \tau_2 M_{-1})$$

As before, the first-order conditions are $U'(\widehat{x}) = 1$ and

$$-\phi + \beta \widehat{V}'(\hat{m}_{1,+1}) = 0. \tag{29}$$

Thus, $\widehat{x} = x^*$. The first-order condition if the deviator is a seller in the first market is $-c'(\widehat{q}_s) + p\phi = 0$. Hence, the deviator produces the same amount as non-deviating sellers so $\widehat{q}_s = q_s = \frac{1-n}{n} q_b$.

Finally, the marginal value of the money satisfies

$$\widehat{V}'(\hat{m}_1) = \phi \left[\frac{(1-n)\, u'(\widehat{q}_b)}{c'(q_s)} + n\right]$$

which means that (29) can be written as

$$\frac{\gamma - \beta}{\beta} = (1-n)\left[\frac{u'(\widehat{q}_b)}{c'(q_s)} - 1\right]. \tag{30}$$

Now if we compare (30) with (19) we find that

$$1 - n = \frac{u'(q_b) - c'(q_s)}{u'(\widehat{q}_b) - c'(q_s)} \tag{31}$$

implying $\widehat{q}_b < q_b$.

The money holdings of the deviator grows at rate γ since from (30) $\widehat{q}_b$ is constant across time. The money holdings of the deviator at the opening of the second market are $\hat{m}_2 = 0$ having bought and $\hat{m}_2 = \hat{m}_1 + \tau_1 M_{-1} + p q_s$ having sold. Thus, hours worked are

$$\widehat{h}_b = x^* + \phi\left[\hat{m}_{1,+1} - \tau_2 M_{-1}\right] = x^* + c'(q_s)\, \widehat{q}_b + \phi(\gamma - 1)(\hat{m}_1 - M_{-1}) \tag{32}$$

$$\begin{aligned}\widehat{h}_s &= x^* + \phi\left[\hat{m}_{1,+1} - (\hat{m}_1 + \tau_1 M_{-1}) - p\widehat{q}_s - \tau_2 M_{-1}\right] \\ &= x^* - c'(q_s)\, q_s + \phi(\gamma - 1)(\hat{m}_1 - M_{-1})\end{aligned} \tag{33}$$

since for a deviator $p\widehat{q}_b = \hat{m}_1 + \tau_1 M_{-1}$. The term $\phi(\gamma - 1)(\hat{m}_1 - M_{-1})$ reflects the fact that the deviator is subject to the inflation tax in a different way than the representative agent.

Since $pq_b = \frac{1}{1-n}(1+\tau_1) M_{-1}$ in equilibrium and $p\widehat{q}_b = \hat{m}_1 + \tau_1 M_{-1}$ it follows that $\phi(\gamma-1)(\hat{m}_1 - M_{-1}) = c'(q_s)(\gamma-1)[\widehat{q}_b - (1-n) q_b]$. Hence, the expected hours worked for a deviator are

$$\widehat{h} = x^* + c'(q_s)\{(1-n)(\widehat{q}_b - q_b) + (\gamma-1)[\widehat{q}_b - (1-n) q_b]\} \tag{34}$$

The endogenous borrowing constraint $\bar{l}$ satisfies $\mathcal{U}(\bar{l}) = \widehat{\mathcal{U}}$. To derive $\bar{l}$ note that for a buyer who took out a loan of size $\bar{l}$ the hours worked at night are

$$\begin{aligned} h_b &= x^* + \phi\left[m_{1,+1} + (1+i)\bar{l} - \tau_2 M_{-1}\right] \\ &= x^* + c'(q_s) q_b + \phi(1+i)\bar{l} - \phi l \end{aligned}$$

Hence from $\mathcal{U}(\bar{l}) = \widehat{\mathcal{U}}$ we have

$$U(x^*) - h_b + \beta V(m_{1,+1}) = U(\widehat{x}) - \widehat{h}_b + \beta \widehat{V}(\hat{m}_{1,+1})$$

where $\widehat{h}_b$ satisfies (32). Since $\widehat{x} = x^*$ we have

$$\widehat{h}_b - h_b = \beta\left[\widehat{V}(\hat{m}_{1,+1}) - V(m_{1,+1})\right] \text{ so}$$

$$\phi(1+i)\bar{l} = \gamma c'(q_s)[\widehat{q}_b - (1-n) q_b] + \beta\left[V(m_{1,+1}) - \widehat{V}(\hat{m}_{1,+1})\right] \tag{35}$$

since $\phi l = nc'(q_s) q_b$.

Finally, the continuation payoffs are

$$\begin{aligned} \widehat{V}(\hat{m}_{1,+1}) &= \sum_{t=0}^{\infty} \beta^t\left[(1-n) u(\widehat{q}_b) - nc(q_s) + U(x^*) - \widehat{h}\right] \\ V(m_{1,+1}) &= \sum_{t=0}^{\infty} \beta^t [(1-n) u(q_b) - nc(q_s) + U(x^*) - h]. \end{aligned}$$

In a steady state the difference is

$$\begin{aligned} V(m_{1,+1}) - \widehat{V}(\hat{m}_{1,+1}) &= \left\{(1-n)[u(q_b) - u(\widehat{q}_b)] + \widehat{h} - h\right\}(1-\beta)^{-1} \\ &= \{(1-n)\Psi + c'(q_s)(\gamma-1)[\widehat{q}_b - (1-n) q_b]\}(1-\beta)^{-1}. \end{aligned}$$

where $\Psi = u(q_b) - u(\widehat{q}_b) - c'(q_s)(q_b - \widehat{q}_b)$. Since

$$\frac{u(q_b) - u(\widehat{q}_b)}{q_b - \widehat{q}_b} > u'(q_b) > c'(q_s)$$

for all $\gamma > \beta$, it follows that $\Psi > 0$. Consequently, (35) can be written as

$$\bar{l} = \frac{\beta}{\phi(1+i)(1-\beta)} \left\{ (1-n)\Psi + c'(q_s)\left(\frac{\gamma-\beta}{\beta}\right)[\widehat{q}_b - (1-n)q_b] \right\}.$$

Unconstrained credit equilibrium. In an unconstrained equilibrium we have unique values of q_b and i. All that is left is to show is that $l \leq \bar{l}$, or

$$\phi(1+i)l = (1+i)nq_b \leq \frac{\beta}{1-\beta}\left\{ (1-n)\Psi + c'(q_s)\left(\frac{\gamma-\beta}{\beta}\right)[\widehat{q}_b - (1-n)q_b] \right\}$$

Since in an unconstrained equilibrium (11) and (20) hold, we need

$$nq_b u'(q_b)(1-\beta) \leq \beta(1-n)\{\Psi + [u'(\widehat{q}_b) - c'(q_s)][\widehat{q}_b - (1-n)q_b]\} \qquad (36)$$

We need to determine the sign of $\widehat{q}_b - (1-n)q_b$. Using (31) we obtain

$$\widehat{q}_b - (1-n)q_b = \frac{\widehat{q}_b u'(\widehat{q}_b) - q_b u'(q_b)}{u'(\widehat{q}_b) - c'(q_s)} + \frac{c'(q_s)(q_b - \widehat{q}_b)}{u'(\widehat{q}_b) - c'(q_s)}$$

Since $q_b \geq \widehat{q}_b$ for all γ, a *sufficient* condition for $\widehat{q}_b - (1-n)q_b \geq 0$ is that $qu'(q)$ is monotonically decreasing in q. This is equivalent to having $R(q) = -qu''(q)/u'(q) \geq 1$ for all $q \in [0, q^*]$. If this condition holds, then $\bar{l} \geq 0$ for any $\gamma \geq \beta$.

Define

$$g(\gamma) \equiv nq_b u'(q_b)(1-\beta)$$
$$f(\gamma) \equiv \beta(1-n)\{\Psi + [u'(\widehat{q}_b) - c'(q_s)][\widehat{q}_b - (1-n)q_b]\}$$

Then, $g(\beta) = nq^*(1-\beta) > 0$ and $f(\beta) = 0$. Thus, the repayment constraint is violated at the Friedman rule.

It can be shown that

$$f'(\gamma) = c'(q_s)[\widehat{q}_b - (1-n)q_b] - \frac{c''(q_s)c'(q_s)^2(1-n)^2 nq_b}{[nu''(q_b)c'(q_s) - u'(q_b)c''(q_s)(1-n)]} > 0$$
$$g'(\gamma) = n(1-\beta)u'(q_b)[1-R(q)]\frac{dq_b}{d\gamma} \geq 0$$

Since $g(\beta) > f(\beta)$, a sufficient condition for uniqueness, should an equilibrium exist, is $g'(\gamma) < f'(\gamma)$ for all γ or

$$n(1-\beta)[u'(q_b) + q_b u''(q_b)]\frac{dq_b}{d\gamma} <$$
$$\beta(1-n)u''(\hat{q}_b)[\widehat{q}_b - (1-n)q_b]\frac{d\widehat{q}_b}{d\gamma} - \beta(1-n)^2 c''(q_s)q_b\frac{dq_b}{d\gamma}.$$

We also have, from (31),

$$\frac{d\widehat{q}_b}{d\gamma} = \frac{u''(q_b) - c''(q_s)(1-n)}{(1-n)\,u''(\widehat{q}_b)}\frac{dq_b}{d\gamma} < 0.$$

Thus after some algebra we have

$$n(1-\beta)\,u'(q_b)\frac{dq_b}{d\gamma} <$$
$$\beta\left[\widehat{q}_b - (1-n)\,q_b - n\left(\frac{1-\beta}{\beta}\right)q_b\right]u''(q_b)\frac{dq_b}{d\gamma} - \beta\widehat{q}_b c''(q_s)(1-n)\frac{dq_b}{d\gamma}$$

The left hand side is negative. The second term on the RHS is positive. The first term will be non-negative for β close to one. Thus, for β sufficiently close to one, if an equilibrium exists it is unique.

To establish existence, note that as $\beta \to 1$, $g(\beta) \to f(\beta)$. Since $f'(\gamma) > g'(\gamma)$ for all γ as $\beta \to 1$, it then follows that for $\beta \in \left(\tilde{\beta}, 1\right)$ an equilibrium exists for all γ greater than a finite value $\tilde{\gamma}$ and it is unique.

Constrained credit equilibrium We now consider $1 \le \gamma < \tilde{\gamma}$. In a constrained equilibrium the defection constraint must hold with equality implying

$$(1+\bar{\imath})\,nc'(\bar{q}_s)\,\bar{q}_b = \frac{\beta}{1-\beta}\left\{(1-n)\,\Psi + c'(\bar{q}_s)\left(\frac{\gamma-\beta}{\beta}\right)[\widehat{q}_b - (1-n)\,\bar{q}_b]\right\} \tag{37}$$

where $\bar{q}_b$ denotes the quantity consumed and $\bar{\imath}$ is the interest rate in a constrained equilibrium. From the first-order conditions on money holdings we have

$$\frac{\gamma-\beta}{\beta} = (1-n)\left[\frac{u'(\bar{q}_b)}{c'(\bar{q}_s)} - 1\right] + n\bar{\imath} \tag{38}$$
$$\frac{\gamma-\beta}{\beta} = (1-n)\left[\frac{u'(\widehat{q}_b)}{c'(\bar{q}_s)} - 1\right] \tag{39}$$

where $\bar{q}_s = \frac{1-n}{n}\bar{q}_b$. Thus, a constrained equilibrium is a list $\{\bar{q}_b, \widehat{q}_b, \bar{\imath}\}$ such that (37)-(39) hold.

We now investigate the properties of (37)-(39). At $\bar{\imath} = 0$, from (38) and (39), $\bar{q}_b = \widehat{q}_b$. Then from (37) we have $\gamma = 1$. This implies there is one and only one monetary policy consistent with a nominal interest rate of zero in a constrained credit equilibrium. Taking the total derivative of (37)-(39) and evaluating it at $\gamma = 1$ (see below for the derivation) we find that

$$\frac{d\bar{\imath}}{d\gamma}|_{\gamma=1} > 0$$

These observations imply that for all $\gamma \in (1, \widetilde{\gamma}]$ we have $\bar{\imath} > 0$. It then follows that a constrained credit equilibrium can exist if and only if $\gamma \in (1, \widetilde{\gamma}]$. However, we cannot show existence of a constrained equilibrium in general.

Finally, we show that consumption can be increasing in γ in the constrained equilibrium. Taking the total derivative of (37)-(39) and evaluating it at $\gamma = 1$ we can show that

$$\frac{d\bar{q}_b}{d\gamma}|_{\gamma=1} > 0$$

if $\beta \geq \frac{2-n}{2-n^2}$ which is satisfied for a wide range of n when β is close to one. Thus, in the constrained equilibrium if agents are sufficiently patient, consumption can be increasing in γ. ■