Math 10350 Fall 07 – Handout 13
(Sections 4.2)

▶ Summation Notation

1. Assuming that the pattern in the sums below, write down (a) the formula for the general term, and (b) the sum using summation notation.

a. $\dfrac{5}{1+1^2}+\dfrac{5}{1+2^2}+\dfrac{5}{1+3^2}+...+\dfrac{5}{1+15^2}$

b. $\dfrac{5}{1+5^2}+\dfrac{5}{1+6^2}+\dfrac{5}{1+7^2}+...+\dfrac{5}{1+13^2}$

c. $\left(\dfrac{1}{n}\right)\sqrt{1-\left(\dfrac{0}{n}\right)^2}+...+\left(\dfrac{1}{n}\right)\sqrt{1-\left(\dfrac{n-1}{n}\right)^2}$

2. (Properties of summation notation) Show the following:

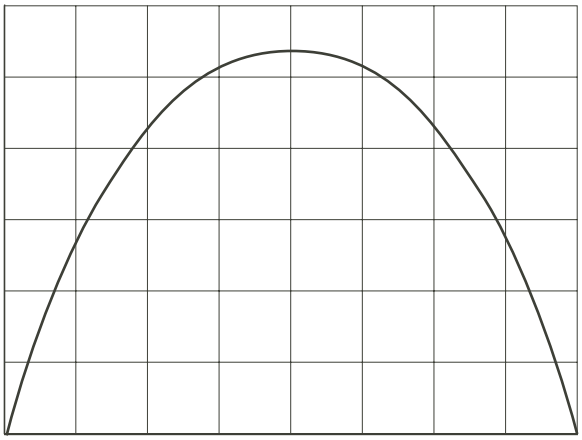
(A) $\displaystyle\sum_{k=1}^{n}(a_k+b_k)=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$

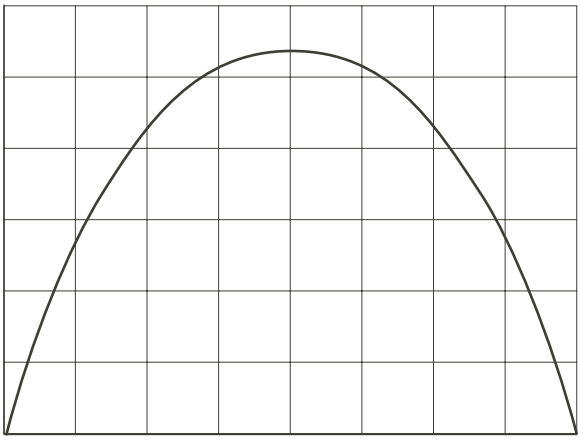
(B) $\displaystyle\sum_{k=1}^{n}(c\cdot a_k)=c\cdot\left(\sum_{k=1}^{n}a_k\right)$

▶ Summation Formulas

$$\sum_{i=1}^{n}c=cn; \qquad \sum_{i=1}^{n}i=\frac{n(n+1)}{2}; \qquad \sum_{i=1}^{n}i^2=\frac{n(n+1)(2n+1)}{6}; \qquad \sum_{i=1}^{n}i^3=\frac{n^2(n+1)^2}{4}$$

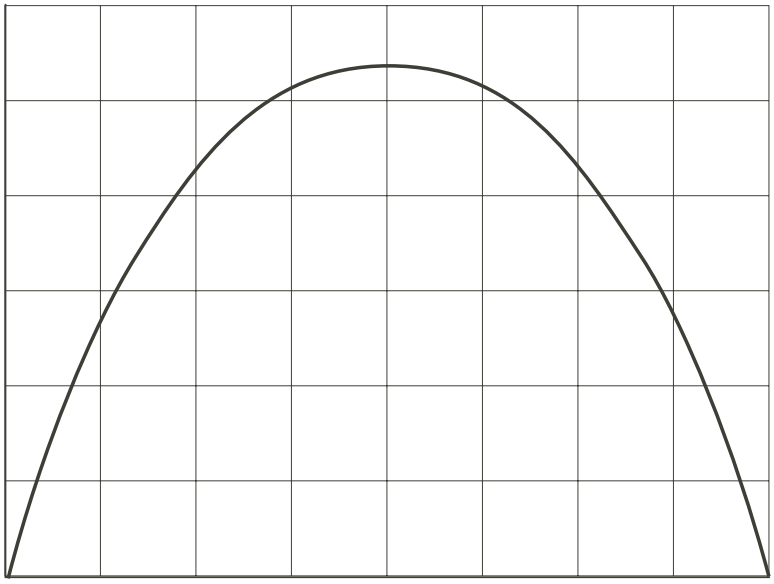
3. Consider the function $f(x)=4x-x^2$.





a. Use upper and lower sums to approximate the area of the region bounded by the graph of $f(x)$ and the x-axis using 8 subintervals of equal width. Such sums are called **Riemann Sum**.

b. Using limit process find the area of the region bounded by the graph of $f(x)$ and the x-axis.



Step 1: Cut up area in n equal strips. Take step size $\Delta x =$ _____. Write down formula for $x_i =$ _____ and range for i.

Step 2: Write down formula for $\Delta A_i =$ _____ = _____

Step 3: Write down the Riemann sum and take limit as $n \to \infty$.