Math 10350 Fall 07 – Handout 12
(Sections 4.1)

▶ **Antiderivatives** (Reversing differentiation)

Definition: We say that $F(x)$ is an **antiderivative** of $f(x)$ provided ________________.

Example 1 Verify that $x^2 + 5$ is an antiderivative of $2x$. Can you write down a few more antiderivative of $2x$? What did you notice? Explain graphically.

Remark: We denote the family of antiderivatives of $2x$ by ________________.

From Example 1, we see that

> **Theorem** If $F(x)$ and $G(x)$ are antiderivatives of the same function throughout an interval, then they differ by a constant c over that interval; that is, for $a < x < b$
> $F'(x) = G'(x) \quad \Rightarrow$
> for some number c.

Notation: If $F(x)$ is an antiderivative of $f(x)$, that is, $F'(x) = f(x)$. Then we may write

$$\int f(x)dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

We call $\int f(x)dx$ the **indefinite integral**.

▶ **Basic indefinite integral formulas**

- For any constant k: $\boxed{\int k\,dx \stackrel{?}{=} \qquad .}$ For Example: $\int 100dx \stackrel{?}{=}$
- Power Rule when $k \neq -1$: $\boxed{\int x^k dx \stackrel{?}{=} \qquad .}$ For Example: $\int x^9 dx \stackrel{?}{=}$
- Power Rule when $k = -1$: $\boxed{\int \frac{1}{x}\,dx = \qquad .}$
- Constant Multiple Rule: $\boxed{\int kf(x)dx = k\int f(x)dx, \text{ any } k}$ For Example: $\int \frac{8}{x^2}\,dx \stackrel{?}{=}$
- Sum Rule: $\boxed{\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx.}$

Example 2 Find each of the following indefinite integral. Check your answer by differentiation.

a. $\int \left(x^7 - 2x^{-4} + \frac{3}{x^2} + e^2 \right) dx$

b. $\int \frac{3x - 10x^2 + \sqrt{x}}{x^3}\,dx$

Example 3 Given that $\int f(x)dx = F(x)+C$ and $G'(x) = g(x)$. Find each of the following indefinite integral in terms of $F(x)$, $G(x)$, and other known functions whenever possible. If not possible state so.

a. $\int [2f(x) + 3x]\, dx$

b. $\int f(x) \cdot g(x)\, dx$

c. $\int \frac{5x^2 - 3x \cdot g(x)}{x}\, dx$

d. $\int \frac{f(x) + 3}{x^2}\, dx$

1. Write down as many integration (anti-differentiation) formula for the trigonometric functions as you can.

2. A ball is projected upward from the ground with an initial velocity of 3 m/sec. Using calculus, write the velocity and position for the ball at time t. You may assume that the acceleration due to gravity is 10 m/s^2.

3. Find the antiderivative F of function f satisfying the given condition:

$$f(x) = \frac{x^3 - 7x^{2/3} + 3}{\sqrt[3]{x}}; \qquad F(1) = 5$$

In other words, solve the initial value problem

$$F'(x) = \frac{x^3 - 7x^{2/3} + 3}{\sqrt[3]{x}}; \qquad F(1) = 5$$

4. Repeat Q3. with $f(x) = \sin x - \cos x$ with $F(0) = 1$.

5. Evaluate the following definite integrals:

a. $\int (1 + 3y - y^2)\, dy$

b. $\int \frac{\tan\theta}{\cos\theta}\, d\theta$

c. $\int \frac{u^{5/2} + 2u^2 + \sqrt[3]{u}}{u^2}\, du$

d. $\int \frac{1 + \cos^2\theta}{\cos^2\theta}\, d\theta$