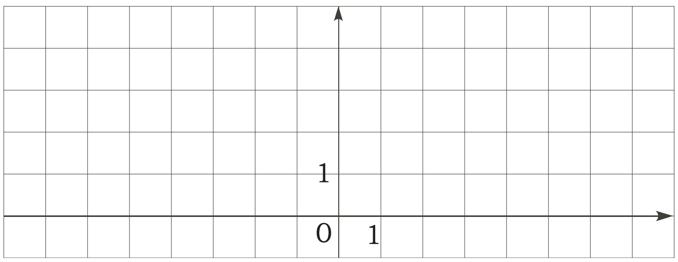
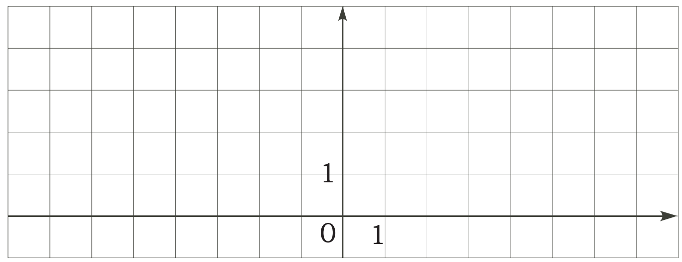
Math 10350 Fall 07 – Handout 10
(Sections 3.6 & 3.7)

1. Sketch the graphs of **two different** functions sharing the same properties below. The graphs should have at least one feature that is markedly different.

- $f'(x) < 0$ on $(-\infty, 0)$ or $(2, \infty)$.
- $f'(x) > 0$ on $(0, 2)$.
- $f(0) = 0$ and $f(2) = 4$.
- $f'(0) = 0$ but $f'(2)$ does not exist.
- $\lim_{x\to+\infty} f(x) = 2 = \lim_{x\to-\infty} f(x)$.





2. Sketch the graph of $g(x) = \dfrac{x}{x^2 - 4}$ by completing the steps below.

a. Find all x-intercepts and y-intercept of the graph of $g(x)$ whenever possible.

b. Find coordinates of all critical points, vertical asymptotes, and places where $g(x)$ are undefined.

c. Determine where $g(x)$ is increasing and where it is decreasing.

d. Determine the concavity and coordinates of inflection points of $g(x)$. $\left(g''(x) = \dfrac{(24 + 2x^2)x}{(x^2 - 4)^3} = \dfrac{24 + 2x^2}{(x^2 - 4)^2} \cdot \dfrac{x}{x^2 - 4}\right)$

e. Find all asymptotes and limit at infinity whenever applicable. Check for any symmetry.

f. Sketch the graph below labeling all important features. Your picture should be large and clear.

3a. Find the absolute (global) maximum and minimum of $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 1$ on the interval $[-1, 1]$.

3b. Using the steps below, find the global maximum and minimum of $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 1$ on $[-1, 3)$.

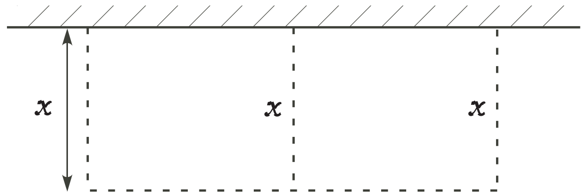
Step 1: Find all critical points in the domain of $f(x)$ and the values of $f(x)$ there. Classify them using first derivative test.

Step 2: Find the values of $f(x)$ at the end-points (if any) of its domain. ____________________

Step 3: If end-point not included, or $\pm\infty$, find all limits of $f(x)$ towards end of interval.

Step 4: Give a rough sketch of the graph of $f(x)$ clearly indicating where the global maximum and minimum are. Stating the global maximum and minimum of $f(x)$ on $[1, 3)$ if any.

4. A landscaper plans to use 120 m of fencing and a very wide straight wall to make two rectangular enclosures with the same dimensions as shown.



a. Write down the possible values of x.

b. Find the maximum value of the total area of the enclosures. What are the dimensions of each enclosure when maximum occurs?

5. The top and bottom margins of a poster are each $6\,cm$ and the side margins are each $4\,cm$. If the area of printed material on the poster is fixed at $384\,cm^2$, find the dimensions of the poster with the smallest area.

6. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3cm and 4cm if two sides of the rectangle lie along the legs.

7. A piece of wire $10\,m$ long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is (a) maximum and (b) minimum.

8. A cylindrical can without a top is made to contain $100\,cm^3$ of liquid. Find the dimension that will minimize the cost of the material to make the can if the material for the side costs $\$2/cm^2$ and the material for the base costs $\$3/cm^2$.

9. Show that of all the isosceles triangles with a perimeter of $30\,cm$, the one with the largest area is equilateral.

10. A house is located at a point H in the woods, 3 miles from the nearest point A on a road. A telephone switching station is located at point B on the road, 6 miles from A. The homeowner wants to run a telephone cable through the woods from H to P (where P is a point between A and B) and then along the road from P to B. The cost of laying the cable through the woods is three times as expensive per mile as it is along the road. Where should the point P be chosen to minimize the cost? (Ans: $3/(2\sqrt{2})$ miles from A)