Math 10350 Fall 07 – Handout 6
(Sections 3.1 & 3.2)

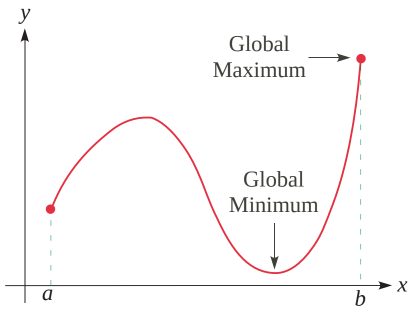

Figure 1

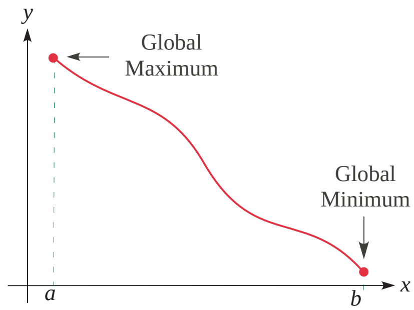

Figure 2

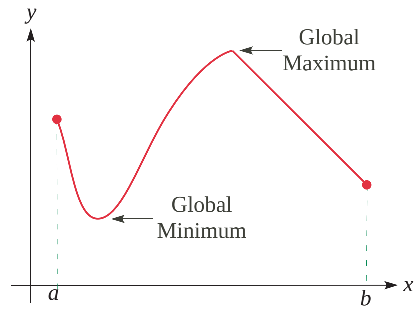

Figure 3

From Figures 1, 2, and 3, we can observe the following fact:

The extreme value theorem

If $f(x)$ is continuous on $[a, b]$ then $f(x)$ has a ____________________

and a ____________________ on $[a, b]$.

Q1: What about when the interval is no longer closed and bounded? Draw some graphs to illustrate how the existence of extrema values could differ.

Q2: If $f(x)$ is continuous, where are the possible places for which global maximum and global minimum of $f(x)$ occur on $[a, b]$? (See Figures 1, 2, and 3.)

A2: On a closed and bound interval $[a, b]$, a continuous function $f(x)$ attains its global maximum and global minimum occur at (1) ____________________, or (2) ____________________, or (3) ____________________.

Definition: Let $f(x)$ be defined at c. Then we say that c is a **critical number** of f if (1) ____________________, or (2) ____________________.

Method for finding global maxima and minima of f on $[a, b]$
1. Find all critical numbers in (a, b).
2. Evaluate f at all critical numbers and at endpoints. Then compare the values of f: **highest** = global maximum and **lowest** = global minimum.

1. Find the absolute (global) maximum and minimum of the function $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 1$ on the interval $[0, 3]$.

2. The lengths of the sides of a triangle are $9 + x^2$, $9 + x^2$ and $18 - 2x^2$ units. Calculate

2a. the area of the triangle in terms of x,

2b. the value of x which makes this area a maximum.

3. Find the global maximum and global minimum of the following functions on the given interval:

(a) $f(x) = 2 + |x - 1|; \quad [0, 2]$ (b) $y = \dfrac{t}{t-2}; \quad [3, 5].$

4. A sales company introduces a new product for which the number of units sold S (in hundreds) is

$$S(t) = 5 - \frac{9}{2+t}$$

where t is time in month.

4a. Find the average value of $S(t)$ during the first year.

4b. During what month does $S'(t)$ equal the average value during the first year?

4c. Give a sketch of the graph of $S(t)$ and illustrate the quantity found in Part (b).

> **The Mean Value theorem**
> If $f(x)$ is continuous on $[a, b]$ and differentiable on the open interval (a, b) then there exists a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

In particular, as a special case with $f(a) = f(b)$, we have:

> **The Rolle's theorem**
> Let $f(x)$ is continuous on $[a, b]$ and differentiable on the open interval (a, b).
>
> If $f(a) = f(b)$ then there exists a number c in (a, b) such that $f'(c) = 0$.

5. Give examples of functions that do not satisfy the Mean Value Theorem (or Rolle's theorem).

6. Verify that the function $f(x) = (x - 2)(x + 3)^2$ satisfies the hypotheses of Rolle's Theorem on $[-3, 2]$. Then find all numbers c that satisfies the conclusion of Rolle's Theorem.

7. Verify that the function $f(x) = \dfrac{x}{x+2}$ satisfies the hypotheses of the Mean Value Theorem on $[1, 4]$. Then find all numbers c that satisfies the conclusion of the Mean Value Theorem.

8. Two bicyclists begin a race at 8:00am. They both finish the race 2 hours and 15 minutes later. Prove that at some time during the race, the bicyclists are traveling at the same velocity.