Math 10350 Fall 07 – Handout 5

Review for Exam 1 (Sections 2.1 - 2.3)

1. Find the derivative of the following functions:

a. $f(x) = \dfrac{3x^3 - \sqrt{x} + \pi x^2}{x^2}$

b. $g(x) = 5x \sin x + \tan x$

c. $y = \dfrac{\cos x}{x^2 + 1}$

d. $w = \dfrac{t + 1}{t^2 + 2}$

2. Find the derivative of the function $f(x) = \sqrt{x + 3}$ using the limit definition of derivative. Does the curve $y = \sqrt{x + 3}$ have any horizontal tangent line?

3. Find the equation of the tangent line to the graph of $y = \dfrac{2}{x^2 + 4}$ when $x = 0$.

4. Let $f(x) = \dfrac{x^2 - 4}{x^2 - x - 6}$. Find the values of the following limits:

a. $\lim\limits_{x \to -2^+} f(x)$

b. $\lim\limits_{x \to -2^-} f(x)$

c. $\lim\limits_{x \to 3^-} f(x)$

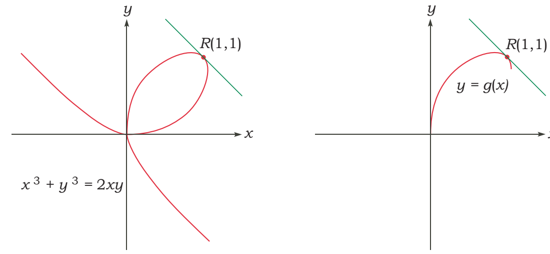
d. $\lim\limits_{x \to 3^+} f(x)$

Discuss the behavior of the graph of $f(x)$ at $x = 2$ and $x = 3$.

1. Find the equation of the tangent line at the point $P(1,2)$ on the circle $x^2 + y^2 = 5$ by solving for y as an appropriate expression of x.

Remark: For a general relation between x and y, it is difficult to write y as a function of x. For example, $x^3+y^3 = 2xy$. To find the slope at $R(1,1)$ on the curve using the above method, we need to find **explicitly** $g(x)$. This is very hard!!

We say that y is an implicit function of x. To find $\dfrac{dy}{dx}$ in such situation we employ a powerful method called **Implicit Differentiation**.



2. Find the equation of tangent line to the circle at $x^2+y^2 = 5$ at the points $P(1,2)$ and $Q(1,-2)$ using implicit differentiation. Explain how implicit differentiation is a better method over that of the previous problem.

3. Verify that the point $(1,1)$ is on the curve $x^3 + y^3 = 2xy$. Find (a) $\dfrac{dy}{dx}$, (b) the slope of the curve $(1,1)$.

4. Find the points on the curve $x^4 = 4(4x^2 - y^2)$ for which the tangent lines to the curve are horizontal.

(Ans: $(2\sqrt{2},4)$; $(2\sqrt{2},-4)$; $(-2\sqrt{2},4)$; $(-2\sqrt{2},-4)$)

5. A spotlight on the ground shines on a wall $12m$ away. If a man $2m$ tall walks from the spotlight toward the building at a speed of $1.6m/s$, how fast is his shadow on the building decreasing when he is $4m$ from the building. (Answer: $0.6\,m/s$)

6. The altitude of a triangle is increasing at a rate of $1cm/min$ while the area of the triangle is increasing at a rate of $2cm^2/min$. At what rate is the base of the triangle changing when the altitude is $10cm$ and the area is $100cm^2$? (Answer: $-8/5\,cm/min$)

7. A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(4,2)$, its x-coordinate increases at a rate of $3cm/s$. How fast is the distance from the particle to the origin changing at this instant? (Answer: $27/(4\sqrt{5})cm/s$)

8. A water tank in the shape of an inverted cone has height 20 feet and diameter of the cone at the top is 30 feet. Water is being pumped into the tank at a rate of $192\pi\, ft^3/min$. How fast is the water level rising when the water is 8 feet deep? (Answer: $16/3\, ft/min$)

9. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is $1\,m$ higher than the bow of the boat. If the rope is pulled in at a rate of $1\,m/s$, how fast is the boat approaching the dock when it is $8\,m$ from the dock?