Math 10350 Fall 07 – Handout 3

Definition 1 A function $f(x)$ is said to be differentiable at $x = c$ provided the following limit exist:

This means that the slope at $x = c$ of the graph is a _____ number.

Graphically, differentiable means that each small segment of the graph of $f(x)$ is almost identical to a straight line. This is illustrated in Figure 1 through 3 below. As you zoom into the point $(c, f(c))$, the segment of the graph of $f(x)$ near point c becomes more and more like its tangent line at $x = c$.

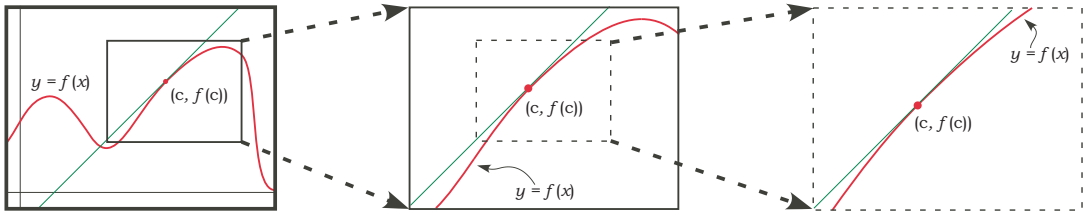


Figure 1. Figure 2. Figure 3.

Remark: We say that a function $f(x)$ is differentiable on (a, b) if $f(x)$ is differentiable for all $x = c$ in (a, b).

Theorem 1 If $f(x)$ be differentiable at $x = c$, then $f(x)$ is _____ at $x = c$.

1. Sketch the graph of $f(x) = |x - 2|$ and decide by visual inspection whether $f(x)$ is differentiable at $x = 2$. Then use the limit definition to decide whether $f(x)$ is differentiable at $x = 2$.

2a. Find the slope of $g(x) = \sqrt{x}$ at a fixed positive value $x = c$ using limits.

2b. Write down the equation of the tangent line to the graph of $y = g(x)$ at $x = 9$.

2c. Write down the derivative (slope function) of $g(x)$.

Notation: If $y = f(x)$ is a differentiable function. Write down all standard notations of the derivative of $y = f(x)$.

Some Common Derivatives:

$\frac{d}{dx}(k) \stackrel{?}{=}$ $\frac{d}{dx}(x^n) \stackrel{?}{=}$ $\frac{d}{dx}(\sin x) \stackrel{?}{=}$ $\frac{d}{dx}(\cos x) \stackrel{?}{=}$

3. If $f(x) = \dfrac{1}{2x+1}$ find the derivative of $f(x)$ by the limit process.

Basic Properties of Derivatives:

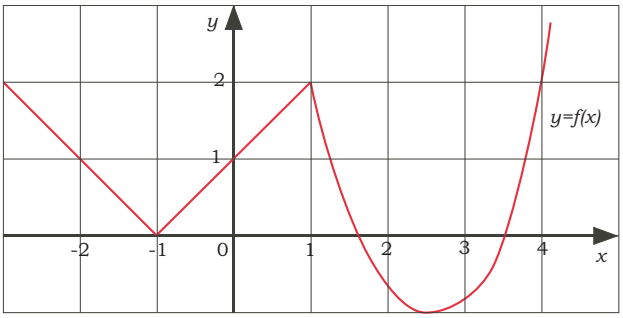
$[f(x) + g(x)]' \stackrel{?}{=}$ $[f(x) - g(x)]' \stackrel{?}{=}$ $[c \cdot f(x)]' \stackrel{?}{=}$

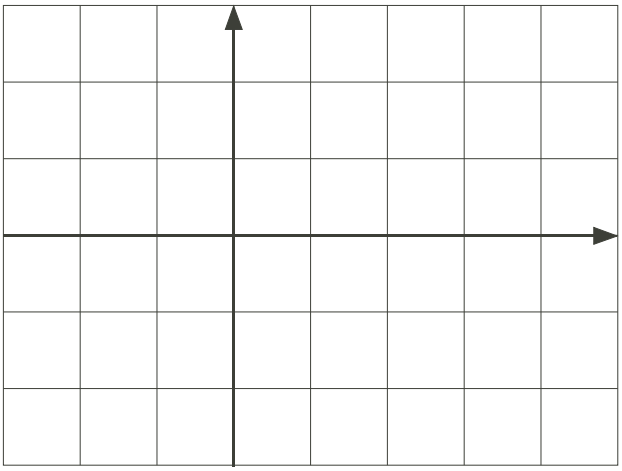
4. Find the derivative of each of the following functions:

a. $f(x) = \sqrt{x} + \dfrac{\pi}{\sqrt{x}}$ **b.** $y = \dfrac{x^3 + 5x + 6}{x}$ **c.** $h(t) = \dfrac{2 + \sqrt{t}}{t^2}$ **d.** $x = 2\sin\theta - 5\cos\theta$

5. The stationary points in the domain of a function $f(x)$ are the values of x such that $f'(x) = 0$. What can you say about the tangent line at stationary points? Find the stationary points of $y = x + \dfrac{1}{x}$.

6. Draw the graph of the graph of f' if the graph of f is given below





7. (2.2/Q64) Find the value of k such that the line $y = -4x + 7$ is tangent to the graph of $f(x) = k - x^2$.

8. It is known that the position (displacement) function for a free-falling object is in the form

$$s(t) = -5t^2 + v_0 t + s_0$$

where v_0 is the initial velocity (m/s) and s_0 is the initial position (m). Using this fact, answer the following questions for a projectile shot from the edge of a 65m cliff with an initial velocity of 60 m/s.

a. What is the velocity after 10 seconds?

b. When does the projectile hit the ground? at what instantaneous velocity?

9. The slope of the curve $y = ax^2 + bx$ at the point $(2, 4)$ is -8. Calculate the values of a and b.

10. Let $f(x)$ and $g(x)$ be differentiable functions. Derive formulas for the derivatives of $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.