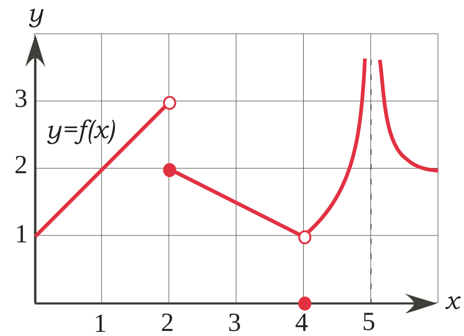
Math 10350 Fall 07 – Handout 2

1. Consider the function $f(x)$ whose graph is given below. Find each of the limits and the values of the function below if they exists. If the limit does not exist, explain why.



a. $\lim_{x\to 1} f(x) =$ ________ and $f(1) =$ ________

Comment:

b. $\lim_{x\to 2} f(x) =$ ________ and $f(2) =$ ________

Comment:

c. $\lim_{x\to 4} f(x) =$ ________ and $f(4) =$ ________

Comment:

d. $\lim_{x\to 5} f(x) =$ ________ and $f(5) =$ ________

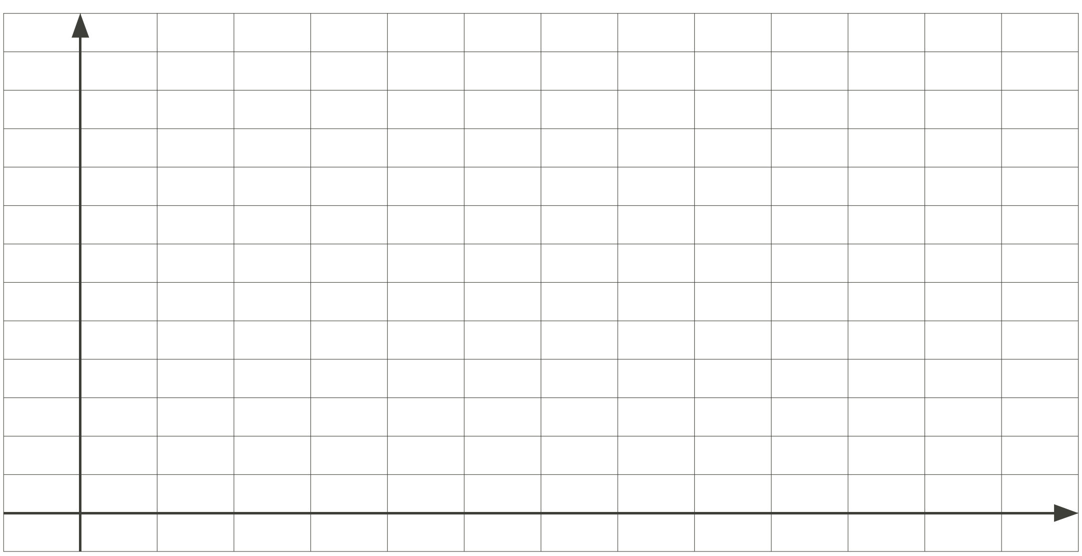
Comment:

Theorem 1 (1) $\lim_{x\to c} f(x)$ exists $\iff$ ______________ and ______________ both exist and are equal.

Moreover, (2) $\lim_{x\to c} f(x) = L \iff$ ______________ $= L =$ ______________.

Theorem 2 A function $f(x)$ is continuous at $x = c \iff f(c)$ is defined and ______________ $= f(c)$.

2. A phone company charges long distance call accounting to the amount of time you use their services. The first minute or less has a base charge of 50 cents. After the first minute, you will be charge 10 cents for every block of 10 seconds or less. Draw a graph that describe your payment for a call that can last up to 2 minutes long in the axes below. Comment on its continuity.



3. Compute the limit $\lim_{x\to 0} \frac{|x|}{x}$ if it exist. If the limit does not exist explain why.

Theorem 3 Let $f(x)$ and $g(x)$ be continuous at $x = c$. Then the following functions are continuous at $x = c$: $f(x) + g(x)$; $f(x) \cdot g(x)$; $\dfrac{f(x)}{g(x)}$ if $g(c) \neq 0$.

Remark: Theorem 3 says that (1) all polynomials are continuous, and (2) all rational functions $\dfrac{p(x)}{q(x)}$ are continuous except at values of $x = c$ for which ______________________________.

4. Find the equations of all the asymptotes for the function $f(x) = \dfrac{x-1}{x^2+5x-6}$. Using limits, determine all values of x for which the function is continuous.

5. Is the function $f(x)$ continuous for all values of x? Explain.

$$f(x) = \begin{cases} \dfrac{\tan 3x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

6. Determine the constants a and b such that the following function is continuous on the entire real line

$$f(x) = \begin{cases} 2 & -\infty < x \leq -1 \\ ax + b & -1 < x < 3 \\ -2 & 3 \leq x < +\infty \end{cases}$$

7. Explain why the function $f(x) = x^3 + x - 1$ has a zero between $[0, 1]$. Can you give an estimate for this zero?

8. Consider the function $f(x) = \begin{cases} 3 + x & -\infty < x < 5 \\ x^2 & 5 \leq x < 11 \\ 5 - 3x & 11 \leq x < +\infty \end{cases}$

Compute the following limits. If the limit does not exist explain why.

a. $\lim_{x \to 5^+} 2f(x) - 5$ **b.** $\lim_{x \to 11} (f(x))^2$

9. A particle is moving on a straight line according to the displacement law:

$$s(t) = 3t^2 - 2t + 5.$$

Here s is in meters and t in seconds.

a. Find the average velocity for this particle during the time interval $2 \leq t \leq 5$.

b. Find the average velocity for this particle during the time interval $2 \leq t \leq 2 + h$. Simplify the expression obtained.

c. Using (b), find the instantaneous velocity of the particle at $t = 2$ using the limit definition.

10. Give a graphical interpretation of the computations in Question 9.