

Conservatism, Growth, and Return on Investment

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Abstract

Return on Investment (ROI) is widely regarded as a key measure of firm profitability. The accounting literature has long recognized that ROI will generally not reflect economic profitability, as determined by the internal rate of return (IRR) of a firm's investment projects. In particular, it has been noted that accounting conservatism may result in an upward bias of ROI, relative to the underlying IRR, and that more conservative accounting may exacerbate that bias. We examine both theoretically and empirically the behavior of ROI as a function of two variables: growth in new investments and accounting conservatism. Higher growth is shown to result in lower levels of ROI provided the accounting is conservative, while the opposite may be true for liberal accounting policies. Conversely, the impact of more conservative accounting on ROI depends crucially on the rate of growth in new investments over the relevant horizon. Taken together, we find that conservatism and growth interact like "strategic substitutes" in their joint impact on ROI.

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1 Introduction

Return on Investment (ROI) is arguably the most prevalent measure of profitability. In financial statement analysis, ROI is a key profitability metric along with the Market-to-Book and the Price-Earnings ratios.¹ For management control purposes, firms continue to rely on the ROI metric in order to evaluate the performance of business units that are organized as profit centers and have authority to make investment decisions.² In the industrial organization literature, ROI is frequently invoked to gauge the competitiveness of particular industries and to justify antitrust measures.³ Finally, in many regulated industries, including utilities and telecommunications, product prices have traditionally been set to satisfy the constraint that the regulated firm earns a targeted return on its investments.

The prevalence of the ROI metric appears to be grounded in the notion that, at least under ideal circumstances, this metric can capture economic profitability as represented by the internal rate of return (IRR) of a firm's projects. To that end, the accounting literature has examined the possibility of accrual accounting rules such that for a given project the resulting ROI is constant over time (and, by implication, the equal to the project's IRR). For instance, Beaver and Dukes (1974) explore such neutral (unbiased) accounting rules in the context of depreciation. In contrast, external financial reporting rules (GAAP) are generally not viewed as unbiased but instead as conservative. While the accounting literature has not settled on a single comprehensive definition of conservatism, a common implication of the different notions of conservatism appears to be that on average book values are understated.⁴ This tendency suggests that, given conservative accounting, the Return-on-Investment metric will be distorted upward relative to the underlying IRR. Furthermore, it has been observed that such distortions do not necessarily revert over time, but can persist indefinitely for firms in a steady state.⁵

¹See, for example, Penman (2003) or Lundholm and Sloan (2004).

²Advocates of so-called Economic Profit Plans have cautioned against ROI as a performance metric because it encourages the maximization of a ratio rather than absolute value maximization (Young and O'Byrne, 2001).

³For instance, Scherer (1982) cites relatively high ROAs in the breakfast cereal industry as a rationale for why the Federal Trade Commission intervened in that industry. Similarly, in the recent discussion about "excessive profits" in the oil refining industry, commentators frequently cite evidence based on ROI measures.

⁴Feltham and Ohlson (1996), Ohlson and Zhang (1998) and Zhang (2000) refer to the accounting as conservative if on average market values exceed book values. In contrast, Basu (1997) and Watts (2003) emphasize asymmetry in the recognition of anticipated losses as opposed to the non-recognition of anticipated gains.

⁵See Zhang (2000) for a fundamental asymptotic result. Penman (2003, Ch. 16), Brief (2002), Gjesdal (2004) derive steady state characterizations Of ROI in finite horizon settings.

Our objective in this paper is to examine analytically and empirically how a firm’s ROI is impacted by two central variables: accounting conservatism and growth. We address the following three basic questions. First, under what circumstances will ROI exceed economic profitability, as measured by the underlying IRR? Second, what is the impact on ROI if either growth or conservatism changes unilaterally, and how does the directional change depend on the specification of the other variable? Finally, how do these two variables interact, i.e., do they offset or reinforce each other in their impact on ROI? In addressing these three questions, our study builds on earlier partial findings which have examined the behavior of ROI in a variety of theoretical, numerical and empirical contexts.

Our model views the firm and its financial statements as an aggregate of individual investment projects undertaken at different dates in the past. These investment projects are assumed to have the same underlying economic profitability, i.e., the same IRR.⁶ Periodic growth is represented as the rate of change in new investments. In effect, a growing firm effectively conducts the same “representative” project, albeit on a larger scale in more recent periods. A major simplification of our model is that growth is treated as an exogenous variable. This assumption seems most plausible in a competitive industry setting, where in equilibrium all firms earn the required cost of capital on their investments, independent of their growth rate. In a competitive industry, aggregate growth in new investment (or decline) will be determined by changes in aggregate demand, yet the growth of individual firms, or the entry of new firms, is essentially indeterminate.⁷

We initially follow earlier literature in calling the accounting *conservative* if investments are written off faster, in a cumulative sense, than they would have been under neutral (unbiased) accounting.⁸ Conversely, *liberal* accounting requires new investments to be amortized more slowly in comparison to neutral accounting. These concepts of conservatism give rise to the following fundamental finding, which we refer to as the “quadrant result”: ROI exceeds the IRR if the accounting is conservative and growth is *moderate* in the sense that

⁶This is consistent with the perspective in Penman (2003), Gjesdal (2004) and Richardson et al. (2005).

⁷A central question in the industrial organization literature is whether in the long run firm profits tend to revert to competitive levels. To answer this question, a variety of profit measurement methodologies have been developed; see, for instance, Mueller (1977) or Pakes (1987). Some authors, including Fisher and McGowan (1983), have argued that it is generally impossible to infer economic profitability from reported accounting rates of return. Our perspective in this paper is to explore how the key variables of growth and conservatism shift the accounting rate of return relative to the underlying economic rate of return.

⁸This representation of conservatism is equivalent to the criterion that at each point in time the fair market value of a firm’s projects exceeds the book value (Feltham and Ohlson, 1996, and Zhang, 2000). In the language of Beaver and Ryan (2004), we consider unconditional conservatism, in contrast to the conditional, i.e., event- dependent, conservatism of Basu (1997) and others.

new investments grew at a rate less than the IRR in each period over the relevant past. With *aggressive* growth (a rate higher than the IRR), in contrast, ROI is predicted to be below the IRR. Liberal accounting reverses the ordering such that moderate (aggressive) growth results in an ROI lower (higher) than the IRR.⁹ For our empirical tests, we accept that GAAP results in conservative accounting, and therefore focus on confirming that ROI, when viewed as a function of growth, will be concentrated in either the North-West or the South-East quadrant.

The quadrant result strongly suggests a partial answer to our second question: given conservative accounting, does faster periodic growth in new investments lead to a monotonic decrease ROI? Such a monotonicity result turns out to be true only if one invokes a stronger notion of conservatism which requires that the recognition of value (viewed in terms of residual income) is delayed not only in a cumulative, second-order sense but in a uniform, first-order sense. For lack of a better term, we refer to this criterion as *neo-conservatism*. This stronger form of conservatism is met, for instance, if straight-line depreciation is applied to projects with uniform cash flows, or if a share of new investments is directly expensed. Given neo-conservatism, it can be shown that higher growth in any past period of the relevant time horizon will ceteris paribus lower current ROI.

Partial expensing of new investments, like those for R&D and other intangible assets under GAAP, is arguably an extreme form of conservatism.¹⁰ More conservative accounting can then be represented by a higher share of directly expensed investments. This notion is closely related to the “C-Score” concept of Penman and Zhang (2002). Holding growth fixed, the natural question in our analysis is whether firms with a higher percentage of expensed investments will ceteris paribus show a higher ROI. Consistent with our earlier classification, this turns out to be true only if growth rates are moderate. For aggressive growth, in contrast, the “numerator effect” in the ROI metric tend to dominate. As a consequence, these firms tend to report lower ROI’s as they expense a higher share of their investments.

Our analysis focuses primarily on characterizing ROI at any given date as a function of conservatism and the pattern of growth over the past T years, where T is the useful life of the representative project. Yet, our analysis also generates predictions regarding the change in ROI across consecutive years. In contrast to Fairfield, Yohn and Whisenant (2003), who test the hypothesis that one-year ahead ROI is decreasing in the growth of current net-operating

⁹Fisher and McGowan (1983), Gjesdal (2004) and others have demonstrated special versions of the quadrant result, for instance, by restricting attention to settings where the firm is in a steady state, i.e., investments grow at a constant rate in each period.

¹⁰Lev and Sougiannis (1996) and Lev (2001) have argued that such accounting diminishes the reliability of financial statements.

assets, we predict and empirically confirm that the relevant criterion for changes in one-year ahead ROI is whether the current growth rate exceeds (is below) the average growth rate over the relevant past periods.

With regard to the interaction between growth and conservatism in their joint impact on ROI, we find that these two variables are “strategic substitutes”. As a consequence, with conservative accounting an increase in growth not only tends to lower ROI, but furthermore this downward effect will be magnified by more conservative accounting rules. If one represents higher degrees of conservatism as a larger share of investments expensed, the cross-partial derivative of ROI in growth and the share of investments expensed is predicted to be negative. Our data analysis supports this negative cross partial interaction.

The remainder of the paper is organized as follows. Section 2 formalizes the elements of our model, including investment projects, growth and conservatism. We then derive a sequence of propositions regarding the joint impact of growth and conservatism on ROI. Several of these predictions lend themselves to empirical tests which are reported in Section 3. We conclude in Section 4.

2 Theory Development

2.1 Transactions and Accrual Accounting

To capture the impact of growth and conservatism on the accounting rate of return, we consider a firm which in each period has access to a representative investment project with identical characteristics. Growth is represented as intertemporal changes in the scale of the representative project, i.e., changes in investment expenditure. Growth is an exogenous variable in our model. This specification is justified for competitive industries, where in equilibrium all firms earn the required cost of capital on their investments, independent of their growth rate. Aggregate growth (or decline) will be determined by changes in aggregate demand in a competitive industry, yet for an individual firm growth is a matter of indifference.

We denote the representative project by $\mathcal{P} = (b^o, c_1^o, \dots, c_T^o)$. It involves an initial investment expenditure of b^o , followed by annual cash inflows of c_t^o over the next T periods. At first, there is no sign restriction on the individual c_t^o 's, however we postulate that the project's internal rate of return, denoted by r , is unique. Depreciation is the only accrual in our model, with depreciation schedules represented by (d_1, \dots, d_T) . Assuming comprehensive income measurement, we have $\sum_{t=1}^T d_t = 1$, and the income of the representative project in period t is $Inc_t^o = c_t^o - d_t \cdot b^o$, with the start-of-period- t book value given by $BV_t^o = (1 - \sum_{i=1}^{t-1} d_i) \cdot b^o$.

Since projects have a useful life of T periods, the firm's overall accounting rate of return

is essentially a weighted average of the rates of return for the representative project, with the weights determined by the growth rates over the last T periods. This feature reflects that the representative project is “scalable” in the sense that any increase in new investment from one year to the next will result in a proportional increase of the associated project cash flows, c_t^o . The growth rate in new investments (and project scale) in year t will be denoted by λ_t . Therefore the firm’s overall accounting rate of return at date T is a function of the accounting rules for the representative project and the growth pattern $\vec{\lambda} = (\lambda_1, \dots, \lambda_{T-1})$ over the past T years:

$$ROI_T(\vec{\lambda}) = \frac{Inc_T^o + Inc_{T-1}^o \cdot (1 + \lambda_1) + \dots + Inc_1^o \cdot \prod_{i=1}^{T-1} (1 + \lambda_i)}{BV_{T-1}^o + BV_{T-2}^o \cdot (1 + \lambda_1) + \dots + BV_0^o \cdot \prod_{i=1}^{T-1} (1 + \lambda_i)}. \quad (1)$$

For future reference, we note that in the special case of constant annual growth, i.e., $\lambda_t = \lambda$ for all t , ROI remains in a steady state in subsequent periods, that is, $ROI_T(\lambda) = ROI_{T+t}(\lambda)$. To formalize the concept of conservatism, we begin with neutral (unbiased) accounting. Following Stauffer (1971) and Beaver and Dukes (1974), a depreciation schedule will be called *neutral* if:

$$\frac{Inc_t^o}{BV_{t-1}^o} = r \quad (2)$$

for all t . It is well known if one merely requires the ratios $\frac{Inc_t^o}{BV_{t-1}^o}$ to be constant over time, then this constant must equal the project’s internal rate of return, r .¹¹ It is also well-known that for any given cash flow pattern $(b^o, c_1^o, \dots, c_T^o)$, there exists one, and only one, neutral depreciation schedule. We denote this schedule by (d_1^*, \dots, d_T^*) . For future reference, it will be useful to note that growth has no impact on $ROI_T(\vec{\lambda})$ provided the accounting is neutral. To illustrate, suppose $T = 2$. If both $\frac{Inc_2^o}{BV_1^o}$ and $\frac{Inc_1^o}{BV_0^o}$ are equal to r , then

$$ROI_2(\vec{\lambda}) = \frac{Inc_2^o + Inc_1^o \cdot (1 + \lambda_1)}{BV_1^o + BV_0^o \cdot (1 + \lambda_1)} = r$$

for any λ_1 . More generally, it will be useful to think of $ROI_T(\vec{\lambda})$ as a weighted average of the “component ratios” $\frac{Inc_t^o}{BV_{t-1}^o}$. For instance, $ROI_2(\vec{\lambda})$ is a weighted average of $\frac{Inc_1^o}{BV_0^o}$ and $\frac{Inc_2^o}{BV_1^o}$, and this average gravitates towards $\frac{Inc_1^o}{BV_0^o}$ as λ_1 gets larger.

Consistent with earlier literature, we initially represent conservatism by the requirement that at each point in time depreciation is accelerated relative to the charges under neutral

¹¹This follows from the observation that, irrespective of the accounting rules, ROI cannot be either consistently above or consistently below the internal rate of return. Otherwise, the present value of the residual incomes, with the capital charge rate given by r , could not be zero.

accounting.

Definition A depreciation schedule (d_1, \dots, d_T) is conservative for the investment project $\mathcal{P} = (b^o, c_1^o, \dots, c_T^o)$ if for all $1 \leq t \leq T - 1$:

$$\sum_{i=1}^t d_i \geq \sum_{i=1}^t d_i^*.$$

The accounting will be called *liberal* if the preceding inequality is reversed and therefore the depreciation charges are backloaded relative to the benchmark of neutral accounting. To operationalize Definition 1 further, it will be convenient to look at the evolution of the residual income numbers over time. For the representative project, let $RI_t^o \equiv Inc_t^o - r \cdot BV_{t-1}^o$. Since r is the project's internal rate of return, the present value of the residual income numbers is zero for any depreciation schedule, i.e., $\sum_{i=1}^T RI_i^o \cdot \gamma^i = 0$, where $\gamma = \frac{1}{1+r}$.

Lemma 1 The depreciation schedule (d_1, \dots, d_T) is conservative if and only if:

$$\sum_{i=1}^t RI_i^o \cdot \gamma^i \leq 0$$

for all $1 \leq t \leq T - 1$.¹²

Conservatism therefore requires that the cumulative project value recognized at any point in time is less than the actual project value, which is zero when cash flows (and residual incomes) are discounted at the internal rate of return. For future reference, we note that this criterion amounts to a “second-order dominance” condition since it speaks to the cumulative value recognized at any point in time. Earlier literature, including Feltham and Ohlson (1996), Ohlson and Zhang (1998) and Zhang (2000), has defined conservatism by the criterion that fair market values exceed book values. In our context, the “fair market value” of the representative project at date t can be represented by the present value of the remaining cash flows, i.e.,

$$MV_t = \sum_{i=t+1}^T \gamma^{i-t} \cdot c_i^o.$$

A market-to-book ratio greater than one is equivalent to the conservatism criterion given in Definition 1. To see this, it suffices to note that MV_t is equal to AV_t plus the sum of future discounted residual incomes, i.e., $MV_t = AV_t + \sum_{i=t+1}^T \gamma^{i-t} \cdot RI_i^o$, irrespective of the accounting rules, provided income measurement is comprehensive. Since $\sum_{i=1}^T RI_i^o \cdot \gamma^i = 0$, we find that $MV_t > AV_t$ if and only if $\sum_{i=1}^t RI_i^o \cdot \gamma^i \leq 0$, i.e., the accounting is conservative according to Lemma 1.

¹²All proofs are provided in the Appendix.

2.2 Conservatism and Growth

Absent growth, there is a strong intuitive argument as to why conservative accounting results in an abnormally high ROI, relative to the benchmark of the internal rate of return r . Compared to neutral accounting, conservatism depresses all book value terms, other than BV^o , in the denominator of (1). At the same time, the “Canceling Errors” Theorem ensures that, because there is no growth, aggregate income is unaffected by conservatism (Greenball, 1969). Yet, the no growth assumption is clearly important to this conclusion. If growth in each period, λ_t , were precisely equal to r , then irrespective of the accounting rules $ROI_T = r$. To see this, it suffices to multiply the difference $ROI_T(\vec{\lambda}) - r$ by the denominator in (1). If $\lambda_t = r$ for all t , the resulting expression is equal to

$$\gamma^{-T} \cdot \left[\sum_{i=1}^T RI_t^o \cdot \gamma^t \right],$$

which is indeed equal to zero since the present value of the residual incomes is zero for any depreciation rule. This observation directly suggests that $ROI_T(\vec{\lambda}) > r$ if growth is constant and less than r , while the reverse inequality holds if growth is constant and exceeds r . The validity of this claim has been demonstrated by Gjesdal (2004).¹³ The following result applies to a larger class of settings where growth may change over time.

Proposition 1 *Conservative accounting implies:*

$$ROI_T(\vec{\lambda}) \begin{cases} \geq r & \text{if } \lambda_t \leq r \text{ for all } 1 \leq t \leq T \\ \leq r & \text{if } \lambda_t \geq r \text{ for all } 1 \leq t \leq T \end{cases} \quad (3)$$

Conversely, with liberal accounting, $ROI_T(\vec{\lambda}) \leq r$ if $\lambda_t \leq r$ for all $1 \leq t \leq T - 1$, while $ROI_T(\vec{\lambda}) \geq r$ for $\lambda_t \geq r$, $1 \leq t \leq T - 1$.

We label Proposition 1 the “quadrant result.” This label suggests itself in the special case of constant annual growth if one visualizes a coordinate system whose axes are λ and ROI_T and whose origin is given by the point (r, r) . The function $ROI_T(\cdot)$ is then always contained in one of the four possible quadrants, depending on (i) whether the accounting is conservative or liberal and (ii) the rate of growth is *moderate*, i.e., $\lambda_t \leq r$, or *aggressive*, i.e., $\lambda_t \geq r$. The proof of Proposition 1 is based on Farkas’ Lemma, which is equivalent to the Duality Theorem of Linear Programming.¹⁴ Farkas’ Lemma says that a collection of inequalities hold if and only if a certain “dual” price vector exists. This equivalence allows

¹³A partial version of this result is also demonstrated in Fisher and McGowan (1983).

¹⁴See, for example, Rockafellar (1970).

Proposition 1 also to be read in the opposite direction: the inequality $ROI_T(\vec{\lambda}) \geq r$ holds for *all* conservative accounting policies only if growth is moderate. Of course, for any *fixed* accounting rules, the inequality $ROI_T(\vec{\lambda}) \geq r$ will obtain for a whole range of growth vectors with annual growth sometimes moderate and sometimes aggressive.

In conjunction with our earlier observation regarding neutral accounting, Proposition 1 implies that the accounting rate of return will always match the economic rate of return, r , in either one of two settings: (i) accounting is neutral or (ii) growth in each period is equal to r . The quadrant result then strongly suggests that faster growth tends to lower $ROI_T(\vec{\lambda})$, provided the accounting is conservative. This turns out not to be true under the assumptions made so far. Suppose, for instance, that there is constant annual growth and project cash flows are distributed uniformly. If $r = .18$ and $T = 3$, the depreciation schedule ($d_1 = .45, d_2 = .18, d_3 = .37$) is conservative, yet ROI_3 is increasing in λ for a range of growth rates $-1 < \lambda < 0$. It should be noted that in this example the residual income numbers, RI_t^o , for the representative project are non-monotonic over time. In particular, RI_2^o is the largest of the three values owing to the small depreciation charge in period 2. This observation motivates the following stronger notion of conservatism, which, for a lack of a better term, we refer to as “neo-conservatism.”

Definition 2 *A depreciation schedule (d_1, \dots, d_T) is neo-conservative for the investment project $\mathcal{P} = (b^o, c_1^o, \dots, c_T^o)$ if the sequence RI_t^o is monotone increasing in t .*

In contrast to the condition posited in Definition 1 (or equivalently in Lemma 1), the monotonicity requirement in Definition 2 says that the residual income sequence start out negative and increases monotonically to positive levels. In that sense, neo-conservatism also says that the value of the representative project (i.e., zero) is recognized with delay. However, the delay in value recognition must not merely hold in a cumulative (second order) sense but in a uniform (first order) sense: up to some critical date \hat{t} , with $1 < \hat{t} < T$, too little value is recognized, while too much is recognized thereafter.¹⁵ In our analysis below, conservatism will be represented by a one-dimensional parameter (such as the percentage of new investments that are directly expensed) and therefore the notion of more conservative accounting rules will have an obvious meaning. Nonetheless, at the current level of generality we can define one depreciation schedule as more conservative than another provided the corresponding residual income sequences, viewed as functions of time, intersect only once,

¹⁵If one imposes the stronger condition of neo-conservatism and assumes that annual growth is constant, i.e., $\lambda = \lambda_t$, then the “quadrant” result of Proposition 1 follows from the following simple argument: $ROI_T(\lambda) \geq r$ is equivalent to $\sum_{i=1}^T RI_i^o \cdot (1 + \lambda)^{T-i} \geq 0$. We know that this inequality holds as an equality at $\lambda = r$. If the sequence RI_i^o is monotone increasing, it will change sign once and therefore Descartes’ “rule of signs” (see, for example, Brealey and Myers, 1991) yields the conclusion.

such that the more conservative schedule yields lower ROI values in earlier periods.

For future reference, we note that in the special case of uniformly distributed cash flows, straight line depreciation leads to linearly increasing residual incomes and therefore such accounting is neo-conservative. This observation does not contradict Penman's (2003, Chapter 16) examples where straight-line depreciation reflects neutral accounting, because in these examples cash flows are not uniform but decreasing over time. It is also worth noting that if a depreciation schedule satisfies neo-conservatism relative to the internal rate of return rate r in the calculation of residual income, the required monotonicity will also hold for any capital charge rate \hat{r} which is less than r .¹⁶

Proposition 2 *Given neo-conservative accounting, $ROI_T(\vec{\lambda})$ is decreasing in each λ_t .*

For an intuitive explanation of this result, consider first a setting in which there is no growth. In that case ROI_T is a weighted average of the individual $\frac{Inc_t^o}{BV_{t-1}^o}$, representing the ROI of different vintages of investments over the past T years. Given neo-conservatism, the sequence $\frac{Inc_t^o}{BV_{t-1}^o}$ is also increasing in t . Therefore older vintage projects contribute an ROI that exceeds r , while new projects do the opposite. Note that these distortions do not average out because the identity

$$\sum_{t=1}^T \left(\frac{Inc_t^o}{BV_{t-1}^o} - r \right) \cdot BV_{t-1}^o \cdot \gamma^t = \sum_{t=1}^T ROI_t^o \cdot \gamma^t = 0$$

implies that the upward distortion (relative to r) of $\frac{Inc_t^o}{BV_{t-1}^o}$ must, in comparison, be much larger for older vintage investments than the downward distortion (relative to r) for new investments. For that reason, neo-conservatism implies that $ROI_T > r$ in the absence of growth. When growth is positive in any given year t , i.e., $\lambda_t > 0$, there will be an even greater weight attached to the lower $\frac{Inc_t^o}{BV_{t-1}^o}$ corresponding to new investments in all period $i > t$.

As observed above, if growth rates have been constant over the past T years, say $\vec{\lambda} = (\lambda, \dots, \lambda)$, ROI will reach a steady state provided growth continues at the same level.

¹⁶In the literature on managerial performance evaluation, Rogerson (1997) and others have advocated the so-called relative benefit depreciation rule as a means of creating goal congruence between owners and managers. As observed in Reichelstein (1997) and Dutta and Reichelstein (2005), relative benefit depreciation amounts to conservative accounting. It is essential to recall, however, that in these models the IRR of the project is unknown to the designer of the residual income performance measure. Instead the designer seeks to motivate the better informed manager to accept those projects for which IRR exceeds the owner's cost of capital, r_c . As a consequence, it is generally impossible to attain neutral accounting. However, for zero-NPV projects, i.e., when $r = r_c$, relative benefit depreciation does indeed result in neutral accounting. Thus, conservatism does not result in these models from an inherent conservatism bias in the depreciation schedule but from information asymmetry about the underlying project profitability.

However, if growth at date T levels off, i.e., $\lambda_T < \lambda$ and the accounting is neo-conservative, then ROI_{T+1} will exceed ROI_T . This reflects that by neo-conservatism the smallest of the component ratio in ROI_{T+1} , i.e., $\frac{Inc_{T+1}^o}{AV_T^o}$, will now receive a comparatively small weight owing to the smaller growth factor λ_T .¹⁷ At the same time, this conclusion will be reversed when current growth accelerates relative to past levels. We state this formally below, without proof.

Corollary 1 *Given neo-conservative accounting, $ROI_{T+1}(\vec{\lambda}, \lambda_T) - ROI_T(\vec{\lambda}) \geq 0$ if and only if $\lambda_T \leq \lambda$.*

Fairfield, Yohn and Whisenant (2003) formulate and test the hypothesis that one-year ahead ROI is decreasing in growth of net-operating assets. In contrast, we predict, and support in Section 3 below, that one-year ahead ROI exceeds (is below) current ROI if the current growth rate is below (exceeds) the average growth rate over the past T periods.

2.3 Constant Growth and Uniform Project Cash Flows

The natural question at this point is how the two variables growth and conservatism interact in their impact on ROI_T . To that end, we now confine attention to settings in which both variables have one-dimensional representations. We consider a parametric class of depreciation schedules with the property that the depreciation charges decline (or grow) at a constant rate. Specifically, we require that $d_t = (1 - \delta) \cdot d_{t-1}$, or equivalently $d_t = (1 - \delta)^{t-1} \cdot d_1(\delta)$, with $d_1(\delta)$ set by the requirement that the sum of the d_t 's be equal to one. Higher values of δ correspond to more conservative accounting. If project cash flows are assumed to be uniform over time - an assumption that will be maintained for the remainder of this subsection - then neutral accounting corresponds to the annuity depreciation method, i.e., $\delta = -r$. The accounting is conservative, and in fact neo-conservative, for any $\delta > -r$. In particular, straight line depreciation ($\delta = 0$) and the most conservative policy of full expensing in period 1 ($\delta = 1$) fall into this range.¹⁸ Conversely, one obtains increasingly liberal policies by letting δ assume large negative values, resulting in backloaded depreciation charges.

Given our assumption of uniform project cash flows, our next result shows that the directional change of $ROI_T(\lambda, \delta)$ in response to higher degrees of conservatism depends critically on the rate of growth being moderate ($\lambda \leq r$) or aggressive ($\lambda \geq r$).

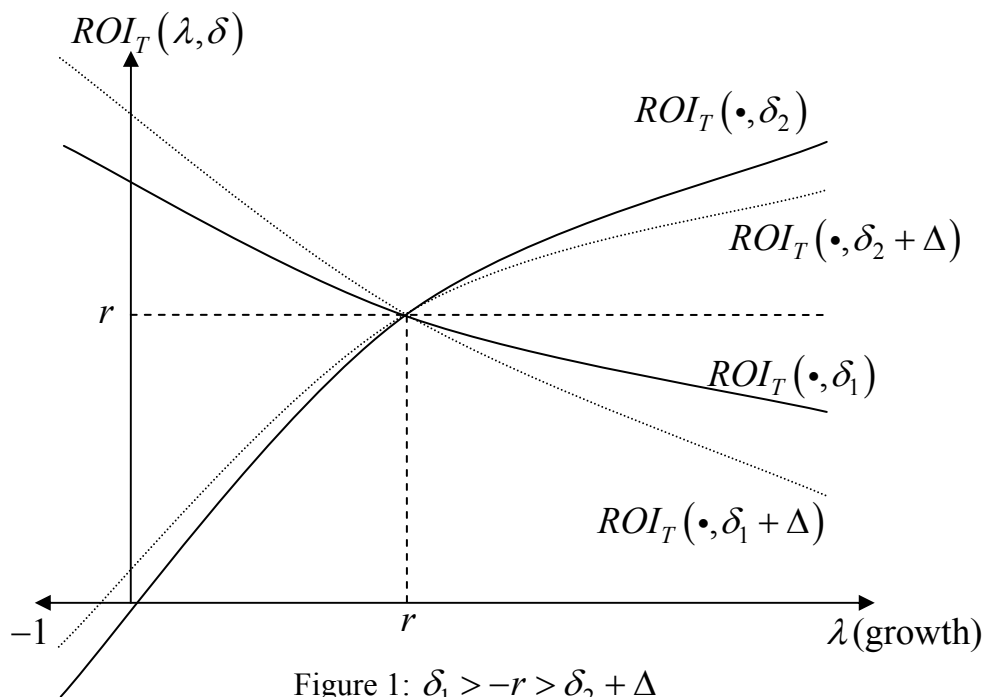
¹⁷Penman (2003) refers to the release of "hidden reserves" as growth slows.

¹⁸When $T = \infty$, one obtains a standard declining balance method, provided $0 < \delta < 1$ and $d_t = \delta \cdot AV_{t-1}^o$. As observed in Beaver and Dukes (1974), such a depreciation policy results in neutral accounting if $c_t = (1 - \delta) \cdot c_{t-1}$.

Proposition 3 $ROI_T(\lambda, \delta)$ is increasing in δ for $\lambda \leq r$, while $ROI_T(\lambda, \delta)$ is decreasing in δ for $\lambda \geq r$.

The finding that higher degrees of conservatism tend to depress ROI_T , given aggressive growth, appears somewhat counterintuitive. One way to visualize this finding is to recall that absent any growth, more conservative accounting decreases the denominator in (1) but leaves the numerator unchanged. If $\lambda = r$, higher degrees of conservatism decrease both the numerator and the denominator in (1) such that the total effect on ROI_T is nil. For aggressive growth, the decrease in the numerator dominates because more conservative depreciation schedules imply that the $\frac{Inc_t^o}{BV_{t-1}^o}$ of younger vintage projects is even further below r and these younger vintage projects receive disproportionately more weight when growth is aggressive.

Proposition 3 establishes a “single-crossing” property: for any two levels of conservatism, δ_1 and δ_2 , the functions $ROI_T(\cdot, \delta_1)$ and $ROI_T(\cdot, \delta_2)$ only cross once, and do so when the periodic growth rate is equal to r . Related to this observation is a partial converse to Proposition 2: given liberal accounting, ROI_T is increasing in the growth rate λ .¹⁹ Figure 1 illustrates these findings. Higher degrees of conservatism essentially rotate the functions $ROI_T(\cdot, \delta)$ in a clockwise fashion around the pivotal point $(\lambda, ROI_T(\lambda, \delta)) = (r, r)$.



¹⁹This result follows directly from the observation that $ROI_T(\lambda, \delta)$ is symmetric in its two variables such that $ROI_T(\lambda, \delta) \equiv ROI_T(-\delta, -\lambda)$ for any $\lambda > -1$ and $\delta < 1$.

Figure 1 suggests that growth and conservatism are “strategic” substitutes in the following sense: starting with modest growth, the decline in ROI due to higher growth rates will be more pronounced the more conservative the accounting. Formally, strategic substitutes require the difference $ROI_T(\lambda, \delta_1 + \Delta) - ROI_T(\lambda, \delta_1)$ to widen, in absolute terms, the further away the growth rate is from r . It turns out that this property of decreasing differences does not hold for depreciation policies that are sufficiently liberal.²⁰ Importantly, however, the property does hold for all conservative policies.

Proposition 3’ *The difference $ROI_T(\lambda, \delta_1 + \Delta) - ROI_T(\lambda, \delta_1)$ is decreasing in λ for any $\delta_1 > -r$ and $\Delta > 0$.*

Unlike our other proofs, we demonstrate Proposition 3’ by numerical simulation. The Appendix reports on the grid of parameter values we covered in our search. It should be noted that the finding in Proposition 3’ is stronger than that in Proposition 3 since decreasing differences implies single crossing but not the other way around. Yet, single crossing holds everywhere in our model, while decreasing differences only apply in the range of conservative policies. Figure 3 in Appendix A provides a 3-dimensional illustration of the resulting ROI function.

Further insight into the structure of ROI_T can be obtained if investments are depreciated according to the straight-line method ($\delta = 0$) over the T -period useful life. This scenario is arguably the most natural for our empirical tests since most firms rely on straight line depreciation for Plant, Property and Equipment assets. Our specification of uniformly distributed cash flows then appears plausible if new investments create fixed production capacity over T periods such that the corresponding “widgets” are sold at constant prices (on average) over the next T periods. Given straight-line depreciation, one obtains the following closed form expression for ROI_T .

$$ROI_T(\lambda, 0) = \lambda \cdot \frac{f(r, T)}{f(\lambda, T)}, \quad (4)$$

where $f(s, T) \equiv -1 + s \cdot T \frac{(1+s)^T}{(1+s)^T - 1}$. It is a direct consequence of Bernoulli’s inequality that $f(s, T) > 0$ for all s and T . Furthermore, $f(s, T)$ is increasing in s and therefore one immediately recovers in this specialized setting our finding in Proposition 1. Beyond the monotonicity result of Proposition 2, we can now determine the curvature of $ROI_T(\cdot, 0)$ and

²⁰Suppose that $T = 5$ and $r = 0.1$. Consider two liberal depreciation policies, $\delta = -5$ and $\delta = -4.95$. It is easily seen via numerical simulations that $ROI_T(\lambda, -5) - ROI_T(\lambda, -4.95)$ is not a monotone function of λ . It crosses 0 from below at $\lambda = r = 0.1$, increases until $\lambda = 1.0343$, and decreases thereafter.

provide upper and lower bounds for this function.

Proposition 4 $ROI_T(\lambda, 0)$ is a decreasing and convex function of λ such that:

$$(i) \lim_{\lambda \rightarrow -1} ROI_T(\lambda, 0) = f(r, T),$$

$$(ii) \lim_{\lambda \rightarrow 0} ROI_T(\lambda, 0) = \frac{2}{(T+1)}f(r, T),$$

$$(iii) \lim_{\lambda \rightarrow \infty} ROI_T(\lambda, 0) = \frac{1}{T}f(r, T).$$

For the special case of uniform cash flows and straight-line depreciation we thus find that the marginal impact of growth on ROI_T diminishes with higher growth rates. For firms that are declining rapidly, $f(r, T) \geq 0$ is an upper bound for ROI_T . Conversely, ROI_T does not tend to zero for rapidly growing firms but is bounded below by a positive constant determined by r and T . The convexity of ROI in λ is well illustrated by the fact that ROI is cut by a factor of $\frac{2}{(T+1)}$ as λ goes from negative one to zero, but is cut only by approximately one half as λ goes from zero to infinity.²¹

To introduce varying degrees of conservatism into this setting, we now allow for a share of new investments to be expensed immediately. In our empirical tests, this share of new investments corresponds to expenditures for R&D and Advertising. We therefore maintain the analytic convenience of straight line depreciation for capitalized investments, yet capture the notion of more conservative accounting by a higher fraction of investments that are directly expensed.²² We represent that fraction by $\beta \in [0, 1)$ and use the notation $ROI_T(\lambda, 0, \beta)$ with $ROI_T(\lambda, 0, \beta = 0) \equiv ROI_T(\lambda, 0)$.

Proposition 5

$$ROI_T(\lambda, 0, \beta) = \frac{1}{1 - \beta} \cdot [ROI_T(\lambda, 0) - \lambda \cdot \beta] \quad (5)$$

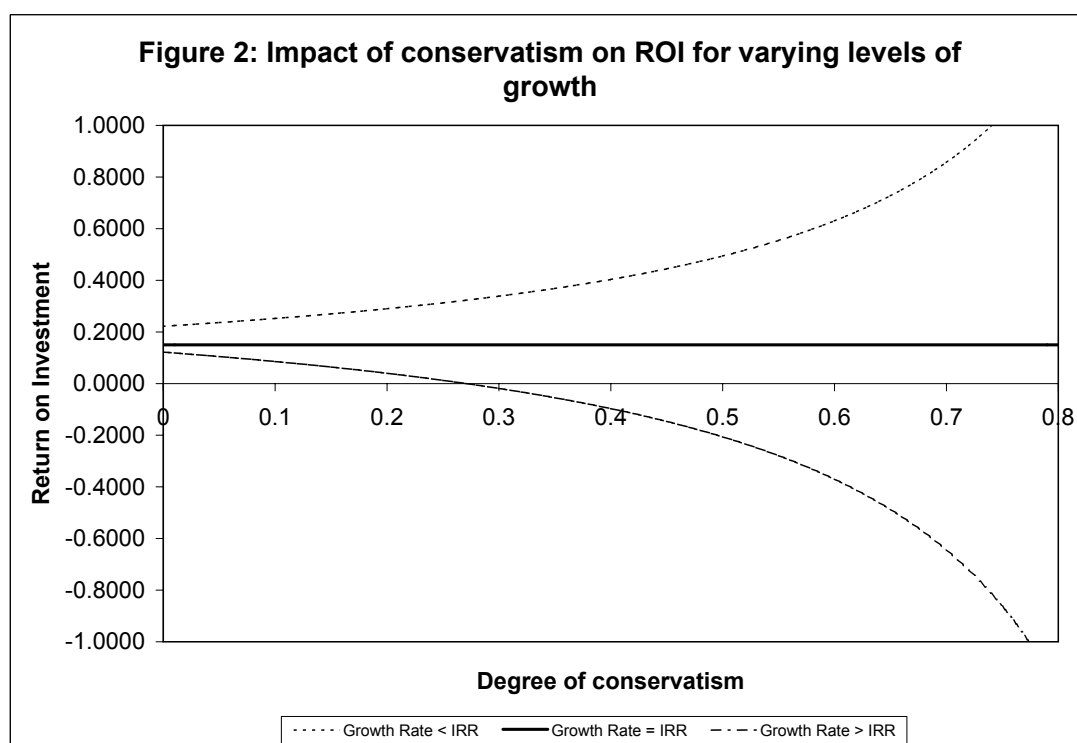
and the cross partial derivative of $ROI_T(\lambda, 0, \beta)$ in λ and β is negative.

As one would expect, direct write-offs of new investments behave qualitatively like more accelerated depreciation schedules, i.e., higher values of δ , in the above analysis. Figure 2 illustrates that in predicting the impact of partial expensing on ROI the constant growth rate

²¹With regard to the behavior of ROI in T, we find that $ROI_T(\lambda, 0)$ is a non-monotonic function of T for $\lambda \neq r$, such that $\lim_{T \rightarrow -1} ROI_T(\lambda, 0) = \lim_{T \rightarrow \infty} ROI_T(\lambda, 0) = r$. The function $ROI_T(\lambda, 0)$ increases (decreases) in T at values of T close to 1 depending on whether $\lambda < (>) r$. Also, for any given λ , $ROI_T(\lambda, 0)$ is convex and increasing in the internal rate of return r , such that $ROI_T(\lambda, 0)$ converges uniformly to zero as r goes to zero, and to infinity as r goes to infinity. Proofs of these claims are available upon request.

²²This specification is consistent with the examples in Penman (2002) and Richardson et al. (2006).

$\lambda = r$ serves as a “bifurcation” point. When periodic growth is less than r , $ROI_T(\lambda, 0, \beta)$ is increasing and convex in β , and is unbounded as β approaches one. On the other hand, for $\lambda > r$, $ROI_T(\lambda, 0, \beta)$ is decreasing and concave in β , and approaches minus infinity as β approaches one. Specifically, for a firm with $\lambda = -.05$, $r = .15$ and $T = 15$, ROI_T increases from 0.22 to 0.36 to 0.49 as a consequence of increasing the fraction of new investments written off from 0 to one-third to one-half. These points are displayed on the upper curve in Figure 2. The lower curve in Figure 2, which corresponds to a value of $\lambda = 0.45$, illustrates the latter scenario.



3 Empirical Analysis

In this section, we empirically test some of the implications of the preceding model exploring the impact of conservatism and growth on return on investment. We do not test all of the empirical implications of the model but focus on those propositions that appear to have the greatest bearing on current empirical research in the field. This section proceeds as

follows: Section 3.1 discusses our empirical proxies for the theoretical constructs, Section 3.2 describes sample formation, and finally Section 3.3 presents the empirical methodology and the results.

3.1 Empirical Proxies

Useful Life

Denoted as T throughout the model, useful life (*Useful Life*, hereafter) is the number of periods for which a given investment continues to produce cash flows for the firm. Empirically, there is no such mandated disclosure by management. Additionally, there is the complication that different assets have different useful lives depending on the type of asset and the circumstances surrounding their use in a particular firm. To measure *Useful Life*, we simply divide the Gross amount of PPE at the firm (Compustat item #7) by depreciation expense (Compustat item #125) in the current year. Although admittedly rough, this measure represents an estimate of the weighted average useful life of all the capitalized assets in the firm. Note that this measure does not include investments that GAAP mandates be immediately expensed such as R&D and advertising expense.²³

Average Growth in Past Investments

To represent past growth, i.e., $\vec{\lambda}$ in the model, we will consider the geometric mean $\mu(\vec{\lambda})$ of the growth history of $\vec{\lambda} = (\lambda_1, \dots, \lambda_{T-1})$. Total investments are calculated as R&D Exp (Compustat item #45) + Adv Exp (Compustat item #46) + Capitalized Expenditures (Compustat item #30). Growth in this variable is calculated as $[(TotalInvestments_t / TotalInvestments_{t-1}) - 1]$. To empirically estimate this construct (*PGrowth*), we calculate the geometric mean of growth in investments over the previous T periods starting in period $t - T + 1$.²⁴ Unlike *PGrowth*, which measures past growth, $Growth_t$ is the growth in investments in the current period and is used in our tests of Propositions 2 and 5.²⁵

Degree of Conservatism

Among the alternative notions of conservatism considered in the accounting literature, the

²³Implicitly, we assume that the useful life of these assets is not significantly different than the average life of the capitalized assets.

²⁴Note that the current year will be referred to as t in the empirical analysis; whereas T denotes the useful life of the assets (as discussed above).

²⁵As a robustness check, we use several other empirical proxies of investment growth which include 1) growth in change in NOA, 2) growth in Gross PPE (Compustat item #7), and 3) growth in capital expenditures, all with qualitatively similar results.

approach pioneered by Basu (1997) has become particularly prominent in recent years; see, e.g., Ball, Robin and Wu (2003) and Bushman and Piotroski (2005). Basu (1997) defines conservatism as “the accountant’s tendency to require a higher degree of verification to recognize good news as gains than to recognize bad news as losses.” Thus, varying degrees of conservatism are captured by the idea of differences in the degree of verification required for gains versus losses (Watts (2003)). Recent research has been critical of this measure (Dietrich, Muller, and Riedl, 2005)) and others have noted that it is negatively correlated with the current M/B ratio (Givoly, Hayn, and Natarajan, 2006). However, Roychowdhury and Watts (2005) argue that the results are consistent when the horizon over which these variables is measured is lengthened. It is important to note that the Basu (1997) measure and the M/B ratio both rely on the use of market prices to infer the degree of conservatism. By contrast, our measure of conservatism is based directly on our construct of β and thereby mirrors our theoretical analysis, which does not rely on market valuations.

Our empirical tests represent different degrees of conservatism by the fraction of investments expensed immediately. In the interest of remaining as consistent with the theory as possible, we employ a measure of conservatism (*Conserv*) that divides the portion of investment immediately expensed by total investments [(R&D Exp + Adv Exp)/ (R&D Exp + Adv Exp + Capitalized Expenditures)].²⁶ Among the many proxies for conservatism that have been employed in the empirical literature, the one that is closest in spirit to ours is the “C-score” computed by Penman and Zhang (2002). Their metric captures conservatism as a notion of reserve creation; it uses R&D expense, advertising expense and LIFO reserves and compares them to *NOA* to obtain a measure of the “quality” of earnings.²⁷ Perhaps the most commonly used measure of conservatism is the Market-to-Book ratio. In unreported tests, we find that our measure exhibits a strong positive correlation with the M/B ratio. Because our measure is new, our empirical tests ultimately become a joint test of the theory as well as the degree to which this measure captures conservatism.

Internal Rate of Return

Our theoretical analysis has treated the internal rate of return (r) as exogenous, with the

²⁶It could be argued that this measure simply measures the degree to which a firm is a member in an industry that employs a great deal of “intangible assets” such as the pharmaceutical or technology industries. We accept this criticism, but argue that this strong correlation with industry does not diminish the measure’s usefulness since these industries are ones in which there is a higher degree of conservatism. Note that our theory does not require that the degree of conservatism be relative to firms within the industry but rather across the economy.

²⁷In related analysis, Monahan (2005) examines the link between growth in R&D expenditures and the association between residual income valuation models and actual equity market values.

sole restriction that it is at least equal to the required rate of return r_c . From an empirical standpoint it is exceedingly difficult to capture the internal rate of return for individual firms at particular points in time. This motivates us to decompose r into two, more easily measured concepts. In particular, we decompose the internal rate of return into the cost of capital, on the one hand, and into “abnormal” profitability, on the other hand. To do this decomposition in a manner that is fully consistent with our prior analysis, we suppose that economic profitability is captured by a variable θ that scales the firm’s (constant) cash flows from any investment. In other words, given the internal rate of return r , the following zero-NPV condition holds:

$$\frac{c \cdot \theta \cdot [1 - (1 + r)^{-T}]}{r} - b_0 = 0, \quad (6)$$

where $\theta \geq 1$ represents the abnormal profitability parameter. A value of $\theta = 1$ corresponds to a competitive industry where all investments yield zero NPV relative to the required cost of capital $r = r_c$. Thus, (6) holds at $\theta = 1$ provided r is replaced by $r = r_c$, while higher values of θ correspond to an internal rate of return in excess of r_c . For our setting in Proposition 5, ROI can now be represented as:

$$ROI_T(\lambda, 0, \beta | r_c, \theta) = \frac{1}{1 - \beta} \cdot [ROI_T(\lambda, 0 | r_c, \theta) - \lambda \cdot \beta],$$

where

$$ROI_T(\lambda, 0 | r_c, \theta) = \left(\frac{\lambda}{f(\lambda, T)} \right) \cdot \left(-1 + r_c \cdot \theta \cdot T \frac{(1 + r_c)^T}{(1 + r_c)^T - 1} \right). \quad (7)$$

Thus we have replaced the internal rate of return by two other constructs - the required rate of return, r_c , and the firm’s level of abnormal profitability, θ .

We proxy for r_c using a firm’s equity cost of capital. The primary reason for this choice (relative to *WACC*, for example) is that our model does not consider debt. We assume that firms are financed entirely by equity, and the use of the equity cost of capital in our empirical analysis is consistent with this presumption. That said, estimating the equity cost of capital is a daunting endeavor. As Fama and French (1997) note, “..two of the ubiquitous tools in capital budgeting are a wing and a prayer.” Nonetheless, accounting researchers have developed a variety of techniques to determine the cost of capital. Recently, Botosan and Plumlee (2005) review five empirical proxies from the literature and evaluate them for consistency and relation to risk. They find that two of them, the r_{DIV} (Botosan and Plumlee, 2002), and r_{PEG} (Easton 2004) dominate the other alternatives. Others however, using different methodologies, come to somewhat different conclusions. For example, Guay, Kothari, and Shu (2005) find that r_{GLS} (Gebhardt, Lee and Swaminathan, 2001) is the best predictor of future realized returns. Appendix B summarizes these three measures of the

equity cost of capital. Because each method requires data from different sources, choosing one method or another may limit the number of observations and possibly bias our sample depending on data that is required for the respective measure (Botosan and Plumlee [2005]). Accordingly, in this paper, we use an arithmetic average of these three measures as our empirical proxy for the cost of capital.²⁸ Because each of the three measures may have a different standard deviation, a simple average may not be sufficient when used in a regression. As in Gaver and Gaver (1993) and Guay (1999) we employ common factor analysis to construct a single mean-zero variable (*COC factor*) that captures variation common to the various cost of capital measures when using them in regression analysis.

To proxy for θ , the spread between the IRR and the required rate of return, we employ a notion of firm-specific abnormal profitability. Empirical estimates of this “abnormal profitability” are again notoriously difficult to obtain. We empirically proxy for this notion by taking an average of the prior three years’ abnormal ROA (AbROA) measured as the difference between ROA of the firm and the industry median for that year. We determine the industry via the use of two digit SIC codes (similar to the approach used by Cheng, 2005).

Other Variables

For our analysis of profitability, we use return on equity, ROE, which is calculated as operating income after depreciation and after interest expense and interest income (Compustat Item #170) deflated by lagged book value of equity (Compustat Item #60). Because taxes are a relevant aspect of any analysis, we multiply our income measures by (1-marginal tax rate) where the tax rates are the same ones used by Nissim and Penman (2001). As a robustness check, we also re-estimate all of our regressions and empirical tests on a pre-tax basis to maintain consistency between our analysis and other papers in the empirical literature. Inferences are unchanged. Note that our theoretical treatment of the profitability measure is indifferent between ROA and ROE because the firms in our model are assumed to have no debt. Accordingly, in the empirical analysis we use both ROE and ROA.²⁹

Our primary empirical measure of return on investment is ROE and we compare it to the equity cost of capital. We also calculate ROA as operating income (Compustat Item

²⁸This approach also has the advantage of dampening random variation caused by different estimation procedures (Dhaliwal et al. (2005) and Hail and Leuz (2004) also use this approach). When not all three measures are available for the same firm-year observation, we take the average of as many different measures as are available to keep the sample size as large as possible.

²⁹For ROE in particular one may wonder whether this metric is affected by dividend payments. Simple algebra shows that *ROE* is invariant to changes in dividend payments provided the following holds: *ROE* is equal to the rate of return, r and a dividend payment of d_{t-1} reduces BVE_{t-1} by d_{t-1} and at the same time reduces income, Inc_t , by $r \cdot d_t$.

#178) deflated by lagged total assets (Compustat Item #6) as an alternative measure of return on investment for all our parametric tests. When using ROA, we compare it with the weighted average cost of capital (*WACC*).³⁰ All the results using ROA and WACC are discussed in the empirical section as well. Sales is Compustat Item #12. Finally, several indicator variables are used. First, *ROEgtIRR* is equal to 1 when ROE is greater than IRR and 0 otherwise. IRR is computed as $COC + AbROA$. However, because estimates of the IRR are noisy and imprecise, we estimate an indicator variable that is equal to 1 when *PGrowth* is greater than IRR by more than 1% (*SEQuad*) and another that is equal to one when *PGrowth* is less than IRR by more than 1% (*NWQuad*). These variables allow us to explore our quadrant result without relying too heavily on the exact empirical estimation of the COC and the IRR.

3.2 Sample Selection

Our empirical tests employ data from several sources. Financial statement data are obtained from the *Compustat* annual database. Data for our cost of capital measures comes from *I/B/E/S* as well as Value Line. Our sample covers all firm year observations with available *Compustat* data and enough data to calculate at least one of the three cost of capital measures we employ and ranges from 1982 to 2002. We exclude all firm-year observations with SIC codes in the range 6000-6999 (financial companies) because the demarcation between operating and financing activities is not clear in these firms. We eliminate firm-year observations with insufficient data on *Compustat* to compute the primary financial statement variables used in our tests. Finally, we eliminate firms where the average compound growth in investments over the past T periods is lower than -50%. Given the average useful life in our sample is 12 years, any firm with an average drop of 50% in growth over 12 years is likely to be a firm in serious decline. These criteria yield a final sample size of 43,680 firm-year observations. The number of observations in any particular test will vary depending on the availability of data necessary for the particular test.

Empirical Methodology and Results

We report results based on the time-series means and t-statistics of annual cross-sectional regressions (Fama and MacBeth, 1973). This approach typically generates a conservative

³⁰*WACC* is calculated as: $\frac{BVE}{BVD+BVE} \cdot r_E + \frac{BVD}{BVD+BVE} \cdot r_D \cdot (1 - \tau)$, where BVE denotes the book value of equity (Compustat Item #60), BVD denotes the book value of debt (Compustat Items #9 + #34), r_E is cost of equity capital (from above), r_D represents the borrowing cost of debt (interest expense) and τ is the applicable income tax rate (Compustat Item #15). The average debt cost and the applicable tax rate are calculated by year similarly to the methodology in Nissim and Penman (2001).

estimate of statistical significance (Loughran and Ritter, 2000). Results are qualitatively similar, albeit more statistically significant, when using pooled OLS regressions. Finally, the test of Proposition 5 requires the use of changes on the left hand side of the regression, with overlapping periods. The use of overlapping observations induces serial correlation in the regression residuals and the standard errors are biased downward if they are not corrected for this induced autocorrelation. We correct for this using the Generalized Method of Moments (GMM) standard errors with the Newey-West correction (Newey-West, 1987), where we set K , the number of over-lapping periods, equal to one.

Although it is important to study our variables in their natural continuous form, we also take steps to ensure that our conclusions are statistically valid. To add statistical assurance to any conclusions based on our continuous regression estimations, we estimate another set of regressions where the continuous value of the independent variable amount is replaced with its annual decile rank. To create decile ranks, all the continuous variables are sorted annually into ten equal-sized groups numbered zero to nine each year and then divided by nine. This second set of regressions present more conservative statistical tests; the only assumption about the regression's functional form is that the relations are monotonic (Iman and Conover, 1979). Accordingly, all of our analysis will be presented using both continuous and rank regressions that are calculated using the Fama and MacBeth (1973) regression methodology.

Descriptive Statistics

Table 1 presents the descriptive statistics for our sample. ROE shows a mean of about 15.4% which is consistent with prior work. Cost of capital has a mean (median) of 13% (12%) and is consistent with the long-run return of the stock market over the past 20 years. Figure 4 shows the distribution of *PGrowth*. It is positively skewed with a mean of 48% but a median of only about 14% because the variable is truncated on the downside at -100% and has unlimited upside. The average useful life of the assets in our sample is 12.7 years. Taken together, many of the descriptive statistics are reasonable in magnitude and do not appear to be subject to extremes. One exception is *PGrowth*, which does not appear to be normally distributed. Panel B of Table 1 presents various descriptive information by *PGrowth* decile.

Test of Proposition 1

Because of the dichotomous nature of the dependent variable, we estimate a pooled logistic regression to empirically test Proposition 1 as follows:

$$ROEgtIRR = \beta_0 + \beta_1 \cdot NWQuad + \beta_2 \cdot Uselife + \beta_3 \cdot AbROA + \beta_4 \cdot Conserv + \beta_5 \cdot PGrowth + \epsilon$$

and

$$ROE_{gtIRR} = \gamma_0 + \gamma_1 \cdot SEQuad + \gamma_2 \cdot Uselfe + \gamma_3 \cdot AbROA + \gamma_4 \cdot Conserv + \gamma_5 \cdot PGrowth + \epsilon.$$

If the quadrant result holds, the expected coefficient on β_1 is positive, whereas γ_1 is expected to be negative. Panel A of Table 2 presents the *NWQuad* results and the coefficient β_1 is positive and statistically significant across both specifications. Similarly, Panel B presents the *SEQuad* result and across both specifications the coefficient γ_1 is negative and statistically significant.³¹

Test of Proposition 2

Given the monotonicity result in Proposition 2, we ask whether ROE is decreasing in the geometric mean of growth in investments over the prior T periods. Specifically, we estimate:

$$ROE_t = \rho_0 + \rho_1 \cdot PGrowth + \rho_2 \cdot Uselfe + \rho_3 \cdot Conserv + \rho_4 \cdot COCfactor + \rho_5 \cdot AbROA + \epsilon_t$$

The coefficient ρ_1 is predicted to be negative. Because the theory analyzes these relations holding all else constant, we include several control variables such as *Uselfe*, *Conserv*, *COCFactor*, and *AbROA*. Throughout the rest of the analysis, we continue to control for these variables to ensure that alternative correlated aspects of the analysis do not drive our main results.

Test of Corollary 1

Next, we explore how changes in ROE relate to changes in current growth relative to past growth. Our prediction is that the larger the deviation in the current period's growth from the (assumed constant) average growth of the prior $T - 1$ periods, the larger the drop in next period's ROE relative to current ROE. To operationalize this prediction empirically, we adopt a changes specification on the LHS in *ROE* and a deviation from prior growth on the RHS of the regression equation:

$$\Delta ROE_{t+1} = \rho_0 + \rho_1 \cdot [Growth_t - PGrowth] + \rho_2 \cdot Uselfe + \rho_3 \cdot Conserv + \rho_4 \cdot COCfactor + \epsilon_{t+1}.$$

Here, $\Delta ROE_{t+1} = ROE_{t+1} - ROE_t$ and $Growth_t - PGrowth$ is the difference between the current period's growth in investments and the geometric mean of growth in the prior $T - 1$ periods. Panel B of Table 1 presents the mean levels of ROE as a function of *PGrowth* deciles. As growth increases we see a general negative trend in ROE. Surprisingly, firms in

³¹Note that because of the requirement that *PGrowth* and IRR deviate by 1%, *NWQuad* = 1 does not immediately imply that *SEQuad*=0.

the first decile of *PGrowth* have lower ROE's and there is a large increase in the 2nd decile. This pattern is not predicted by our theory. The firms in this decile have a mean decline in *PGrowth* of about 12%; coupling that with an average *Uselife* of 12 years makes it clear that these are firms in severe decline. Thus, ex-post, although not predicted by the above model, it appears reasonable that these firms would not be as profitable and display lower levels of ROE.

Finally, Proposition 2 is tested statistically using the empirical equations above and the results are provided in Table 3. Panel A presents the results of the *ROE* level and Panel B presents the future change in *ROE*. In Panel A, the coefficient on *PGrowth*, 1 is negative and statistically significant with a negative t-value of 4.96 in Model 1. Model 2 uses ranks and finds similar results. As mentioned earlier, the coefficient in this regression can be interpreted to give a better sense of the economic significance of the effect. The coefficient of -0.037 indicates that the difference in ROI is about 4% when moving from the top to bottom deciles of *PGrowth* holding everything else constant. Panel B presents the results for the changes specification. Similar to the levels result, the coefficient on ρ_1 is negative and statistically significant across both continuous and rank specifications. In addition, the coefficient on the rank regression is similar in economic magnitude to the levels specification at 3%.³²

Test of Proposition 4

After imposing straight-line depreciation, the convexity result of Proposition 4 states that ROE is not only a decreasing but also a convex function of past growth. Thus, a simple linear approximation will not adequately capture the impact of growth on conservatism. Accordingly, we estimate an empirical relation similar to the one above, but include a quadratic term to capture the convexity:

$$ROE_t = \rho_0 + \rho_1 \cdot PGrowth + \rho_2 \cdot Uselife + \rho_3 \cdot Conserve + \rho_4 \cdot COCfactor \\ + \rho_5 \cdot Conserv \cdot PGrowth + \rho_6 \cdot ABROA + \rho_7 \cdot [PGrowth]^2 + \epsilon_t$$

If the relationship between growth and *ROE* is convex as predicted by the model, then we would expect the coefficient on ρ_7 to be positive in this regression. Note that we estimate the regression with and without the cross-partial effects (ρ_5) as well. The results are presented on Table 4. In both empirical estimations, the bolded coefficient ρ_7 is positive and

³²As mentioned earlier, we also examine these same tests and use ROA instead of ROE and WACC instead of COC. Our results are qualitatively similar. The coefficient on *PGrowth* and the current deviation from *PGrowth* is negative in all specifications.

statistically significant.³³ The main effect on ρ_1 is still negative; however, the cross-partial result on ρ_5 is no longer negative and significant, indicating that the non-linearity in the squared term explains some of the effect captured in Proposition 2.³⁴

Test of Proposition 5

Proposition 5 predicts that increases in conservatism will further decrease the negative slope of the previously discussed relation between growth and *ROE*. This proposition is formally tested as follows:

$$ROE_t = \rho_0 + \rho_1 \cdot PGrowth + \rho_2 \cdot Uselfe + \rho_3 \cdot Conserv + \rho_4 \cdot COCfactor \\ + \rho_5 \cdot AbROA + \rho_6 \cdot Conserv \cdot PGrowth + \epsilon_t$$

We expect the coefficient ρ_6 to be negative indicating the slope will continue to decrease as the degree of conservatism (*Conserv*) increases. Finally, although not predicted by the theory, we explore whether the empirical changes specification from Corollary 1 holds in this case. Specifically, we test whether the cross-partial result is also found in a deviation from prior growth. Accordingly, we estimate the following regression:

$$\Delta ROE_{t+1} = \rho_0 + \rho_1 \cdot [Growth_t - PGrowth] + \rho_2 \cdot Uselfe + \rho_3 \cdot Conserv \\ + \rho_4 \cdot COCfactor + \rho_5 \cdot Conserv \cdot [Growth_t - PGrowth] + \epsilon_{t+1}.$$

Once again, the cross-partial result on the coefficient ρ_5 is also expected to be negative. Looking at Panel B of Table 1, firms in the first decile of *PGrowth* have lower ROE's across all three levels of *Conserv* and there is a large increase in the 2nd decile. Consistent with the theory, firms with higher *Conserv*'s (measured with deciles), have lower mean and median levels of ROE; this becomes apparent in deciles 5 - 10.

Table 5 presents descriptive statistics consistent with Figure 1. Starting with the low *Conserv* column, we find that the mean *ROE* falls from 19.3% to 14.5% when moving from low to high *PGrowth*. This drop of 4.8% is consistent with our theory. Further, we find that as conservatism increases, the difference increases to 7.3%, then to 12.3%, as we move

³³Note that we do not estimate rank regressions for this particular empirical specification. The process of creating decile ranks after squaring is ordinal because it preserves the monotonic relationship. Squaring a variable is a simple monotonic transformation and recall that this is the primary attribute of rank regressions (Iman and Conover, 1979). Thus, the rank variable will have the exact values as the primary value after decile ranking and adds no new information or variation to the regression estimation.

³⁴When estimating this regression using ROA, we do not find statistically significant results. The coefficient on the squared term is not statistically significant in either specification.

from low to high *Conserv* terciles respectively (these differences are statistically significant in unreported tests). Table 6 presents the parametric tests of Proposition 5. Again, Panel A presents the level of ROE and Panel B presents the changes in ROE.

In Model 1, the cross-partial variable is added and the coefficient on ρ_6 is found to be negative and significant as predicted by the theory. However, in the ranks regression (Model 2), the interaction term is insignificant; thus, there are mixed results across the two specifications. Panel B presents the changes regression and, once again, the interaction term is negative and significant as predicted by our theory across both specifications. The main effect continues to be significant at the 1% level with a negative t-statistic of 3.20 on ρ_1 in Model 3. Similarly, the rank coefficient remains significant and negative in Model 4. Moreover, in unreported tests, when using ROA instead of ROE, ρ_1 and ρ_5 are negative and statistically significant in both of the empirical specifications. Further, the results are larger in economic significance.

4 Concluding Remarks

ROI is arguably the most widely-used measure of firm profitability. In this paper, we have examined the fundamental issue of how ROI relates to the underlying economic profitability of a firm’s investment projects. Our main conclusion is that accounting conservatism and past growth in investments jointly determine how ROI compares to the underlying internal rate of return. Given conservative accounting, faster growth tends to depress ROI and this decline will be more pronounced for more conservative accounting rules. Conversely, the impact of higher degree of conservatism on ROI will depend on whether past growth rates are above or below a critical level, given by the internal rate of return of the firm’s projects. A major contribution of our analysis is the derivation of a closed-form expression for the steady-state ROI expressed as a function of several key variables: conservatism, growth in new investments, the useful life of assets, and the internal rate of return of projects available to the firm.

Our analytical results suggest a series of hypotheses which we test using a large-scale panel dataset data that spans a 20-year time period. Our choice of empirical proxies for the variables mirrors the nature of the constructs used in deriving the analytical results. Overall, we find consistent support for our predictions regarding the behavior of accounting rates of return, under a variety of specifications (including both levels and changes). In particular, return on equity, our primary proxy for ROI, is found to satisfy the “quadrant result,” as well as to behave in a manner predicted by the model with regard to growth, accounting conservatism, and the interaction of these variables.

The thrust of our analysis has been to understand distortions in the accounting rate of return relative to the underlying economic profitability. Our analysis may also lend itself to a reverse approach: infer economic profitability from observed ROI's in conjunction with other firm characteristics including growth, the useful life of assets, the proportion of intangible assets etc. This type of inference has long been a major challenge to economists seeking to assess the competitiveness of particular industries.

Our modeling framework has envisioned a representative firm with exogenously specified levels of economic profitability and exogenous growth rates in investments. We have thus abstracted away from issues related to the specific structure of the product markets that the firm is competing in. In future work, it would be of interest to develop a richer model of imperfect competition that entails such elements as the formation of product prices, new investments and entry or exit decisions. As a general matter, such an expanded model should make it possible to relate observed industry characteristics to the accounting profitability for firms in that industry.

While this paper has focused on the accounting rate of return, it is natural to ask how our findings extend to other key financial ratios such as the Market-to-Book or the Price-Earnings ratio. Extending our analysis along these lines would not only be of interest in its own right but would also facilitate the comparison to the rapidly growing literature on conservatism in accounting, which for the most part has examined conservatism in its relation to equity values.

TABLE 1
Descriptive Statistics

Panel A: Descriptive Statistics of Entire Sample

Variable	Mean	Std Dev	25%	50%	75%
<i>ROE_t</i>	9.3%	23.9%	2.8%	11.7%	18.7%
<i>ΔROE_{t+1}</i>	(0.046)	0.353	(0.114)	(0.012)	0.060
<i>COC</i>	13%	5%	10%	12%	15%
<i>PGrowth</i>	0.48	20.40	0.06	0.14	0.31
<i>Conserv</i>	31%	23%	14%	27%	45%
<i>Uselife</i>	12.7	7.5	7.3	11.4	15.9

The full sample consists of 43,680 firm-year observations from 1982 to 2002. The variables are defined as follows: *ROE* is calculated as operating income after depreciation and after interest expense and interest income (Compustat Item #170) deflated by lagged book value of equity (Compustat Item #60). *Uselife* is the gross amount of PPE (Compustat item #7) divided by depreciation expense (Compustat item #125) in the current year. *PGrowth* is the geometric mean of growth in investments over the previous *Uselife* periods starting in period $t - 1$ where $Total\ Investments = R\&D\ Exp\ (Compustat\ item\ \#45) + Adv\ Exp\ (Compustat\ item\ \#46) + Capitalized\ Expenditures\ (Compustat\ item\ \#30)$. Growth is calculated as $[(Total\ Investment_t / Total\ Investment_{t-1}) - 1]$. $Growth_t$ is the growth in investments in the current period. $Conserv = [(R\&D\ Exp + Adv\ Exp) / (R\&D\ Exp + Adv\ Exp + Capitalized\ Expenditures)]$. *AbROA* is measured as an average of the prior three years abnormal *ROA* calculated as *ROA* for firm i in year t less the industry median for the same year where the industry is determined using 2 digit SIC codes. *COC* is the arithmetic average of r_{DIV} , r_{PEG} , and r_{GLS} (See Appendix B for specifications). *COCfactor* is obtained using common factor analysis on the three measures of *COC* listed above. *Sales* is Compustat Item #12. All financial statement variables are winsorized at the 1% and 99% levels.

Panel B: Mean (medians) by Growth Decile

	<i>PGrowth</i> Decile									
	1	2	3	4	5	6	7	8	9	10
<i>PGrowth</i>	-12%	1%	5%	8%	12%	17%	23%	33%	54%	340%
	-8%	1%	5%	8%	12%	16%	22%	30%	49%	116%
<i>ROE</i>	8%	11%	12%	12%	12%	11%	10%	8%	6%	3%
	9%	12%	12%	13%	13%	13%	12%	12%	10%	8%
<i>COC</i>	15%	13%	13%	13%	13%	13%	13%	13%	13%	14%
	13%	12%	12%	12%	12%	12%	12%	12%	12%	13%
<i>Conserv</i>	30%	28%	29%	30%	31%	32%	35%	35%	37%	35%
	26%	25%	26%	27%	29%	28%	31%	33%	35%	33%
<i>Uselife</i>	12.5	15.8	15.8	15.0	14.1	13.0	11.4	10.5	9.5	9.0
	11.1	14.3	14.4	13.6	12.8	11.8	10.4	9.4	8.2	7.0
<i>MVE</i>	1,531	3,024	4,138	4,203	3,738	2,898	2,421	1,763	1,370	916
	237	601	758	750	587	487	400	334	232	194
<i>BM</i>	0.69	0.67	0.64	0.61	0.59	0.59	0.57	0.55	0.55	0.57
	0.57	0.60	0.57	0.55	0.53	0.50	0.47	0.45	0.43	0.42
<i>Sales</i>	1,756	3,427	4,601	4,842	3,846	2,797	2,010	1,402	868	476
	352	853	1,052	1,008	744	549	405	292	193	128

TABLE 2
Tests of Quadrant Result using Logistic Regressions

Panel A:

$$ROE_{gt}COC = \beta_0 + \beta_1 NWQuad + \beta_2 Uselfe + \beta_3 AbROA + \beta_4 Conserv + \beta_5 PGrowth + \epsilon$$

	β_0	β_1	β_2	β_3	β_4	β_5	$Pr > \chi^2$
Coefficient	0.171	0.340					291
(P-value)	< .0001	< .001					< .0001
Coefficient	0.675	0.082	0.010	-0.141			2145
(P-value)	< .0001	0.0004	< .0001	0.0180			< .0001
Coefficient	0.436	0.084	0.018	-0.061	0.449	0.014	2234
(P-value)	< .0001	0.0003	< .0001	0.3812	< .0001	0.0413	< .0001

Panel B:

$$ROE_{gt}COC = \gamma_0 + \gamma_1 SEQuad + \gamma_2 Uselfe + \gamma_3 AbROA + \gamma_4 Conserv + \gamma_5 PGrowth + \epsilon$$

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	$Pr > \chi^2$
Coefficient	0.161	-.0375					347
(P-value)	< .0001	< .0001					< .0001
Coefficient	0.782	-.357	0.010	-0.139			2158
(P-value)	< .0001	< .0001	< .0001	< .0409			< .0001
Coefficient	0.544	-0.351	0.017	-0.050	0.446	0.016	2246
(P-value)	< .0001	< .0001	< .0001	0.4335	< .0001	0.0218	< .0001

TABLE 3
Time-Series Means and t-Statistics for Coefficients from Annual
Cross-Sectional Regressions of *ROE* on Growth

Panel A:

$$ROE_t = \rho_0 + \rho_1 P\text{Growth} + \rho_2 U\text{selife} + \rho_3 \text{Conserv} + \rho_4 \text{COC factor} + \rho_5 \text{AbROA} + \epsilon_t$$

	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	Adj. R^2
Model 1	0.112	-0.012	0.000	-0.009	-0.060	0.539	28.1%
	21.22	-4.83	2.57	-1.33	-12.97	17.25	16.43
Ranks Replacing Continuous Values							
Model 2	0.069	-0.037	0.024	0.003	-0.117	-1.949	24.6
	8.62	-4.87	3.72	0.64	-15.46	22.12	19.92

Panel B:

$$\Delta ROE_{t+1} = \rho_0 + \rho_1 [\text{Growth}_t - P\text{Growth}] + \rho_2 U\text{selife} + \rho_3 \text{Conserv} + \rho_4 \text{COC factor} + \epsilon_{t+1}$$

	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	Adj. R^2
Model 3	-0.028	-0.004	0.001	-0.019	-0.004	1.5%
	-4.56	-3.90	2.67	-3.18	-1.01	4.93
Ranks Replacing Continuous Values						
Model 4	-0.020	-0.031	0.026	-0.007	0.001	1.3%
	-2.76	-7.76	3.66	-1.40	-0.13	4.80

TABLE 4
Time-Series Means and t-Statistics for Coefficients from Annual
Cross-Sectional Regressions of *ROE* on Growth and Convexity

$$ROE_t = \rho_0 + \rho_1 P\text{Growth} + \rho_2 U\text{selife} + \rho_3 \text{Conserv} + \rho_4 \text{COCfactor} + \rho_5 \text{Conserv} \cdot P\text{Growth} + \rho_6 \text{AbROA} + \rho_7 [P\text{Growth}]^2 + \epsilon_t$$

	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	Adj. R^2
Mean Coefficient	0.118	-0.032	0.000	-0.010	-0.059		0.548	0.004	28.4%
t-statistic	20.40	-6.42	1.77	-1.40	-12.91		16.74	2.67	16.84
Mean Coefficient	0.118	-0.032	0.000	-0.008	-0.059	-0.003	0.549	0.004	28.6%
t-statistic	20.87	-6.45	1.78	-1.03	-12.96	-0.30	16.82	2.92	16.81

TABLE 5
Mean (Median) *ROE*_t Levels by *Conserv* and *Growth* terciles.

		<i>Conserv</i>		
		Low	Medium	High
<i>PGrowth</i>	Low	0.193	0.217	0.152
		(0.204)	(0.213)	(0.185)
	Medium	0.192	0.196	0.115
Tercile		(0.203)	(0.209)	(0.174)
	High	0.145	0.144	0.029
Difference Between		0.048	0.070	0.123
High and Low <i>PGrowth</i>		(0.020)	(0.030)	(0.080)

TABLE 6
Time-Series Means and t-Statistics for Coefficients from Annual
Cross-Sectional Regressions of ROE and ΔROE
on Growth and Conservatism

Panel A: $ROE_t =$

$$\rho_0 + \rho_1 PGrowth + \rho_2 Uselfe + \rho_3 Conserv + \rho_4 COCfactor + \rho_5 AbROA + \rho_6 Conserv \cdot PGrowth + \epsilon_t$$

	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	Adj. R^2
Model 1	0.111	-0.010	0.000	-0.002	-0.595	-0.544	-0.020	28.3%
	20.99	-3.06	2.48	-0.33	-12.94	17.13	-2.98	16.56
Ranks Replacing Continuous Values								
Model 2	0.114	-0.063	0.039	0.001	-0.194	3.18	0.005	24.6%
	8.34	-3.73	3.95	0.61	-15.33	27.48	0.41	20.07

Panel B: $\Delta ROE_{t+1} = \rho_0 + \rho_1 [Growth_t - PGrowth] + \rho_2 Uselfe + \rho_3 Conserv + \rho_4 COCfactor + \rho_5 Conserv \cdot [Growth_t - PGrowth] + \epsilon_{t+1}$

	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	Adj. R^2
Model 3	-0.028	-0.002	0.001	-0.019	-0.004	-0.012	1.6%
	-4.60	-3.20	2.70	-3.18	-1.04	-2.54	5.27
Ranks Replacing Continuous Values							
Model 4	-0.017	-0.014	0.025	-0.009	-0.001	-0.019	1.3%
	-2.46	-2.66	3.63	-1.96	-0.15	-3.53	4.74

Appendix A

Proof of Lemma 1:

Direct substitution yields

$$\sum_{i=1}^t RI_i \cdot \gamma^i = \gamma \cdot [c_1^0 - (d_1 + r) \cdot b^o] + \gamma^2 \cdot [c_2^0 - (d_2 + r \cdot (1 - d_1)) \cdot b^o] + \dots \quad (8)$$

$$+ \gamma^t \cdot [c_t^0 - (d_t + r \cdot (1 - d_1 - \dots - d_{t-1})) \cdot b^o]. \quad (9)$$

Collecting coefficients for each of the variables (d_1, \dots, d_t) on the right-hand side of the above equation, the coefficient for d_i is:

$$\gamma^i - r \cdot [\gamma^{i+1} + \dots + \gamma^t] = \gamma^{i+1} - r \cdot [\gamma^{i+2} + \dots + \gamma^t] = \dots = \gamma^t.$$

Thus,

$$\sum_{i=1}^t RI_i^0 \cdot \gamma^i = \sum_{i=1}^t \gamma^i \cdot c_i^0 - [(1 - \gamma^t) + \gamma^t \cdot (d_1 + \dots + d_t)] \cdot b^0$$

By definition of neutral accounting

$$\sum_{i=1}^t \gamma^i \cdot c_i^0 - [(1 - \gamma^t) + \gamma^t \cdot (d_1^* + \dots + d_t^*)] \cdot b^0 = 0.$$

Therefore $\sum_{i=1}^t d_i \geq \sum_{i=1}^t d_i^*$ is equivalent to $\sum_{i=1}^t RI_i^0 \cdot \gamma^i \leq 0$. ■

Proof of Proposition 1:

The expression for ROI at date T is given in (1). Therefore $ROI_T(\vec{\lambda}) \geq r$ is equivalent to:

$$RI_T^o + (1 + \lambda_1) \cdot RI_{T-1}^o + (1 + \lambda_1) \cdot (1 + \lambda_2) \cdot RI_{T-2}^o + \dots + \prod_{i=1}^{T-1} (1 + \lambda_i) \cdot RI_1^o \geq 0, \quad (10)$$

where $RI_t^o \equiv c_t^o - d_t \cdot b^o - r \cdot b^o (1 - \sum_{j=1}^{t-1} d_j)$ denotes the residual income of the representative project $\mathcal{P} = (b^o, c_1^o, \dots, c_T^o)$. Lemma 1 shows that for any conservative depreciation schedule (d_1, \dots, d_T) :

$$\sum_{i=1}^t RI_i^o \cdot \gamma^i \leq 0 \quad (11)$$

for all $1 \leq t \leq T - 1$. The claim therefore amounts to showing that for any $T - 1$ tuple $(RI_1^o, \dots, RI_{T-1}^o)$ satisfying (11), inequality (10) will be met if $\lambda_t \leq r$. Since $\sum_{i=1}^T RI_i^o \cdot \gamma^i = 0$, inequality (10) can equivalently be written as:

$$\begin{aligned}
& RI_{T-1} \cdot [(1 + \lambda_1) - \gamma^{-1}] + RI_{T-2} \cdot [(1 + \lambda_1)(1 + \lambda_2) - \gamma^{-2}] + \dots \\
& + RI_1 \cdot \left[\prod_{i=1}^{T-1} (1 + \lambda_i) - \gamma^{-(T-1)} \right] \geq 0.
\end{aligned} \tag{12}$$

The inequalities in (11) can be represented in matrix form as:

$$\Gamma \cdot \vec{RI}^0 \leq 0 \tag{13}$$

where

$$\Gamma = \begin{bmatrix} \gamma & & & & \\ \cdot & \gamma^2 & & & \\ \cdot & \cdot & \gamma^3 & & \\ \cdot & \cdot & \cdot & \ddots & \\ \cdot & \cdot & \cdot & \cdot & \\ \gamma & \gamma^2 & \gamma^3 & \dots & \gamma^{T-1} \end{bmatrix}$$

and $\vec{RI}^0 = (RI_1^0, \dots, RI_{T-1}^0)$. By Farkas' Lemma (see Rockafellar [1970]), (12) holds for any vector satisfying (13) if and only if there exists a non-negative row-vector $v' = (v_1, \dots, v_{T-1})$ such that:

$$v' \cdot \Gamma = \begin{pmatrix} \gamma^{1-T} - \prod_{i=1}^{T-1} (1 + \lambda_i) \\ \vdots \\ \gamma^{-1} - (1 + \lambda_1) \end{pmatrix}. \tag{14}$$

Since Γ is a diagonal matrix, the system of equations in (14) can be solved explicitly, yielding:

$$\begin{aligned}
v_{T-1} &= \gamma^{-(T-1)} [\gamma^{-1} - (1 + \lambda_1)] \\
v_{T-2} &= \gamma^{-(T-2)} (1 + \lambda_1) [\gamma^{-1} - (1 + \lambda_2)] \\
&\vdots \\
v_1 &= \gamma^{-1} \cdot \prod_{i=1}^{T-1} (1 + \lambda_i) [\gamma^{-1} - (1 + \lambda_{T-1})]
\end{aligned}$$

It follows that $v_t \geq 0$ if and only if $\gamma^{-1} \geq (1 + \lambda_t)$, or equivalently, $r \geq \lambda_t$.

If the annual growth rates all exceed r , the claim is that inequality (12) reverses. The same line of arguments as before applies with the vector $w' \geq 0$, required by Farkas' Lemma, given by:

$$w_t = -v_t \geq 0.$$

Finally, for liberal accounting, the vector of residual income numbers satisfies:

$$\Gamma \cdot \vec{RI}^0 \geq 0.$$

Therefore the inequalities are “flipped,” such that (10) holds provided $\lambda_t \geq r$ and the opposite is true for $\lambda_t \leq r$. ■

Proof of Proposition 2:

To demonstrate that $ROI_T(\vec{\lambda})$ is monotone decreasing in each λ_t , we first establish the following technical result.

Claim: The function

$$H(\vec{\lambda}) = \frac{\sum_{i=0}^{T-1} a_i(\vec{\lambda})}{\sum_{i=0}^{T-1} \omega_i \cdot a_i(\vec{\lambda})}$$

is monotone decreasing in λ_t provided: (i) $\omega_{i+1} \geq \omega_i$ and (ii):

$$\frac{\frac{\partial}{\partial \lambda_t} a_i(\vec{\lambda})}{a_i(\vec{\lambda})} \leq \frac{\frac{\partial}{\partial \lambda_t} a_{i+1}(\vec{\lambda})}{a_{i+1}(\vec{\lambda})} \quad (15)$$

for all $0 \leq i \leq T - 2$.

Proof of Claim: The numerator of the derivative of $H(\vec{\lambda})$ with respect to λ_t is given by:

$$\sum_{i=0}^{T-1} \sum_{j=i+1}^{T-1} (\omega_j - \omega_i) \left[\frac{\partial}{\partial \lambda_t} a_i(\vec{\lambda}) \cdot a_j(\vec{\lambda}) - \frac{\partial}{\partial \lambda_t} a_j(\vec{\lambda}) \cdot a_i(\vec{\lambda}) \right].$$

Therefore $\frac{\partial}{\partial \lambda_t} H(\vec{\lambda}) \leq 0$ whenever conditions (i) and (ii) are met.

To apply the above Claim, we recall that $Inc_t^o = c_t^o - d_t \cdot b^o$ and set:

$$\begin{aligned} a_0(\vec{\lambda}) &= Inc_T^o \\ a_1(\vec{\lambda}) &= Inc_{T-1}^o \cdot (1 + \lambda_1) \\ &\vdots \\ a_{T-1}(\vec{\lambda}) &= Inc_1^o \cdot \prod_{i=1}^{T-1} (1 + \lambda_i). \end{aligned}$$

Recalling that $BV_t^o = (1 - \sum_{i=1}^{t-1} d_i) \cdot b^o$, we also set:

$$\omega_0 = \frac{BV_{T-1}^o}{Inc_T^o}, \quad \omega_1 = \frac{BV_{T-2}^o}{Inc_{T-1}^o}, \dots, \omega_{T-1} = \frac{BV_0^o}{Inc_1^o}.$$

Straightforward differentiation shows that the functions $a_t(\vec{\lambda})$ in (15) satisfy the elasticity conditions in part (ii) of the Claim. Finally, by neo-conservatism, RI_t^0 is increasing in t . It follows that

$$ROI_{t+1}^0 - r = \frac{RI_{t+1}^0}{BV_t^0} > ROI_t^0 - r = \frac{RI_t^0}{BV_{t-1}^0}$$

since $BV_{t-1}^0 \geq BV_t^0$. We conclude that

$$ROI_1^0 \equiv \frac{1}{\omega_{T-1}} \leq ROI_2^0 \equiv \frac{1}{\omega_{T-2}} \leq \dots \leq ROI_T^0 = \frac{1}{\omega_0}$$

implying that $\omega_0 \leq b_1 \leq \dots \leq o_{T-1}$, as recognized by condition (i) in the Claim. ■

Proof of Proposition 3: We can express $ROI_T(\lambda, \delta)$ as:

$$\frac{r(1+r)^T(\lambda+\delta)[1-(1-\delta)^T][(1+\lambda)^T-1]-\lambda\delta[(1+r)^T-1][(1+\lambda)^T-(1-\delta)^T]}{[(1+r)^T-1][\lambda(1+\lambda)^T[1-(1-\delta)^T]-\delta(1-\delta)^T[(1+\lambda)^T-1]}}.$$

Let $NROI_T(\lambda, \delta)$ represent the numerator, and $DROI_T(\lambda, \delta)$ the denominator of this expression (we will ignore the $[(1+r)^T-1]$ term in $DROI_T(\lambda, \delta)$ since it is not germane for this proof). The derivative of $ROI_T(\lambda, \delta)$ with respect to δ is

$$\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta) = \frac{g(r, T, \lambda, \delta)}{(DROI_T(\lambda, \delta))^2} \quad (16)$$

where

$$\begin{aligned} g(\cdot) = & DROI_T(\lambda, \delta) \cdot [r(1+r)^T[(1+\lambda)^T-1] \cdot [1-(1-\delta)^T + (\lambda+\delta)T(1-\delta)^{T-1}] \\ & - \lambda[(1+r)^T-1] \cdot [(1+\lambda)^T-(1-\delta)^T + T\delta(1-\delta)^{T-1}]] - NROI_T(\lambda, \delta) \cdot \\ & [\lambda(1+\lambda)^T T(1-\delta)^{T-1} - (1-\delta)^T[(1+\lambda)^T-1] + \delta T(1-\delta)^{T-1}[(1+\lambda)^T-1]]. \end{aligned}$$

Extensive simplification yields

$$g(r, T, \lambda, \delta) = [\lambda(1+\lambda)^T[(1+r)^T-1] - r(1+r)^T[(1+\lambda)^T-1]] \cdot H(\delta, \lambda, T), \quad (17)$$

where

$$H(\cdot) \equiv T\delta(1-\delta)^{T-1}[(1+\lambda)^T-1](\delta+\lambda) + \lambda[1-(1-\delta)^T][(1-\delta)^T-(1+\lambda)^T].$$

For $\lambda = r$, it follows immediately that $\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta) = 0$. Ignoring for now the $\lambda = 0$ case, we can rewrite the first expression in (17) as:

$$[(1+r)^T-1][(1+\lambda)^T-1] \left[\frac{\lambda(1+\lambda)^T}{(1+\lambda)^T-1} - \frac{r(1+r)^T}{(1+r)^T-1} \right].$$

Since the function $\frac{s(1+s)^T}{(1+s)^{T-1}}$ is monotone increasing in s , for $s > -1$ (a direct consequence of Bernoulli's Inequality), it follows that the entire expression is negative when $0 < \lambda < r$, and positive otherwise. To prove the claim, it is therefore sufficient to show that the function $H(\cdot) > (<)0$ for $\lambda < (>)0$. We next present (without elaboration) a series of facts about $H(\cdot)$ that establish this claim.

- i) $H(\delta, \lambda = -1, T) = (1 - \delta)^T [T\delta - 1 + (1 - \delta)^T] > 0$;
- ii) $H(\delta, \lambda = -\delta, T) = 0$;
- iii) $H(\delta, \lambda = 0, T) = 0$;
- iv) $H_\lambda(\delta, \lambda = -1, T) = (1 - \delta)^{T-1} [1 - \delta(1 + T) - (1 - \delta)^{T+1}] < 0$;
- v) $H_\lambda(\delta, \lambda = -\delta, T) = 0$;
- vi) $H_\lambda(\delta, \lambda = 0, T) = T^2\delta^2(1 - \delta)^{T-1} - [(1 - \delta)^T - 1]^2 < 0$;
- vii) $\text{Sign}[H_{\lambda\lambda}(\cdot)] = \text{Sign}[a + b\lambda]$, $b < 0$, $a > b$;
- viii) $H_{\lambda\lambda}(\delta, \lambda = -\delta, T) = (1 - \delta)^T [T\delta + 2 - \delta] + \delta(1 + T) - 2 \geq 0$ (≤ 0) as $\delta \geq 0$ ($\delta \leq 0$).

It follows that if $\delta > (<) 0$, then $H(\cdot)$ has a strict local minimum (maximum) at $\lambda = -\delta$. In other words, the function $H(\cdot)$ is tangent to the x-axis at $\lambda = -\delta$ but switches from positive to negative just once, at $\lambda = 0$, as was to be shown.

Finally, note that at $\lambda = 0$ and at $\lambda = -\delta$, $H(\cdot) = 0$. However the denominator of $\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta)$ is $(DROI_T(\lambda, \delta))^2$, where $DROI_T(\lambda, \delta) = \lambda(1 + \lambda)^T [1 - (1 - \delta)^T] - \delta(1 - \delta)^T [(1 + \lambda)^T - 1]$, which also equals 0 at $\lambda = 0$ and $\lambda = -\delta$. By l'Hospital's rule we find that for $\lambda = 0 < r$,

$$\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta) \rightarrow \frac{[(1 - \delta)^{T-1} \delta^2 T^2 - [(1 - \delta)^T - 1]^2] \cdot [-1 + (1 + r)^T - rT(1 + r)^T]}{[-1 + (1 - \delta)^T + \delta T(1 - \delta)^T]^2} > 0$$

as required. Similarly, for $\lambda = -\delta$, $\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta)$ tends to:

$$\begin{aligned} & - \left[\delta(1 - \delta)^T [(1 + r)^T - 1] + r(1 + r)^T [(1 - \delta)^T - 1] \right] \cdot \lim_{\lambda \rightarrow -\delta} \left(\frac{H(\delta, \lambda, T)}{(DROI_T(\lambda, \delta))^2} \right) \\ & = [(1 + r)^T - 1] [(1 - \delta)^T - 1] \cdot \left[\frac{-\delta(1 - \delta)^T}{(1 - \delta)^T - 1} - \frac{r(1 + r)^T}{(1 + r)^T - 1} \right] \\ & \quad \cdot \text{sign}\{(1 - \delta)^T \cdot [T\delta + 2 - \delta] + \delta(1 + T) - 2\}. \end{aligned}$$

As noted in viii), the latter expression has the same sign as δ . Since $-\delta > -1$, the first expression is negative if and only if $0 < -\delta < r$. This leaves three possible scenarios:

- a) $\delta > 0$ and $\lambda = -\delta < 0 < r$ implies $\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta) > 0$;
- b) $\delta < 0$ and $0 < \lambda = -\delta < r$ implies $\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta) > 0$;
- c) $\delta < 0$ and $0 < r < \lambda = -\delta$ implies $\frac{\partial}{\partial \delta} ROI_T(\lambda, \delta) < 0$.

In all cases, we therefore get the hypothesized sign, thus concluding the proof. ■

Proof of Proposition 3’:

For the case of $T = 2$, $ROI_T(\lambda, \delta)$ can be reduced to the following expression:

$$\frac{\delta[-\lambda(1+r)^2 - r(3+2r)] + (2+\lambda)r(3+2r)}{[3+2\lambda - \delta(2+\lambda)](2+r)}. \tag{18}$$

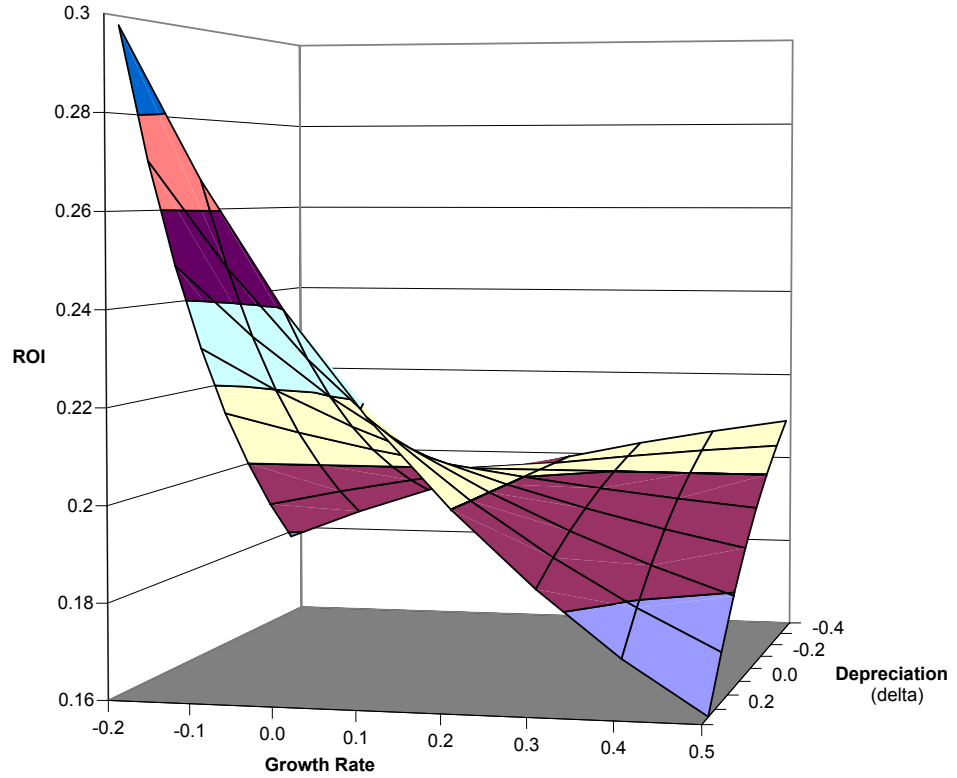
It can be shown, using conventional analytic methods, that the cross-partial of (18) in λ and δ is negative for values of $\delta > -r$.³⁵ To verify the result for higher values of T , we use numerical methods to evaluate the cross-partial derivative of ROI in δ and λ . We employed MATLAB to search over the entire grid of possible values for the four exogenous variables that determine ROI_T , with the depreciation variable, δ , restricted to the range of conservative policies (i.e., $\delta > -r$). The specific grid points used in the evaluation were as follows:

T :	3	4	5	7	10	15	20	25				
λ :	-0.90	-0.60	-0.30	-0.15	0.00	0.05	0.10	0.25	0.50	1.00	3.00	5.00
r :	0.02	0.05	0.07	0.10	0.15	0.20	0.25					
δ :	-r	-0.5r	0.0	0.1	0.2	0.3	0.5	0.7	0.9			

We considered every possible combination of the above points, a total of 5,544 evaluations in all. For every one of them, we found that the cross-partial $\frac{\partial^2}{\partial\delta\partial\lambda}ROI_T(\lambda, \delta)$ was strictly negative. This is of course equivalent to the presence of decreasing differences, thus confirming our conjecture. ■

³⁵The proof is available on request. We can actually show a stronger result: for the case of $T = 2$, the cross-partial is globally negative.

Figure 3: Return on Investment as a function of depreciation policy and growth in investment



Proof of Proposition 4:

As noted in connection with Proposition 3, uniformly distributed cash flows and an IRR of r imply:

$$b_0 = \frac{c \cdot [1 - (1 + r)^{-T}]}{r}. \tag{19}$$

Thus the steady state book value at the beginning of period t becomes

$$BV_{t-1} = \frac{b_0}{T} \sum_{j=1}^T (1 + \lambda)^{j-1} \cdot j = \frac{c \cdot [1 - (1 + r)^{-T}]}{T \cdot r} \cdot \frac{[1 - (1 + \lambda)^T + \lambda T(1 + \lambda)^T]}{\lambda^2} \tag{20}$$

Similarly, net income in period t is given by the difference between cash flows and depreciation, and equals:

$$\frac{c[(1 + \lambda)^T - 1]}{\lambda} - \frac{c \cdot [1 - (1 + r)^{-T}][(1 + \lambda)^{T-1}]}{r \cdot \lambda \cdot T}. \tag{21}$$

Combining (20) and (21) and simplifying them leads to the key of variable of interest, the steady state accounting rate of return:

$$ROI_T(\lambda, 0) = \lambda \cdot \frac{f(r, T)}{f(\lambda, T)}, \quad (22)$$

where $f(s, T) \equiv -1 + s \cdot T \frac{(1+s)^T}{(1+s)^{T-1}}$. We first prove that ROI_T is decreasing and convex in λ . Since $f(r, T) > 0$ for all $r > 0$, it is sufficient to examine the behavior of $\frac{\lambda}{f(\lambda, T)}$. After differentiating and carrying out substantial simplification, we obtain:

$$\frac{\partial}{\partial \lambda} \left(\frac{\lambda}{f(\lambda, T)} \right) = \frac{\lambda^2 T^2 (1 + \lambda)^{T-1} - [(1 + \lambda)^T - 1]^2}{[1 - (1 + \lambda)^T + \lambda T (1 + \lambda)^T]^2}. \quad (23)$$

Both the numerator and denominator of (23) equal zero at $\lambda = 0$. Repeated application of l'Hospital's rule then yields that the limit of (23), as λ goes to zero, is $\frac{-(T-1)}{3(T+1)} < 0$.

For $\lambda \neq 0$, consider the T non-negative numbers, $\{1, (1 + \lambda), (1 + \lambda)^2, \dots, (1 + \lambda)^{T-1}\}$. From the fundamental inequality that links arithmetic and geometric means, we know that

$$\frac{1}{T} [1 + (1 + \lambda) + (1 + \lambda)^2 + \dots + (1 + \lambda)^{T-1}] > [1 \cdot (1 + \lambda) \cdot (1 + \lambda)^2 \dots (1 + \lambda)^{T-1}]^{1/T} \quad (24)$$

Therefore,

$$\frac{1}{T} \left[\frac{(1 + \lambda)^T - 1}{(1 + \lambda) - 1} \right] > (1 + \lambda)^{\frac{T-1}{2}}. \quad (25)$$

Squaring the inequality in (25) shows that (23) is negative for all values of $\lambda > -1$. We have thus shown that ROI_T decreases monotonically in λ . To show convexity, we differentiate (23) again with respect to λ . The resulting second derivative of ROI_T is the product of the following two expressions:

$$\frac{T(1 + \lambda)^{T-2}}{[1 - (1 + \lambda)^T + \lambda T (1 + \lambda)^T]^3} \quad (26)$$

and

$$\begin{aligned} & [1 - (1 + \lambda)^T + \lambda T (1 + \lambda)^T] [T \lambda^2 (T - 1) + 2 \lambda T (1 + \lambda) - 2(1 + \lambda) [(1 + \lambda)^T - 1]] \\ & - 2 \lambda (1 + \lambda) (1 + T) [\lambda^2 T^2 (1 + \lambda)^{T-1} - (1 + \lambda)^{2T} - 1 + 2(1 + \lambda)^T]. \end{aligned} \quad (27)$$

For $\lambda \rightarrow 0$, it can be shown that the product of (26) and (27) converges to the following, positive expression:

$$\frac{T^2 + T - 2}{9(T + 1)} > 0.$$

Also note that for $\lambda \neq 0$, equation (26) is strictly positive everywhere. In order to demonstrate convexity, it is therefore sufficient to show that (27) is strictly positive for $\lambda \neq 0$. Letting $z = (1 + \lambda)$, (27) can be rewritten and reduced as follows:

$$\left[\begin{array}{l} 2z^{2T+2} - T^2(1+T)z^{T+3} + (T+1)(3T+4)(T-1)z^{T+2} - T(3T-1)(T+2)z^{T+1} \\ +T(T+1)^2z^T + (T+1)(T+2)z^2 - 2T(T+1)z + T(T-1) \end{array} \right]. \quad (28)$$

We need to show that the polynomial in (28), $p(z)$ say, is strictly positive for $z \neq 1$. To that end, note that $p(0) = T(T-1) > 0$. Also, $p(\cdot)$ has exactly six sign changes; from Descartes' "rule of signs", it follows that $p(\cdot)$ has at most 6 positive roots.

It can be shown, however, that $p(z)$ is of the form³⁶

$$p(z) = \sum_{k=6}^{2T+2} a_k (z-1)^k.$$

In other words, the number 1 is a root of $p(z)$ with multiplicity 6, and therefore is its unique root. As $p(0) > 0$, $p(\cdot)$ can never cross into negative territory, yielding us the desired result of convexity.

To establish the limit results i)-iii), we note that the limit result for $\lambda \rightarrow -1$ is obvious. The result for $\lambda \rightarrow 0$ follows from repeated applications of l'Hospital's rule, as shown below:

$$\begin{aligned} \lim_{\lambda \rightarrow 0} \frac{\lambda}{f(\lambda, T)} &= \lim_{\lambda \rightarrow 0} \frac{(1+\lambda)^T - 1 + \lambda T(1+\lambda)^{T-1}}{-T(1+\lambda)^{T-1} + T(1+\lambda)^T + \lambda T^2(1+\lambda)^{T-1}} \\ &= \lim_{\lambda \rightarrow 0} \frac{2(1+\lambda) + \lambda(T-1)}{-T+1+2T(1+\lambda) + \lambda T(T-1)} \\ &= \lim_{\lambda \rightarrow 0} \frac{2+\lambda+T\lambda}{1+T+T\lambda+\lambda T^2} = \frac{2}{1+T}. \end{aligned}$$

The limit result as $\lambda \rightarrow \infty$ follows from applying l'Hospital's rule once more to the final expression above. Proofs regarding the behavior of ROI_T in r and T are available from the authors upon request. ■

Proof of Proposition 5:

If a β -fraction of new investments is expensed, steady state depreciation is given by:

$$(1 - \beta) \cdot b_0 \cdot \frac{1}{T} \sum_{j=1}^T (1 + \lambda)^{j-1} = (1 - \beta) \cdot b_0 \cdot \frac{1}{T} \frac{(1 + \lambda)^T - 1}{\lambda}. \quad (29)$$

³⁶We do not include the details here for reasons of space. The technique involves considering $z = (\lambda + 1)$, and demonstrating that the coefficient on each of the first six powers of λ (i.e., λ^0 through λ^5) equals zero. A full proof is available on request.

The starting book value is simply scaled by $(1 - \beta)$, while cash flows and the specification of the internal rate of return (\hat{r}) equation are unaltered. Income in the numerator of ROI_T is therefore given by :

$$c \cdot \left[\frac{(1 + \lambda)^T - 1}{\lambda} - \beta(1 + \lambda)^T \frac{[1 - (1 + r)^{-T}]}{r} - \frac{(1 - \beta)[1 - (1 + r)^{-T}][(1 + \lambda)^T - 1]}{T\lambda r} \right], \quad (30)$$

while book value in the denominator becomes:

$$\frac{1 - \beta}{T} \cdot \frac{c[1 - (1 + r)^{-T}]}{r} \cdot \frac{[1 - (1 + \lambda)^T + \lambda T(1 + \lambda)^T]}{\lambda^2}.$$

Simplification of the resulting ratio then yields $ROI_T(\lambda, 0, \beta)$:

$$\begin{aligned} &= \lambda \cdot \left[\frac{[(1 + \lambda)^T - 1][rT - 1 + (1 + r)^{-T}] + \beta[1 - (1 + r)^{-T}][(1 + \lambda)^T - 1 - \lambda T(1 + \lambda)^T]}{(1 - \beta)[1 - (1 + r)^{-T}][1 - (1 + \lambda)^T + \lambda T(1 + \lambda)^T]} \right] \\ &= \frac{\lambda \cdot [(1 + \lambda)^T - 1][1 - (1 + r)^T + rT(1 + r)^T] + \beta[(1 + r)^T - 1][(1 + \lambda)^T - 1 - \lambda T(1 + \lambda)^T]}{(1 - \beta) \cdot [(1 + r)^T - 1][1 - (1 + \lambda)^T + \lambda T(1 + \lambda)^T]} \\ &= \frac{\lambda \cdot [(1 + \lambda)^T - 1][1 - (1 + r)^T + rT(1 + r)^T]}{[1 - (1 + \lambda)^T + \lambda T(1 + \lambda)^T][(1 + r)^T - 1]} \cdot \frac{1}{1 - \beta} - \frac{\lambda \cdot \beta}{1 - \beta} \\ &= \frac{\lambda \cdot f(r, T)}{f(\lambda, T)} \cdot \frac{1}{1 - \beta} - \frac{\lambda \cdot \beta}{1 - \beta} \\ &= \frac{1}{1 - \beta} \cdot [ROI_T(\lambda, 0) - \lambda \cdot \beta]. \end{aligned}$$

The claim regarding the negative cross-partial derivative of $ROI_T(\lambda, 0, \beta)$ follows immediately upon recalling that $ROI_T(\lambda, 0)$ is everywhere decreasing in λ because setting $\delta = 0$ corresponds to conservative accounting. ■

Appendix B

Alternative Methods for Calculating the Cost of Capital

Method 1: Target Price Method (r_{DIV})

The target price method, introduced in Botosan and Plumlee (2002), employs a short-horizon form where the infinite series of future cash flows is truncated at the end of year five by inserting a forecasted terminal value. This yields the equation below. The primary assumption underlying this method is that analysts' forecasts of dividends per share during the forecast horizon and stock price at the end of the forecast horizon capture the market's expectation of those values.

$$P_0 = \sum_{t=1}^5 (1 + r_{DIV})^{-t} (dps_t) + (1 + r_{DIV})^{-5} \cdot P_5$$

where:

- P_0 = price at time $t = 0$.
- P_5 = price at time $t=5$.
- r_{DIV} = estimated cost of equity capital.
- dps_t = dividends per share.

Dividend forecasts for the current fiscal year ($t = 1$), the following fiscal year ($t = 2$), and the long run ($t = 5$), as well as maximum and minimum long-run target price estimates are collected from forecasts published by *Value Line* during the third quarter of the calendar year. Since *Value Line* does not provide dividend forecasts for years $t = 3$ and $t = 4$, we interpolate between the year $t = 2$ and $t = 5$ dividend forecasts using an implied straight-line rate of growth in dividends from year $t = 2$ to year $t = 5$.

Our forecast of terminal value (P_5) is the 25th percentile of *Value Line*'s forecasted long-run price range, although our conclusions are robust to the use of the 50th percentile or the minimum value. We use the 25th percentile to adjust for an apparent optimistic bias in analysts' forecasts of target price. Current stock price (P_0) equals the stock price reported on *CRSP* on the *Value Line* publication date or closest date thereafter within 3 days of publication.

Method 2: Industry Method (r_{GLS})

The industry method, introduced by Gebhardt, Lee and Swaminathan [2001], employs a residual income valuation model derived from a 12-year forecast horizon. The following

model results:

$$P_0 = b_0 + \sum_{t=1}^{11} \gamma_{GLS}^t \cdot (ROE_t - r_{GLS}) \cdot b_{t-1} + \frac{\gamma_{GLS}^{12}}{r_{GLS}} \cdot (ROE_{12} - r_{GLS}) \cdot b_{11}$$

where

- $ROE_t = \frac{eps_t}{b_{t-1}}$ is the forecasted return on equity for period t .
- eps_t = forecasted earnings per share in year t ,
- b_t = book value per share in year t ,
- r_{GLS} = estimated cost of equity capital,
- $\gamma_{GLS} \equiv \frac{1}{1+r_{GLS}}$.

Method 3: PEG Ratio Method (r_{PEG})

The PEG method, introduced by Easton (2004) proceeds as follows:

$$r_{PEG} = \sqrt{\frac{eps_2 - eps_1}{P_0}}$$

Our method is similar, but we use long-run earnings forecasts (eps_5 and eps_4) in place of eps_2 and eps_1 (consistent with Botosan and Plumlee (2005)). Accordingly, the empirical specification of the equation we employ to estimate r_{PEG} is given by:

$$r_{PEG} = \sqrt{\frac{eps_5 - eps_4}{P_0}}$$

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