MATH 228: Introduction to Linear Algebra and Differential Equations

Practice Exam II

I-9. If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$ with $\mathbf{u} \cdot \mathbf{v} = 2$ and $\|\mathbf{u} + \mathbf{v}\| = 3$, calculate $\|\mathbf{u} - \mathbf{v}\|$.

I-10. Show that the range of the linear operator defined by

$$\begin{array}{rcrcrcr} w_1 & = & x_1 & + & 4x_2 & - & 3x_3 \\ w_2 & = & 2x_1 & + & 6x_2 & + & 5x_3 \\ w_3 & = & x_1 & + & 6x_2 & - & 14x_3 \end{array}$$

is not R^3, and find a vector that is not in the range.

I-12. Determine whether the set of 2×2 matrices A with $\det A = 0$ is a subspace of the vector space of all 2×2 matrices.

I-13. Determine the values of x, if any, for which the set of vectors $\{(1, x, x), (x, 1, x), (x, x, 1)\}$ is linearly dependent.

I-14. Find the coordinate vector of $1 + 3x + 5x^2$ relative to the basis $\{1 + x + x^2, 1 - x + x^2, 1 - x - x^2\}$ of P_2.

I-15. Find a basis for the nullspace, row space, and column space of the matrix $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 2 & 2 & 0 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix}$.

I-17. Determine whether the set of vectors in $\mathbb{R}^4$, $\{(3, 1, 1, 2), (-4, 0, 1, 1), (1, 2, -1, 3), (-1, -4, 6, -4)\}$, is linearly independent.

I-18. Use the Wronskian to determine whether the set $\{x^2, 1 + x^2, x^3\}$ is linearly independent.

I-20. Use the information in the table to determine whether the linear system $A\mathbf{x} = \mathbf{b}$ is consistent, and, if so, determine the number of parameters in its general solution.

	size of A	rank(A)	rank[A|$\mathbf{b}$]
a)	4×3	3	3
b)	3×5	3	4
c)	5×4	3	3
d)	6×9	5	5
e)	2×7	1	2

II-1. Let V be the vector space $C[0, 2\pi]$ with inner product defined by $\langle f, g\rangle = \int_0^{2\pi} f(x)g(x)\,dx$. Use the Gram-Schmidt process to find an orthonormal basis for the subspace of V spanned by 1, $\sin(x)$, x.

II-2. Let $W = \{(1, 2, 3, 4), (1, 1, 1, 1), (1, 4, 7, 10)\}$. Find a basis for $W^{\perp}$.

II-3. Find the QR-decomposition of $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

II-4. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 15 \\ 0 \end{bmatrix}$. Determine the least squares solution of the linear system $A\mathbf{x} = \mathbf{b}$, as well as the projection of $\mathbf{b}$ on the column space of A.

II-5. Let $A = \begin{bmatrix} -2 & 3 & 3 \\ -1 & 5 & 6 \\ 1 & -4 & -5 \end{bmatrix}$. Find the eigenvalues and bases for the eigenspaces of A^{20}.

I I-6.Let A be a 6×6 matrix with characteristic equation $\lambda^3(\lambda+2)^2(\lambda-5) = 0$. Give the possible dimensions for the eigenspaces of A.