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Quantum mechanics in phase space. (English summary)

Perspectives on quantization (*South Hadley, MA*, 1996), 47–62, *Contemp. Math.*, 214, *Amer. Math. Soc.*, *Providence*, *RI*, 1998.

22E30 (58G40 81R30 81S10)

This paper gives a very readable account of the author's generalization (known in the literature now as the Hall transform) of the Segal-Bargmann transform to the setting of compact groups as well as ramifications of this transform in quantum mechanics. The Hall transform is a unitary isomorphism of a space of square-integrable functions over a compact group onto a space of square-integrable holomorphic functions over the complexification of the group. It is, as explained in this paper, the intertwining operator which relates two representations of the quantum mechanics of a system whose classical configuration space is the compact group. Sections 1, 2 and 4 describe (two forms of) the Hall transform and its properties. Section 3 explains how the complexified group can be viewed as the relevant phase space, by identifying it with the cotangent bundle of the group. Section 5 describes an interesting and surprising uniform (and sharp) bound on the probability density corresponding to a "wave function" on the phase space. Section 6 describes Toeplitz operators on spaces of holomorphic functions (these are multiplication operators followed by projection onto the holomorphic subspace) and also describes results showing that the position and momentum operators of the Schrödinger representation correspond, via the Hall transform, to Toeplitz operators on the space of holomorphic square-integrable functions on the complexified group. In the final section, quantization using Toeplitz operators on the Segal-Bargmann space is compared with geometric quantization and the advantages of the first method are pointed out. This paper contains no proofs but the reader is provided a good guide to all the necessary references and more, including several other recent papers by the author [Comm. Math. Phys. **184** (1997), no. 1, 233–250; MR1462506 (98g:81108); B. C. Hall and A. N. Sengupta, J. Funct. Anal. **152** (1998), no. 1, 220–254; MR1600083 (99a:58020)].

{For the entire collection see MR 98g:00029}

Reviewed by *Ambar Sengupta*