

Corrections to “Lie Groups, Lie Algebras, and Representations: An Elementary Introduction,” by Brian C. Hall

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1. SUBSTANTIVE CORRECTIONS

Chapter 1

1. Page 6, last sentence in first paragraph of Section 1.2.4. Replace $(A^*)_{jk} = \overline{A_{jk}}$ with $(A^*)_{jk} = \overline{A_{kj}}$.
2. Page 18, last full paragraph. In the sentence beginning “Note that...,” replace

$$U_1 U_2 A U_2^{-1} U_2^{-1}$$

with

$$U_1 U_2 A U_2^{-1} U_1^{-1}.$$

3. Page 23, Exercise 2. Replace $(B^{tr})_{kl} = B_{kl}$ with $(B^{tr})_{kl} = B_{lk}$.
4. Page 23, Exercise 3. Replace $(A^*)_{kl} = \overline{A_{kl}}$ with $(A^*)_{kl} = \overline{A_{lk}}$.

Chapter 2

1. Page 36, fourth displayed equation. Replace $O(1/m^2)$ by $O(1/m)$. Equation should read $\left(e^{\frac{X}{m}} e^{\frac{Y}{m}}\right)^m = \exp\left(X + Y + O\left(\frac{1}{m}\right)\right)$.
2. Page 44, last line of the proof of Theorem 2.18. Replace “belongs to \mathfrak{h} ” with “belongs to \mathfrak{g} .”
3. Page 59, Exercise 2. In the expression $\|X\|^2 = \sum_{j,k=1}^n |\langle u_j, Au_k \rangle|^2$, replace A with X on the right-hand side.

Chapter 3

1. Page 63, fourth line of Section 3.1. Replace “and” with “=”: “ $\Phi(\exp X) = \exp(\phi(X))$.”
2. Page 66, Third line in the proof of Theorem 3.2. Replace ϕ with Φ : “Define $\Phi : H \rightarrow G$ by ...”
3. Page 69, last several lines. Replace X by Y four times. Specifically, the second-to-last displayed equation should read

$$\phi((e^{\text{ad}_X} e^{\text{ad}_Y})(Y)) = e^{\text{ad}_{\phi(X)}} e^{\text{ad}_{\phi(Y)}}(\phi(Y)),$$

and in the last displayed equation, first two lines, the X just before the dt should be Y .

4. Page 71, Equation (3.12). In the second line, omit the factor of $1/m$ in front of the sum.

5. Page 86, first line of second paragraph. Replace \mathfrak{h} with H : “Let \mathfrak{h}' be the Lie algebra of H .”
6. Page 89, Exercise 3. The derivative on the left-hand side of the displayed inequality should be evaluated at $t = 0$.

Chapter 4

1. Page 92, last line. Replace “Theorem 6” with “Theorem 2.21.”
2. Page 93, sixth line in the proof of Proposition 4.5. Replace V with W : “Since W is invariant under $\pi(X_i)$.”
3. Page 99, proof of Proposition 4.8. In the first two paragraphs of the proof, replace the portion beginning with “Let k_0 be the largest value of k ...” with the following. Let k_0 be the smallest value of k for which $a_k \neq 0$ and consider

$$\pi_m(X)^{m-k_0}w.$$

Since (by (4.5)) each application of $\pi_m(X)$ lowers the power of z_1 by 1, $\pi_m(X)^{m-k_0}$ will kill all the terms in w except $a_{k_0}z_1^{m-k_0}z_2^{k_0}$. On the other hand, we compute easily that

$$\pi_m(X)^{m-k_0}(z_1^{m-k_0}z_2^{k_0}) = (-1)^{m-k_0}(m-k_0)!z_2^m.$$

We see, then, that $\pi_m(X)^{m-k_0}w$ is a *nonzero* multiple of z_2^m . Since W is assumed invariant, W must contain z_2^m .

Chapter 5

1. Page 136, second line in Equation (5.7). Replace $\pi(X_i)\pi(X_j)$ with $\pi(X_j)\pi(X_i)$.
2. Page 136, last sentence in the proof of Lemma 5.14. Replace “ $n - 1$ factors” and “ n factors” by “ $N - 1$ factors” and “ N factors,” respectively.
3. Page 142, second full sentence. Replace “ $Y_1 \otimes I$ ” with “ $Y_2 \otimes I$ ”: “the right arrow indicates the action of $Y_2 \otimes I + I \otimes \bar{Y}_2$.”
4. Page 143, just after Definition 5.21. Replace \mathfrak{t} with \mathfrak{h} : “We can define an action of W on \mathfrak{h} .”
5. Page 143, first paragraph after Definition 5.21. Replace “ $B = AZ$ with $Z \in Z$ ” with “ $B = AC$ with $C \in Z$,” and then replace Z with C in the displayed equation.
6. Page 146, displayed equation in middle of page. In last line of the display, replace $\mu(AHA^{-1})$ with $\mu(A^{-1}HA)$.
7. Page 147, line 6. Replace X with v : “there exists a nonzero v in V .”

Chapter 6

Chapter 7

1. Page 231, last paragraph. Replace “reflection about the root $\alpha_3 = \alpha_1 + \alpha_2$ ” with “reflection about the line perpendicular to the root $\alpha_3 = \alpha_1 + \alpha_2$.”

2. Page 240, Exercise 6. In (7.51), replace $\chi(a)$ with $X(a)$. In the note at the end, replace χ_μ with X_μ .

Chapter 8

1. Page 248, last displayed equation. Equation should read

$$w_\alpha \cdot H_\beta = 2 \frac{w_\alpha \cdot \beta}{\langle \beta, \beta \rangle} = 2 \frac{w_\alpha \cdot \beta}{\langle w_\alpha \cdot \beta, w_\alpha \cdot \beta \rangle} = H_{w_\alpha \cdot \beta}.$$

2. Page 254. Replace $2 \langle \alpha, \mu \rangle / \langle \mu, \mu \rangle$ with

$$2 \frac{\langle \mu, \alpha \rangle}{\langle \alpha, \alpha \rangle}$$

five times.

3. Page 255, Definition 8.24. Replace $2 \langle \alpha_k, \mu_l \rangle / \langle \mu_l, \mu_l \rangle$ with

$$2 \frac{\langle \mu_k, \alpha_l \rangle}{\langle \alpha_l, \alpha_l \rangle}.$$

4. Page 255, first paragraph after Definition 8.24. Replace $\mu_k = \mu / \langle \alpha_k, \mu \rangle$ with

$$\mu_k = \frac{1}{2} \mu \frac{\langle \alpha_k, \alpha_k \rangle}{\langle \mu, \alpha_k \rangle}.$$

5. Page 274, last full paragraph. Replace $\langle \mu_k, \alpha_l \rangle = \delta_{kl}$ with $2 \langle \mu_k, \alpha_l \rangle / \langle \alpha_l, \alpha_l \rangle = \delta_{kl}$.

Appendix A

1. Page 290. Combine Exercises 10 and 11 into a single exercise.

Appendix B

1. Page 293, third paragraph. Replace $(A^*)_{kl} = \overline{A_{kl}}$ with $(A^*)_{kl} = \overline{A_{lk}}$.
2. Page 300, first paragraph after Definition B.11. Replace “eigenvectors” with “eigenvalues”: “a weight is a set of simultaneous eigenvalues for the operators in \mathcal{A} .”

Appendix C

1. Page 303, last two lines. Replace U with V : “another neighborhood V of m .”
2. Page 313, last displayed equation. Replace $(L_{e^t X})$ with $(L_{e^t X})_*$ in the second line.
3. Page 315, first full paragraph. Replace Π with G : “i.e., a continuous map of G into some $\mathrm{GL}(n; \mathbb{C})$.”
4. Pages 315-316. Replace the representation Π with Σ in Theorem C.9 and its proof. Replace the Lie algebra representation π by σ in the proof of Theorem C.9. Then add a factor of 2π in the second displayed equation on p. 316 (where now π is a number, not a representation):

$$e^{2\pi n \sigma(B)} = \Sigma(e^{2\pi n B}) = I.$$

Appendix D

1. Page 324, second to last line. Replace ψ by A : “the map A must satisfy (D.2).”
2. Page 327, displayed equation prior to (D.8). Add a minus sign to the first term:

$$-(\phi \circ \pi_2(X)) \otimes u + \phi \otimes (\pi_1(X)u).$$

3. Page 327, Equation (D.8). Add a minus sign to the first term and modify the parentheses:

$$-\phi(\pi_2(X)v)u + \phi(v)\pi_1(X)u.$$

Appendix E

1. Page 341, second paragraph. The displayed set of inclusions should read

$$\begin{aligned} \text{root lattice} &\subset \text{analytically integral elements} \\ &\subset \text{algebraically integral elements.} \end{aligned}$$

2. TRIVIAL CORRECTIONS

Preface

1. Page VII, line 6. Add “the” before “Lie algebra.” Sentence should read “to the complexification of the Lie algebra of a compact ...”
2. Page VII, first line of second to last full paragraph. Add “a” before “minimum”: “keeping the mathematical prerequisites to a minimum...”

Chapter 1

1. Page 5, last displayed equation. Replace x with y in the middle item: $\langle A(A^{-1}x), A(A^{-1}y) \rangle$.
2. Page 16, fifth line from bottom of page. Omit the first occurrence of “SO(3).”
3. Page 18, second paragraph in Section 1.6.1. Replace “B.6 of B” with “Section B.6 of Appendix B.”
4. Page 20, paragraph before Proposition 1.17. Add “a”: “If P is a self-adjoint matrix...”
5. Page 25, Exercise 12. Replace the reference to “Theorem 1.9” with “Proposition 1.9.”

Chapter 2

1. Page 36, just before Theorem 2.11. Add “a” before “matrix”: “Recall that the trace of a matrix is defined as ...”
2. Page 38, last paragraph. Replace “Proposition 2.27” with “Theorem 2.27.”
3. Page 47, second displayed equation. Omit one d/dt in first line.
4. Page 61, Exercise 19. In the third displayed equation, replace $\text{Ad}(e^X)$ with Ad_{e^X} .

5. Page 61, Exercise 24. Replace “as defined in the previous exercise” with “as defined in Exercise 20.”

Chapter 3

1. Page 69, fourth line of proof of Corollary 3.4. Add “a”: “if ϕ is a Lie algebra homomorphism.”
2. Page 70, line 2. Add “of the”: “Before coming to the proof of the Baker–Campbell–Hausdorff formula.”
3. Page 72, first paragraph after Lemma 3.6. Replace 0 with Y : “and the limit in $\Delta(X/m, Y)$.”
4. Page 88, Exercise 2, last line. Change “an” to “a”: “with n a nonzero integer.”

Chapter 4

1. Page 92, paragraph after Definition 4.3. Replace “In Section 5.4” with “In Section 4.4.”
2. Page 113, first line of Definition 4.24. Replace “ Π be” with “ Π is”: “ Π is a representation of G .”
3. Page 115, second displayed equation. Replace $\phi(v)$ with $\phi(u)$ in middle item.
4. Page 116, middle of first paragraph. Add “for $\mathfrak{so}(3)$ ”: “the basis F_1, F_2, F_3 for $\mathfrak{so}(3)$, then direct computation shows...”

Chapter 5

1. Page 133, last full paragraph. The word “each” is repeated.

Chapter 6

1. Page 159, first paragraph. Replace “It can be shown that $\mathfrak{k} = \mathfrak{k}_1 \oplus \mathfrak{k}_2$ ” with “It can be shown that the Lie algebra \mathfrak{k} of K decomposes as $\mathfrak{k} = \mathfrak{k}_1 \oplus \mathfrak{k}_2$.”
2. Page 170, Step 7. Add “is”: “if there is another eigenvector in V^α ...”
3. Page 173, third line of last paragraph in Section 6.5. Omit “an”: “these quantities must be integers.”
4. Page 179, second line from bottom. Omit “this”: “Theorem 6.33 shows that the Lie algebra definition...”
5. Page 184, line 6. Replace “roots” with “root”: “(In other root systems,...”

Chapter 7

1. Page 198, third and fourth lines in the proof. Replace k with l : “For each k , choose...”
2. Page 204, third line in Subsection 7.3.3. Add “for”: “the set of weights for V_μ/U_μ .”

3. Page 205, second to last line. Replace “element” with “operator”: “This operator is going to describe...”
4. Page 207, last full paragraph. Add “the proof of”: “the proof of Proposition 7.17 would tell us...”
5. Page 220, first paragraph in Section 7.5.2. Replace X with X^* : “where X^* is the usual matrix adjoint X .”

Chapter 8

1. Page 244, third line from bottom. Replace R with W : “every element of W maps R onto itself.”

Appendix A

1. Page 281, first sentence in Section A.2. Replace $g_1 * g_2$ with $g * h$: “instead of the more cumbersome $g * h$.”
2. Page 283, first sentence of Section A.2.9. Replace “where m is positive” with “where n is positive.”
3. Page 288, fifth line from bottom. Add “on G/N ”: “the definition of the product operation on G/N .”

Appendix B

1. Page 293, fourth line in Section B.2. Replace “ $A \rightarrow CBC^{-1}$ ” with “ $B \rightarrow CBC^{-1}$.”
2. Page 294, proof of Proposition B.4. Replace v_λ by V_λ : “let v be an element of V_λ .”
3. Page 294, first paragraph in Section B.3. Replace “the equation $Av = \lambda v$ ” with “the equation $Av = v$.”
4. Page 294, fifth line from bottom of page. Replace “in” with “is”: “only if $(A - \lambda I)$ is noninvertible.”
5. Page 297, last paragraph in Section B.5. Add “a”: “if A is a linear operator.”
6. Page 299, second to last line in first paragraph. Replace “and” with “an”: “making such an identification.”
7. Page 300, third line. Replace V with v : “simultaneous eigenvalues associated to v .”
8. Page 300, Definition B.12. Add “is”: “and \mathcal{A} is some collection of linear operators.”

Appendix C

1. Page 308, in Theorem C.2. Add “ $\rightarrow \mathcal{M}$ ” in two places: “ $\gamma_1 : (-a_1, b_1) \rightarrow \mathcal{M}$ ” and “ $\gamma_2 : (-a_2, b_2) \rightarrow \mathcal{M}$.”
2. Page 312, second paragraph in Section C.2.5. Replace “that” with “the”: “This construction illustrates the advantage.”

3. Page 314, second paragraph of Section C.3. In the displayed equation, there is one occurrence of u_1 that should be u_2 .
4. Page 317, second to last paragraph. Replace “homomorphism” with “homomorphisms”: “the associated group homomorphisms Σ and Π are equal.”

Appendix D

1. Page 326, third paragraph after Theorem D.5. Replace l'' with l' : “the matrix entries $\langle u_l, B_k v_{l'} \rangle$.”
2. Page 328, last paragraph in proof. Add “all”: “After all, the orthogonal projection operator...”

Appendix E

1. Page 338, second paragraph. First sentence should read, “We use the isomorphism ϕ of $\mathfrak{su}(2)$ and $\mathfrak{so}(3)$...”