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DETECTING EARTH-MASS PLANETS WITH GRAVITATIONAL MICROLENSING

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ABSTRACT

We show that Earth-mass planets orbiting stars in the Galactic disk and bulge can be detected by monitoring microlensed stars in the Galactic bulge. The star and its planet act as a binary lens which generates a light curve that can differ substantially from the light curve due only to the star itself. We show that the planetary signal remains detectable for planetary masses as small as an Earth mass when realistic source star sizes are included in the light curve calculation. These planets are detectable if they reside in the "lensing zone," which is centered between 1 and 4 AU from the lensing star and spans about a factor of 2 in distance. If we require a minimum deviation of 4% from the standard point-lens microlensing light curve, then we find that more than 2% of all $M_\oplus$ planets and 10% of all 10 $M_\oplus$ in the lensing zone can be detected. If a third of all lenses have no planets, a third have 1 $M_\oplus$ planets, and the remaining third have 10 $M_\oplus$ planets then we estimate that an aggressive ground-based microlensing planet search program could find one Earth-mass planet and half a dozen 10 $M_\oplus$ planets per year.

Subject headings: gravitational lensing — stars: planetary systems

1. INTRODUCTION

The recent discovery of several giant planets (Mayor & Queloz 1995; Marcy & Butler 1996) has confirmed the existence of planets orbiting main-sequence stars other than the Sun. Two of these first three giant planets have orbits that were unexpected, and this together with the surprising discovery of planets in a pulsar system (Wolszczan & Frail 1992) demonstrates the importance of observational studies of extrasolar planetary systems. Indirect ground-based techniques that detect the reflex motion of the parent star through accurate radial velocity measurements or astrometry are likely to have sensitivity that extends down to the mass of Saturn ($\sim 100\ M_\oplus$) (Butler et al. 1996) or even down to 10 $M_\oplus$ (Shao & Colavita 1992) with interferometry from Keck or the VLT. There is great interest in searching for planets with masses similar to that of Earth, and NASA's new ExNPS program (Elachi 1995) seeks to build a spacecraft capable of imaging nearby Earth mass planets in the infrared. In order to ensure the success of such a mission, we will need to have at least a rough idea of how prevalent planets with masses close to that of Earth really are.

A ground-based gravitational microlensing survey system sensitive to planets down to 1 $M_\oplus$ has been proposed by Tytler (1995). This project would involve both a microlensing survey telescope to detect microlensing events in progress and a world-wide network of follow-up telescopes that would monitor the microlensing light curves on an approximately hourly timescale in search of deviations due to planets. Existing microlensing surveys (Alcock et al. 1993; Aubourg et al. 1993; Udalski et al. 1993; Alard 1995) have recently demonstrated real-time microlensing detection capability (Alcock et al. 1996a; Udalski et al. 1994), and two world-wide microlensing follow-up collaborations (Albrow et al. 1995; Pratt et al. 1995) are now in operation, but to detect Earth-mass planets, more capable survey and follow-up systems will be required.

In this paper, we provide the theoretical basis for this enterprise by calculating realistic microlensing light curves and detection probabilities for planets as small as 1 $M_\oplus$. Previous authors (Mao & Paczyński 1991; Gould & Loeb 1992; Bolatto & Falco 1994) have considered the deviations from the single lens light curve due to planets using the point source approximation. This is a poor approximation for planets in the 1–10 $M_\oplus$ mass range, so we have calculated planetary-binary lensing event light curves for realistic finite size source stars, and we show that planets in the 1–10 $M_\oplus$ mass range can cause deviations from the standard single lens light curve with amplitudes larger than 10% that last for 2 hr or more. We calculate planetary detection probabilities based upon a set of assumed event detection criteria and a simple planetary system model loosely based upon the solar system.

2. MICROLENSING

The only observable feature of a microlensing event is the time variation of the total magnification of all the lens images due to the motion of the lens with respect to the observer and source. The characteristic transverse scale for a lens of mass M is given by the Einstein ring radius which is the radius of the ring image obtained when the source, lens, and observer are colinear. It is given by

$$R_{\rm E} = 2\sqrt{\frac{GMD}{c^2}} = 4.03\ {\rm AU}\sqrt{\left(\frac{M}{M_\odot}\right)\left(\frac{D}{2\ {\rm kpc}}\right)}\,, \qquad (1)$$

where D is the "reduced distance" defined by $1/D = 1/D_{ol} + 1/D_{ls}$. D_{ol} and D_{ls} are the distances from the observer to the lens and from the lens to the source, respectively. For a point mass lens, the amplification of a microlensing event is given by

$$A = \frac{u^2+2}{u\sqrt{u^2+4}}\,; \quad u = \sqrt{u_{\rm min}^2 + [2(t-t_0)/\hat{t}]^2}\,, \qquad (2)$$

where u is the separation of the lens from the source-observer line of sight in units of $R_{\rm E}$ and t_0 and $\hat{t}$ refer to the time of peak amplification and the Einstein diameter crossing time, respectively.

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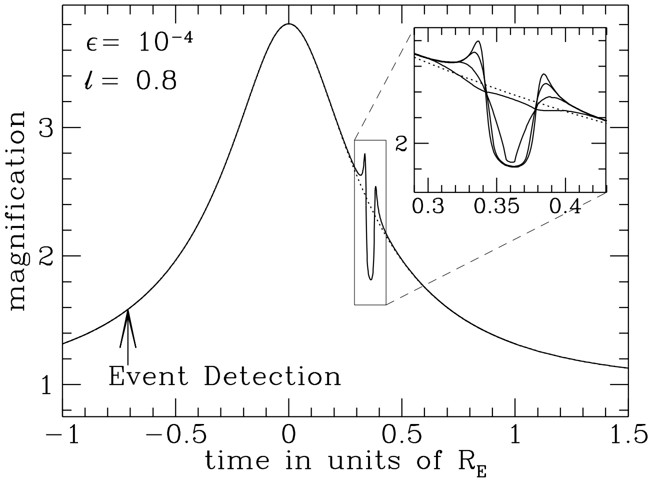

FIG. 1*a*

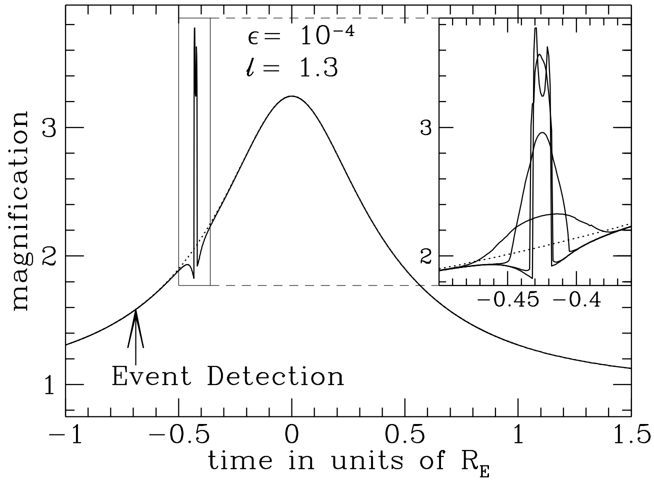

FIG. 1*b*

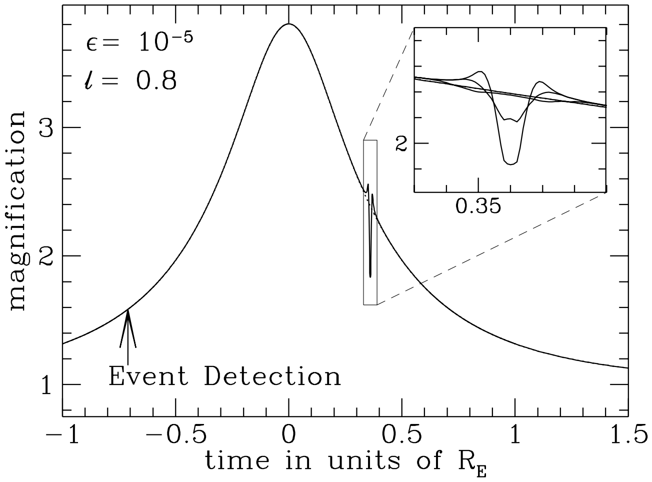

FIG. 1*c*

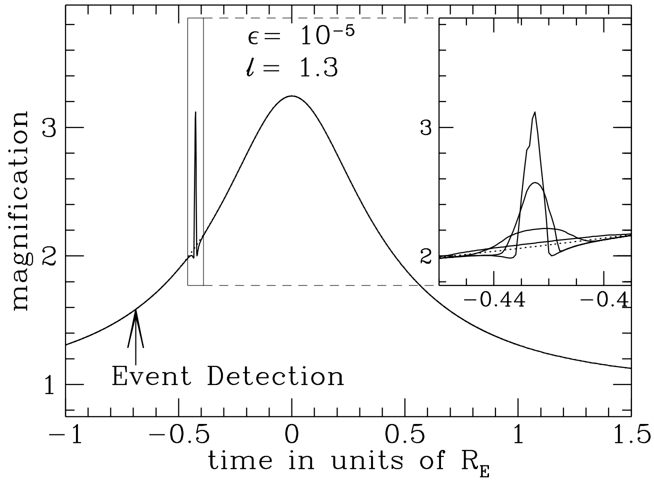

FIG. 1*d*

FIG. 1.—Microlensing light curves that show planetary deviations are plotted for mass ratios of $\epsilon = 10^{-4}$ and 10^{-5} and separations of $l = 0.8$ and 1.3. The main plots are for a stellar radius of $r_s = 0.003$ while the insets show light curves for radii of 0.006, 0.013, and 0.03 as well. (The amplitude of the maximum light curve deviation decreases with increasing r_s.) The dashed curves are the unperturbed single lens light curves, $A_0(t)$. For each of these light curves, the source trajectory is at an angle of $\sin^{-1} 0.6 = 36°.9$ with respect to the star-planet axis. The impact parameter $u_{\min} = 0.27$ for the $l = 0.8$ plots and $u_{\min} = 0.32$ for the $l = 1.3$ plots.

In a "planetary lensing event," the majority of the light curve is described by equation (2), but in the region of the planetary deviation we must consider the binary lens case (Schneider, Ehlers, & Falco 1992; Rhie 1996). If ω and z denote (in complex coordinates) the source and image positions in the lens plane, the binary lens equation is given by

$$\omega = z - \frac{1-\epsilon}{\bar{z} - \bar{x}_s} - \frac{\epsilon}{\bar{z} - \bar{x}_p}\,, \tag{3}$$

where ϵ is the fractional mass of the planet, and x_s and x_p are the positions of the star and planet, respectively. We work in units of the Einstein radius, $R_{\rm E}$, of the total mass M. Equation (3) has three or five solutions (z) for a given source location, ω.

The Jacobian determinant of the lens mapping (eq. [3]) is

$$J = 1 - |\partial_z \bar{\omega}|^2\,; \quad \partial_z \bar{\omega} = \frac{1-\epsilon}{(z - x_s)^2} + \frac{\epsilon}{(z - x_p)^2}\,, \tag{4}$$

and the total magnification of a point source is obtained by summing up the absolute value of the inverse Jacobian determinant calculated at each image:

$$A = \sum_i |J_i|^{-1}\,. \tag{5}$$

The curve defined by $J = 0$ is known as the critical curve, and the lens mapping (eq. [3]) transforms the critical curve to the caustic curve in the source plane. By equation (5), a point source that lies on a caustic will have an infinite magnification. (The singularity at $J = 0$ is integrable, so finite sources always have finite magnifications.) When the source star is in the region of the caustic curve, the magnification will differ noticeably from the single lens case, allowing a "planetary" signal to be detected. If the planet mass is of order 1–10 $M_\oplus$, the total extent of the caustic curves are comparable to the size of the source star, and the point source approximation is not appropriate.

3. PLANETARY LIGHT CURVES

Because lensing conserves surface brightness, the magnification of an image is just the ratio of the image area to the source area (which is given by eq. [4] for a point source).

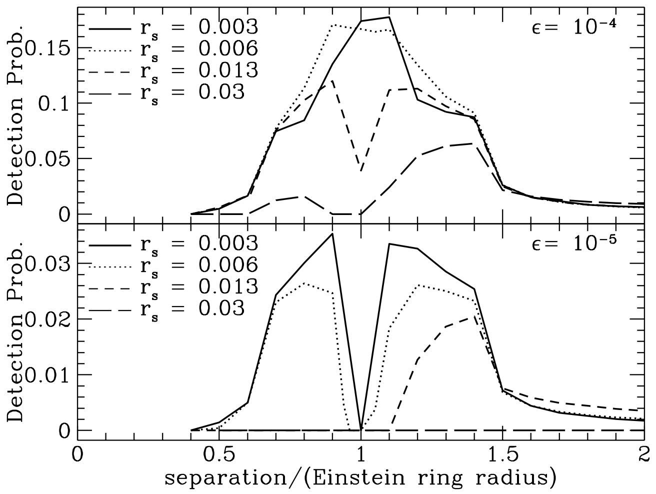

FIG. 2.—The planetary deviation detection probability is plotted for different values of the planetary mass ratio, ϵ, and the stellar radii, r_s. A planet is considered to be "detected" if the light curve deviates from the standard point lens light curve by more than 4% for a duration of more than $\hat{t}/400$. Note the y-axis scale is different for the upper and lower panels of this plot. Only the portion of the light curve after the alert trigger at $A = 1.58$ is assumed to be searched for planetary deviations.

For a finite size source, we calculate the lens magnification in the image plane where it is given by the sum of the image area weighted by the limb darkened source profile assumed to have the form: $I(\theta)/I(0) = 1 - 0.6(1 - \cos\theta)$. This avoids the magnification singularities on the caustics in the source plane. We integrate over the images as follows: First, we determine the location of the "center" of each image, which is usually the image of the center of the star. If a portion of the stellar disk is inside a caustic when the center is outside, then we must also include an additional double image of the included portion of the star. We then cycle over the included images and build a numerical integration grid centered on each image. These grids are expanded until all the grid boundary points are outside of the images. Sometimes these image grids can include more than one image, and in these cases, the redundant grids are dropped from the calculation.

The source radius projected to the lens plane is given by $r_s \equiv R_s D_{ol}/[R_E(D_{ol} + D_{ls})]$ normalized to the Einstein ring radius. Figure 1 shows some examples of planetary light curves calculated for source size $r_s = 0.003$, planetary mass fraction $\epsilon = 10^{-5}$ and 10^{-4}, and separations of $l \equiv |x_p - x_s| = 0.8$ and 1.3. For a typical Galactic lens of $0.3\ M_\odot$, the mass fractions $\epsilon = 10^{-5}$ and 10^{-4} correspond to planet masses of 1 and 10 $M_\oplus$, respectively. The insets show the effects of varying the source size r_s over the values 0.003, 0.006, 0.013, and 0.03. The r_s values of 0.003 and 0.006 correspond to a main-sequence turn-off source star lensed by lenses in the disk and bulge, respectively, while values of 0.013 and 0.03 correspond to a clump giant source lensed by lenses in the disk and bulge. (We have assumed $R_s = 3\ R_\odot$ for turn-off stars and $R_s = 13\ R_\odot$ for clump giants.)

Figures 3*a*–3*b* (Plates 11–12) show two-dimensional plots of the magnification ratio (A/A_0) of the planetary binary lens case to the single lens case for the planetary parameters $\epsilon = 10^{-4}$, $r_s = 0.003$, and $l = 0.8$ and 1.3. Source trajectories are represented by straight lines across these figures, and the $\epsilon = 10^{-4}$ light curves shown in Figure 1 correspond to source paths which cross close to the center of these two-dimensional plots at an angle of $\sin^{-1} 0.6 = 36^\circ.9$ from the horizontal lens axis.

4. PLANETARY DETECTION PROBABILITIES

Let us define a reasonable set of planetary detection criteria: First of all, the microlensing event must be discovered by the microlensing survey system, and then the planetary deviation must be detected by the microlensing follow-up system. The follow-up system is assumed to observe each lensed star about once per hour with an accuracy of 0.5%–1% so that moderate amplitude deviations can be detected. Then, we require that the light curve deviate from the single lens light curve by more than 4% for a period longer than $\hat{t}/400$, which is about 2.4 hr for a typical event lasting $\hat{t} = 40$ days. This deviation must occur after the event has been detected by the survey system, which we take to be after magnification $A = 1.58$ has occurred. (This is the 0.5 mag event detection threshold.) Using these detection criteria, we examine all detectable (i.e., $A_{\max} > 1.58$) events for a fixed lens star-planet separation, l, and determine what fraction of these events pass the selection criteria. This involves examining all possible lines (or light curves) on the two-dimensional deviation plots like the ones shown in Figure 2. (These simulated light curves are assigned uniform distributions in $u_{\min}$ and angle.) Our probability results for the 4% threshold are shown in Figure 3 for fractional masses of $\epsilon = 10^{-4}$ and 10^{-5} and a variety stellar radii. We can see from Figure 3 that the detection probability is highest for separations close to the Einstein ring radius, but for $l = 1$ detection is difficult if r_s is large or if ϵ is small. A similar effect also occurs for $l < 1$ and moderately large r_s. This can be understood by considering the amplification contours in the source plane. From Figure 2*a* we can see that the $l < 1$ lens system has a region of negative deviation in the center of two regions containing the caustics that have a positive deviation. A large source star will cover much of this region so that the positive and negative deviations will tend to cancel in the integral over the entire source. For $l = 1$, the positive and negative deviation regions are more closely packed together and this effect is even stronger.

5. A MODEL PLANETARY SYSTEM

In order to translate the results displayed in Figure 3 to the probability of detecting planets, we must make some assumptions about the planetary systems that we are searching for. For simplicity, let us define a simple "factor of 2" model planetary system that has a distribution of star-planet separations which is uniform in log (l) and has on average one planet for every factor of 2 in separation from the parent star in the region of interest. The region of interest, or the "lensing zone," is the interval in l where the lensing detection probability is relatively high: $0.6 < l < 1.5$. For stellar lenses in the disk or bulge in the mass range 0.1–1 $M_\odot$, the lensing zone will cover about a factor of 2 in transverse distance somewhere in the range 0.6–6.0 AU. A virtue of the "factor of 2" model is that the probability distribution of transverse star-planet separations is not changed by orbital inclination or phase (although the individual transverse star-planet separations for a given system will depend on orbital inclination or phase). For a planetary system like our own, where the planets' semimajor axes fall in the range 0.4–40 AU, it would be very rare for the orbital parameters to conspire to

TABLE 1

PLANETARY DETECTION PROBABILITIES

r_s	ϵ	$P(2\%)$	$P(4\%)$	$P(10\%)$	$P(20\%)$
0.003……	10^{-4}	0.188	0.144	0.094	0.052
0.006……	10^{-4}	0.238	0.159	0.085	0.043
0.013……	10^{-4}	0.201	0.118	0.052	0.014
0.03 …….	10^{-4}	0.120	0.035	0.012	0.000
0.003……	10^{-5}	0.060	0.034	0.014	0.004
0.006……	10^{-5}	0.052	0.026	0.005	0.002
0.013……	10^{-5}	0.019	0.008	0.001	0.000
0.03 …….	10^{-5}	0.002	0.000	0.000	0.000

NOTE.—Planetary detection probabilities P are shown as a function of the deviation threshold for different values of the source star radius r_s, and planetary mass fraction ϵ. Idealized "factor of 2" planetary systems with one planet per factor of 2 in distance from the lens star are assumed. A planet is considered to be detected if it deviates from the single lens light curve by more than the threshold for a period of time longer than $\hat{t}/400$. The r_s values of 0.003 and 0.006 correspond to a turn-off source star with disk and bulge lenses, respectively, while the r_s values of 0.013 and 0.03 correspond to a giant source with disk and bulge lenses.

move the outer planets inside the lensing zone, so our assumption that the planetary system always extends through the lensing zone is reasonable. We have used the "factor of 2" model to calculate the probability of planet detection for a variety of stellar radii and detection thresholds assuming that the planets in a given system have a single mass. The results are summarized in Table 1.

Table 1 can be used to estimate the number of planets that might be detected with a second generation microlensing survey and follow-up system similar to that discussed by Tytler (1995). Such a system might be able to discover 200 lensed turn-off stars ($0.003 \le r_s \le 0.006$) and 50 lensed clump giants ($0.013 \le r_s \le 0.03$) per year.[3] We will assume that practical difficulties such as weather serve to reduce the actual detection probability to 50% of the theoretical values shown in Table 1. Then with planetary detection, thresholds of 4% for turn-off stars and 2% for clump giants (which are brighter), we would expect to detect about 19 10 $M_\oplus$ planets or three 1 $M_\oplus$ planets per year if every lens system had a "factor of 2" planetary system with planets of these masses. (These numbers are roughly independent of the fraction of the lenses in the bulge or disk.) If a third of all lenses have no planets, a third have 1 $M_\oplus$ planets and the remaining third have 10 $M_\oplus$ planets, then we would expect to detect six 10 $M_\oplus$ planets and a single 1 $M_\oplus$ planet every year. More than half of the 10 $M_\oplus$ planetary light curves and a third of the 1 $M_\oplus$ light curves would have deviations larger than 10%. Clearly, a null result from an 8 yr survey of this magnitude would be a highly significant indication that planetary systems like our own are rare.

6. DISCUSSION

In the previous section, we have shown that a significant number of planets with masses down to 1 $M_\oplus$ can be detected via gravitational microlensing if microlensing events toward the Galactic bulge are monitored approximately hourly with photometric precision of 0.5%–1.0%, which is readily achievable in crowded stellar images.

We can also use the results of our probability calculations to help determine the optimal planetary search strategy. For example, given a large number of events to monitor for planetary deviations and a limited amount of observing time, how long should we follow each event? The probabilities given in Table 1 assume that each event is followed from event detection at $A = 1.58$ until A drops to 1.13, but if we stop the follow-up observations when $A > 1.34$, then we will only be sensitive to planetary deviations from planets in the interval $0.62 < l < 1.62$. Integration over the curves in Figure 3 indicates that this will reduce the chance of detecting a planet by 5%–10% (for the "factor of 2" model), but the total number of observations required drops by 27%. Thus, if the capacity of the follow-up system is saturated, it is best to concentrate follow-up observations on events with $A > 1.34$. This effect is basically geometric: planets that are outside the lensing zone ($l > 1.6$) tend to give rise to "isolated" events that are not associated with a stellar lensing event detected by the survey system.[4] It is optimal to search for planets at $l < 1.6$ where they would "modulate" a detectable stellar lensing light curve.

Our results also suggest that it will be easier to detect Earth-mass planets by monitoring turn-off star lensing events than giant star events. (Gould & Welch 1996) have shown that combined infrared and optical observations may allow the detection of Earth-mass planets in giant star lensing events, however.)

We have established that low-mass planets can be detected, but we should also address what we can learn about each planet that is discovered through microlensing. Planetary light curve deviations would be detected in real time so that observations can be repeated every few minutes during the planetary deviation. The lens parameters l (the separation perpendicular to the line of sight in units of R_E) and the mass ratio ϵ can generally be determined from gross features of the light curve. We can easily determine l (up to a two-fold ambiguity) from the amplification that the unperturbed light curve would have in the deviation region, and the mass ratio ϵ can be determined from the timescale of the planetary deviation. The two-fold ambiguity in l is also easily resolved in most cases by the shape of the light curve deviation, as can be seen in Figures 1 and 2. For $l < 1$, the deviation region consists of positive deviation regions surrounding the two caustic curves with a long trench of negative deviations in between. This leads to light curves with regions of large negative perturbations surrounded by regions of smaller positive perturbations. For $l > 1$, the situation is reversed and the dominant perturbation is a central positive one that has regions of small negative perturbations on either side of it.

Another parameter that may be measured is the angular Einstein ring radius of the planet itself. This comes about because the ratio of the radius to the angular radius of the star is the parameter that describes the finite source effects. For planets of Earth mass, the finite source effects are almost always important, so in principle, this parameter may be measurable in most events.

In summary, we have calculated realistic light curves for microlensing events where the lens star has a low-mass planetary companion, and we have shown that planets with

[3] These estimates are based upon an assumed 2 m survey telescope which could monitor 30 million bulge stars over a 250 day bulge season with an assumed lensing rate of $\Gamma = 2.4 \times 10^{-5}$ events yr^{-1} and a 50% detection efficiency (Alcock et al. 1996b).

[4] Isolated planetary lensing events might be detected by microlensing surveys, but the detection efficiency and variable star background rejection would be quite poor.

masses as small as 1 $M_{\oplus}$ can be detected via gravitational microlensing. Thus, gravitational microlensing is the only ground-based method that has been shown to be sensitive to Earth-mass planets.

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