

MODELING AND SIMULATION UNCERTAINTY IN MULTIDISCIPLINARY DESIGN OPTIMIZATION

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ABSTRACT

This paper is intended to contribute to the ongoing discussion of selected concepts related to the topic of technical risk or uncertainty in the model-based design of physical artifacts. The paper focuses on the use of analytic models and numerical simulation in the multidisciplinary design optimization process. It considers how issues of physical process variability, information uncertainty and the use of models and simulations influence the design decision process. This paper only qualitatively addresses these issues but the goal is to provide a focus for discussion of concepts associated with information uncertainty as applied to model-based multidisciplinary design and optimization.

INTRODUCTION

Consider first some issues and concepts related to the physical system or artifact being designed. The artifact can be defined, using the concepts introduced by Herbert Simon¹, in terms of its inner and outer environment. “An artifact can be thought of as a meeting point – an “interface” in today’s terms – between an “inner” environment, the substance and organization of the artifact itself, and an “outer” environment, the surroundings in which it operates”. The engineer is tasked with establishing a complete definition of the design and in many cases, manufacturing details (i.e. the inner environment) that can cope with the outer environment in order to achieve a predetermined set of goals. Many of the issues that are often generically referred to as “uncertainty” are related to the ability of the artifact to achieve those goals and are due to characteristics associated with both the inner and outer environments. The process of design

is associated with the engineer making decisions based upon information – from a variety of sources – related to this interface.

The following paper addresses a number of issues associated with uncertainty as related to physical artifacts. The basic distinction is made between the artifact and sources of uncertainty as viewed in the “real world” versus the role of uncertainty in analysis, modeling and simulation. Both these issues influence the decisions made during the system design process. Many of the issues addressed below have resulted from a variety of research efforts related to the development and application of methods for multidisciplinary design optimization. Some of the concepts presented below have also been adapted from the considerable body of research related to experimental uncertainty.²⁻⁵ The recent work by Oberkampf, et.al⁶ has also served to provide some important additional concepts and terminology. Though Reference 6 addresses the estimation of “total uncertainty” in modeling and simulation, and that is just one of a number of issues related to the role of uncertainty in multidisciplinary design optimization, the careful and detailed framework it provides is an important contribution to this area. The current paper does not provide a review of literature in this growing field but does include a selected Bibliography that may assist those interested in pursuing related efforts.

An important issue associated with the subject topic of this paper is the confusion and ambiguity associated with many of the terms used herein. Unfortunately, there are a set of terms that can assume both general “secular” definitions and more specific “technical” definitions. These include uncertainty, error, variability,

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bias, precision, and tolerance. In this paper we hope that the particular context will help the reader to differentiate between the secular and technical definitions. In many cases it was found that the terminology proposed by Oberkampf, et.al.⁶ was very effective in expressing the issues of concern and was adopted, or adapted, whenever possible.

UNCERTAINTY IN THE REAL WORLD

In order to describe how “uncertainty” (and all those concepts associated with it, e.g. risk, robustness, quality, ...) influences the engineering process of system design, consider the variability that exists in physical artifacts and in the way in which the artifacts interact with their outer environments. In order to study the perceived, and real, variations that occur in physical artifacts, one needs to be careful how these observations are made, described and interpreted.

First any measurable characteristic of a physical artifact is always subject to the uncertainty in the measurement process. As a simple example in this discussion consider a particular model/make of an automobile and a small set of characteristic “states” of the automobile that may be of interest to a designer (and a customer). One characteristic is weight and the other is time-to-accelerate to 60 mph. Both of these characteristics can be measured for a single vehicle on repeated trials or for a group of vehicles using either single measurements or repeated trials. The differences in the measured values of these two characteristics would depend upon a variety of factors.

- a) Variations in the measurement process due to differences in technique, equipment or other sources (these would occur even if the physical artifact/s involved were absolutely “identical” and the measurement events (i.e. the acceleration test) were performed the exact same way. This measurement uncertainty is a topic of study in itself and for the remainder of this discussion, it will be assumed that this source of uncertainty has been eliminated.
- b) Variations in the “outer” environment during the measurement. These may be due to differences in wind direction or road conditions during the acceleration test. One would typically like to eliminate these as part of the measurement process but they will be present in all cases. Due to changes that occur in time, no two “measurements” on the same or similar artifacts are carried out under the exact same conditions.
- c) As in case b) where the outer environment will continuously change, there will occur

continuous change in the inner environment. The same artifact is not the “same” for two different measurements. One might believe that the vehicle weight would not change significantly (measurably) from one weight measurement to the next but in reality it would (albeit very small, e.g. consumption and evaporation of fuel, accumulation of residue). If a variety of acceleration tests are conducted without changing tires, the effect on acceleration times could be much more pronounced.

- d) Variations associated with tests conducted on different realizations of the same product. These represent variations in the “inner environment” between different realizations of nominally the same artifact. No two cars are created exactly the same and this is one of the factors of significant interest in the design process.

From the perspective of the designer, item d) above is often considered to be the issue of primary concern. What are the primary sources of these variations in the inner environment? Is it possible to reduce the sources of variation in the artifact’s performance that are associated with the “inner environment” through decisions made in the design process? The designer is also often concerned with how to reduce the sensitivity of the system’s behavior or performance to variations in the outer environment. Is it possible to design the systems so that it is less sensitive to the outer environment variability?

Consider if a particular characteristic of an artifact is measured using a sample of 32 different realizations of the “same” artifact. Figure 1, shown below, might be representative of the results of such a series of “measurements”. As shown in the figure, not all of the results were the same. These results can be described using a variety of statistical techniques and one usually attempts to present this stochastic information in terms of a mean or nominal value and some measure of the variation. The mean and the variation are “characteristics” of the artifact associated with both its inner environment and its outer environment.

It is both the mean and variability that the designer would like to be able to predict, and/or control, as part of the design process. As will be emphasized below, the information available in this figure is only available after the artifact has been realized and, in most cases, that is “too late” to influence the design process. Developing techniques to develop accurate a priori estimates of these characteristics is an important goal in model-based design.

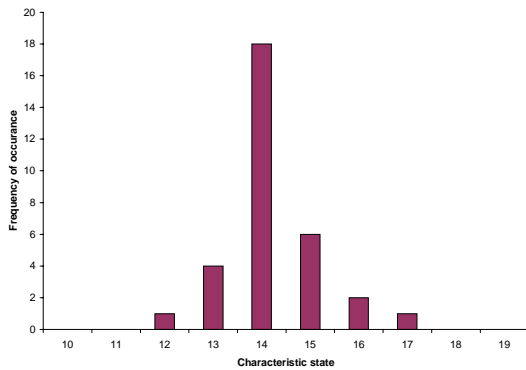


Figure 1. Sample population of a characteristic state for a “real” system

UNCERTAINTY FROM THE SYSTEMS DESIGNER’S PERSPECTIVE

From the designer’s perspective the artifact exists only as an abstraction. The description of the artifact is always incomplete and any information related to the artifact’s characteristics or behavior is approximate prior to its physical realization. Dealing with this incomplete description of the artifact and the approximate nature of the information associated with its characteristics and behavior are key issues in the design process. The following discussion does not address uncertainty in the outer environment and its influence on the designer. This too is a critical issue and worthy of considerable attention.

It is difficult to effectively generalize the design process due to the great diversity in the manner in which the process is carried out and the resulting artifacts. There are differences between the evolutionary design of a simple component and the revolutionary design of a complete complex system. The current discussion, though potentially applicable over this wide range, is intended to focus on the emerging process of model-based design using the extensive capabilities now available with modeling, simulation and digital computing. The discussion will focus on how engineers develop and interpret the information that is used to guide and validate decisions made during the design process.

In the following one should assume that the basic concept for an artifact has been established. With the selection of this basic concept the engineer also has to identify (though not necessarily quantify) a finite set of “design variables” that will eventually be used to uniquely specify the design. (Thus the current focus is only one small step in the overall system design process.) This set of design variables will evolve with

the design throughout the design process. The identification and quantification of this set of design variables are central to this process. It is this quantitative description of the artifact, based upon information developed using numerical models or simulation, that is the focus of this discussion. Though there is great interest in providing quantitative descriptions as early in the design process as possible, this depends upon the availability and the level of development of models and analysis methods related to the class of artifacts being considered.

As the level of abstraction of the artifact changes and more and more detail is required to define it, the number of design variables will grow considerably. Design variables typically are associated with the type of material used in the artifact and geometric description of the form that the material assumes as part of the artifact. The engineer can select materials that possess desirable qualities and then attempt to “shape” them into useful forms to achieve a given purpose. Eventually the engineer will be required to specify (i.e. quantify) each of the design variables in the most refined representation of the artifact. This description often takes the form of a “detailed engineering drawing” that includes materials information, all necessary geometric information needed for fabrication including manufacturing tolerances.

Decisions associated with quantifying (or selecting) the design variables are usually based upon an assessment of a set of behavioral variables, also referred to herein as system states. The behavioral variables or system states are used to describe the artifact’s characteristics. The list of these characteristics also increases in detail as the level of abstraction of the artifact decreases. The designer uses the behavioral variables to assess the suitability of the artifact and these variables represent the engineer’s attempt to predict the future. They are based upon information about the artifact gained from a variety of sources.

There are two primary sources of information available to engineers during the design process - both of which are used in most cases. These two sources are: 1) archived experience, and 2) engineering analysis, modeling and simulation.

Engineers often gather experiential information from empirical data or knowledge bases. Interpolating or extrapolating from information on similar design concepts can also help provide the designer with the confidence to make a decision based upon the success of earlier, similar designs. Often this type of information is incorporated into rules-of-thumb, design handbooks or design guidelines. If an engineer wishes

to include a bolted connection in a mechanical component, rarely would she design the bolt from scratch. The bolts would be selected from a finite set of those available and the selection would be based upon the characteristics of the bolts as provided by the “bolt manufacturer”. It would be the designer’s role to estimate the loads, temperatures and other conditions that would be imposed on the bolts and then to select the bolts that would meet those requirements. The selection decision would most likely allow for certain factors of safety or other means to include the information uncertainty in the process. The use of empirical information requires the designer to make numerous assumptions concerning the suitability of the available information and its applicability to the current situation. These assumptions add to the uncertainty in the design process.

There are also many decisions made in the design process that are based upon individual or corporate experience that is not formally archived in a database. This type of information is very valuable in the design of artifacts that are perturbations (evolutionary design) of existing successful designs but has severe limitations when considering the design of new or revolutionary designs. Many organizations have or are currently trying to archive this type of experience in a formal manner. Though this may be very useful information, quantifying the uncertainty associated with it, in a way that will assist in quantifying the risk associated with the entire product, is usually not possible.

The second type of information available to the designer is that based upon analysis, modeling and simulation. As engineering systems become more complex and greater demands are placed upon artifact performance, time-to-market and cost, this source of information will become even more important in the design process. With the explosive growth in digital computing and in particular with the development of many model-based tools such as FEA for stress, deformation and thermal environment prediction and CFD for fluid-structure interactions, the opportunity to perform complex numerical simulations using desktop computers is a possibility for many engineering designers. It should be emphasized that the information provided by these models or simulations carries with it some level of uncertainty and the use of that information introduces a level of risk to the decisions made based upon this information. Quantifying that uncertainty and understanding the role it plays in the product design process is an issue attracting much discussion.

MULTIDISCIPLINARY SYSTEM MODELING AND ANALYSIS

To introduce the idea of model based design for a multidisciplinary system (and there are very few systems that are not inherently multidisciplinary) the following discussion will use a number of concepts adapted from terminology used in multidisciplinary design optimization, MDO. In this context the process used to develop the quantitative description of the artifact using numerical modeling and simulation is referred to as a System Analysis, SA. The SA for a coupled, non-hierarchic system is illustrated schematically below using a $N \times N$ or N^2 diagram.

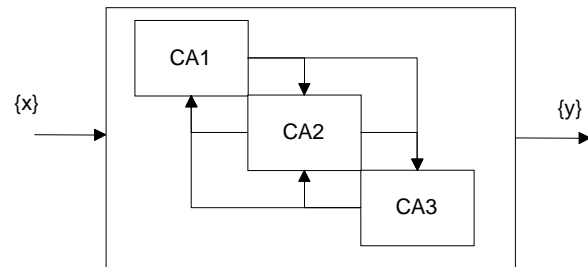


Figure 2. Coupled System Analysis, SA

The representation in Figure 2 assumes that a number of different analytic or numerical models are used to represent the artifact and its behavior at a given level of abstraction. These individual models or analyses are referred to as Contributing Analyses, CAs. At a given level of abstraction the artifact is defined using a vector of design variables, $\{\mathbf{x}\}$. The behavioral characteristics, or system states, of the artifact at the corresponding level of abstraction are expressed in a vector of states, $\{\mathbf{y}\}$. The purpose of the SA is to provide a means of computing $\{\mathbf{y}\}$ for a given vector $\{\mathbf{x}\}$.

Each individual CAs typically require a subset $\{\mathbf{x}_i\}$ of the total system design variable vector, $\{\mathbf{x}\}$, and is used to compute a number of the elements $\{\mathbf{y}_i\}$ of the vector of system states $\{\mathbf{y}\}$. In the case of the completely coupled system shown in the diagram above, each of the CAs are the sources of information that are needed in another one of the CAs, as indicated by the arrows. The arrows illustrate that information is both fed-forward and fed-back during the execution of the SA of a complex, multidisciplinary system. This requires that this process is iterative in nature. The formulation of the system analysis is quite complex and the system analysis itself evolves along with the artifact during the design process.

Early in the process when the representation of the artifact is very abstract, the individual CAs may be

simple “textbook” analyses but as the process continues, the individual CAs can become much more detailed and time-consuming such as the FEA or CFD analyses suggested above. Thus the time necessary to perform the SA depends upon the detail of the models, the complexity of the analyses and convergence characteristics of the iterative process associated with SA.

Reference 6 presents a much “finer grain” discussion of issues related to either an individual CA or the complete SA. It proposes that the development of a SA or CA is composed of a number of distinct phases, these are:

- 1) conceptual modeling of the physical system,
- 2) mathematical modeling of the conceptual model,
- 3) discretization and algorithm selection for the mathematical model,
- 4) computer programming of the discrete model,
- 5) numerical solution of the computer program model,
- 6) representation of the numerical solution.

Each of these phases introduces issues of variation, uncertainty and error (special terms discussed with some detail later) into the process. The final statement in Reference 6 stresses the complexity and importance of these issues,

“..this formal recognition of the sources of nondeterminism and error shows the compounding effect and rapid growth of each source through the modeling and simulation process. Some have referred to this growth of total uncertainty through the modeling and simulation process as “overwhelming.” However, it is an issue that must be faced by analysts and decision makers who use the results of modeling and simulation.”

It may be impractical to pursue the detailed process suggested in Reference 6 for all model-based design activities but it is critical to realize that the issues introduced therein must be addressed in many cases and its importance will continue to be increased.

Returning to the simple representation of the system analysis discussed earlier, the schematic representation shown in Figure 2 actually lacks an important feature that has been added to Figure 3 below.

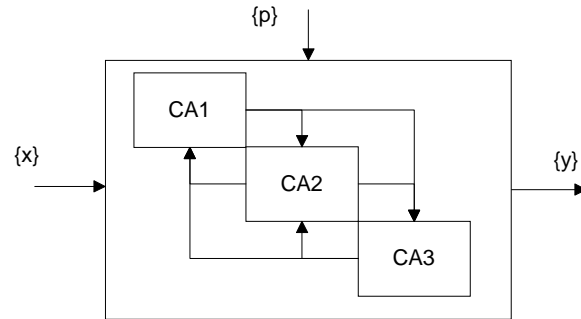


Figure 3. System Analysis augmented with input parameters

As illustrated there is additional information that is typically required to perform the system analysis. This information is represented in the vector $\{p\}$, parameter vector. The elements of the parameter vector generally represent two types of information. The first type is information related to the outer environment in which the artifact will exist and/or operate. This is often outside of the direct “control” of the designer but is information necessary to perform the analyses contained within the SA. The second type is information about the artifact that is not yet available due to the level of abstraction or model detail represented in the SA but is required in order to perform some calculation within the SA. For example the weight of a particular “out-sourced” component that will be selected at a later point in the design process but is needed to predict vibration response. Thus some information must be assumed related to certain features of the inner environment at different stages in the design process and this can be included in $\{p\}$.

Given a specific design as represented by the design vector, $\{x\}$, and a set of parameters, $\{p\}$, one can perform the system analysis in order to develop the corresponding vector $\{y\}$. This process involves making calculations within individual CAs and exchanging information between CAs until convergence to a consistent solution is achieved. The resulting vector $\{y\}$ will completely describe the artifact, at least at the level of detail provided by all the CAs included in the SA. It should be noted that in some situations the SA may also include certain “design rules” and thus the distinction between analysis and design can become somewhat clouded. The issue of convergence of the SA so that a given $\{x\}$ and $\{p\}$ will yield a unique $\{y\}$ is an issue for further discussion but the existence of a unique solution to the SA is assumed at this point.

At least for the current discussion one should assume that the SA is primarily intended to provide information that will be used by the engineer to make design

decisions. These decisions usually involve altering the design vector $\{x\}$ in order to meet certain constraints on the selected system states. This typically means achieving some level of desired “performance” as well as avoiding certain types of failure. How this is accomplished is up to the designer/s and depending upon the complexity of the system this can be a very challenging task. The MDO discipline has as one of its goals the rational automation of this decision process in a manner such that not only are feasible designs achieved (i.e. ones that meet all requirements or constraints) but optimum designs are identified.

Returning to the question of uncertainty, the designer is tasked with determining how well the SA represents the “real world” and assessing the risk associated with making decisions based upon the information provided by the SA. There are a number of important questions to be asked in this regard.

What is the uncertainty associated with the models and techniques used in the SA itself?

How can one predict the inevitable variability, or probabilistic behavior, that the artifact will demonstrate when actualized?

Can the designer predict and compensate for the stochastic nature of the artifact’s behavior in the design of the artifact?

The remaining discussion will attempt to briefly address the first two questions. The third is an ongoing challenge for future research. Recall that it is both the mean characteristics and variability of the actual artifact that the designer would like to be able to predict prior to “fabricating” the artifact. The designer would like to be able to accurately predict the nominal value of each of the states used to define the characteristics of the artifact as well as provide a quantitative characterization of the stochastic behavior of these characteristics.

The answer to first question, “What is the uncertainty associated with the SA itself?” depends upon a number of factors, many of which have been detailed in Reference 6 from the perspective of the analyst. The following attempts to address this question from the perspective of the designer as their perspective may not be exactly the same, in that the designer will eventually be required to make decisions based upon the this information. Consider a situation in which the level of abstraction of the artifact is defined, thus the individual CAs are identified and the elements of $\{x\}$, $\{p\}$ and $\{y\}$ are also defined. By selecting specific values for all of the elements in $\{x\}$ and $\{p\}$ one can usually

uniquely define a corresponding set of states $\{y\}$ (neglecting systems that demonstrate random or chaotic behavior and thus state prediction takes on a different meaning). Normally, if this process is repeated with the same values for $\{x\}$ and $\{p\}$, it would yield the same $\{y\}$.

Consider the “real world” situation discussed earlier. A system analysis could be repeated twenty times and this deterministic analysis procedure, using mean values of all uncertain design variables and parameters, would yield the same result in each case. If the result for the selected characteristic state is included on the histogram presented earlier, it would yield the result shown in Figure 4.

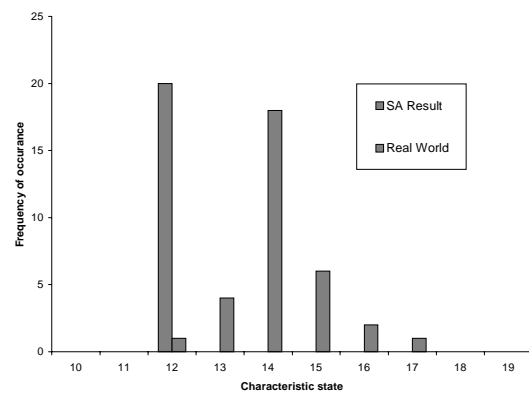


Figure 4. Comparison of SA result to artifact response

In this case all the SAs provide the same result - as expected. If the system analysis contained CAs that included some “random” characteristics such as curve fitting schemes that are initiated from random numbers or other numerical processes with inherent random characteristics, it is possible that the SA itself could yield different results for the same $\{x\}$ and $\{p\}$. If that is the case, the characteristic state may not be the same for each SA performed and that would need to be considered as another source of SA uncertainty but that situation is not considered here. The difference between the mean value of the characteristic state of the “real” system and the value resulting from the SA is referred to herein as a bias error.

Invoking the concepts introduced in Reference 6, bias error would be a combined effect of “uncertainty” and an “acknowledged error”. Uncertainty was defined as resulting from a lack of knowledge or incomplete information. Uncertainty as defined in Reference 6 is normally not stochastic (in contrast to their concept of variability which is inherently stochastic) and for a specific application of an SA, it would most likely be constant. An acknowledged error is a “recognizable deficiency” that is “not due to a lack of knowledge” and

it is usually reproducible and deterministic. In the current context bias error would be one recognized by the analyst (though not necessarily easily quantifiable) and introduced by the modeling or simulation process. The bias error should be differentiated from an unacknowledged error such as a “mistake” and these are not considered herein, though they can be significant in particular in the complex software often associated with design optimization.

Bias errors result from the inherent limitations associated with the analytic models and numerical methods used in the SA. The magnitude and “direction” of this error is the cumulative result of many assumptions associated with the individual CAs within the SA. The magnitude of the bias error would most likely change if the SA was performed for another “design”, in other words if an alternative design (i.e. different values of $\{x\}$) was considered. Since the assumptions and limitations in the analysis and models most likely depend upon characteristics of the design, the bias error would change (e.g. the applicability of the thin walled assumption in a stress calculation or the value assumed for the thermal conductivity of a material.) Though the existence of the bias error is recognized, its value is not known a priori and it can only be altered by modifying the model or the simulation process.

In the coupled system analysis it is important to realize that the effect of bias errors in individual CAs propagates through the entire SA. If CA2 requires information developed in CA1, bias errors in states computed in CA1 will influence the calculations in CA2. Thus if one changes a model or analysis procedure within one CA, it will influence the results of other CAs to which it provides state information and thus influence the overall result from the SA. The propagation of bias error through the SA is an issue of concern. It should be stressed that one never actually knows the bias error until the artifact has been realized and unless a particular phenomena is encountered in actual operation, the presence of a bias error may never be known. Thus predicting the bias error during the product design process requires the designer to make assumptions about the magnitude and “direction” of the bias errors for all of the state information that will influence the design. Bias errors can be influenced by changing the model (e.g. including more or more appropriate finite elements in a stress calculation) or by changing the method of analysis (e.g. using FEM analysis in lieu of simple beam-bending formula). Considerable engineering judgement and experience can be required in order to quantify bias errors for a CA within a SA. One might assume that in the future there

will be a need to provide bias error estimates as part of the system analysis.

The next question posed above “How can one predict the inevitable “random”, or stochastic behavior, that the artifact will demonstrate when actualized?” has received considerable attention in the recent past. The most straightforward way to begin to understand the influence of the variability using the system analysis is illustrated schematically below using the same N^2 diagram used earlier.

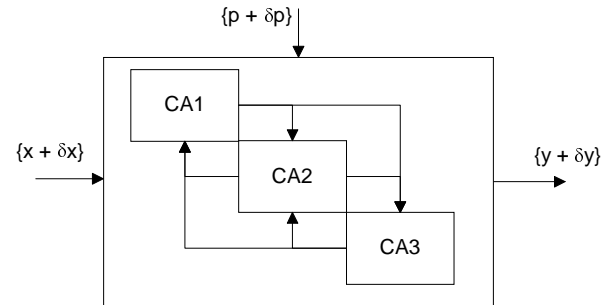


Figure 5. System analysis with input and output variability

In this case it is recognized that some or all of the design variables $\{x\}$ or parameters $\{p\}$ could have associated with them some variation or “variability” (δ) from a mean value⁶. Changing some or all of the elements in these vectors will result in changes in the resulting states $\{y\}$. If one were to perform a Monte Carlo simulation in which variations in the design variables and parameters are selected from an appropriately selected population of random numbers, the resulting state information could be represented on a histogram as illustrated below in Figure 6.

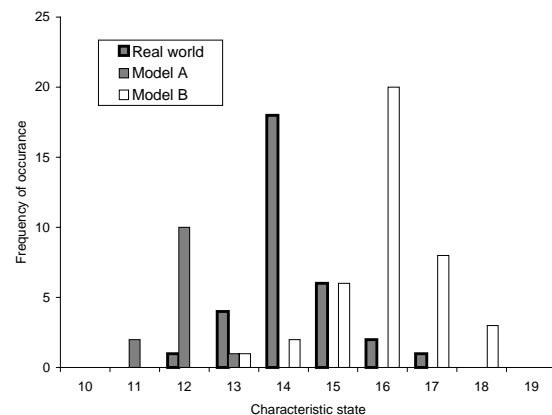


Figure 6. Characteristic state information resulting from multiple system analyses and two different models

In this case three “sets” of information have been presented. The “real world” distribution is the same as

that presented earlier. The two other sets are used to represent two different SAs that differ in the detail or type of models used. The variations associated with Model A and Model B result from the use of random variations about the nominal mean value of design variables and parameters and repeated system analyses. In the case of Model A there appears to be a negative bias error and for Model B a positive bias error. Neither model has been particularly effective in predicting either the mean value or the variation in this particular characteristic or state. It must be emphasized that in reality the designer never knows the “real world” information before the design has been realized. Thus the designer would be faced with selecting between Model A and Model B and the somewhat sparse information provided with each.

The variation about the mean value of the characteristic state for each model results from the perturbations about the mean values of the design variables used in the SA. It is an attempt on the part of the designer to quantify the “variability” that is “the inherent variation associated with the physical system or the environment under consideration.”⁶ This variability is stochastic in nature and is often characterized by a probability or frequency distribution if adequate samples are available.

The magnitude and character of the variations in the state will depend upon the form of the assumed probability distribution for the design variables and parameters. In this coupled SA, random variations in a single design variable or parameter will result in random variations in many of the system states as these variations can influence the states computed in a single CA but those states are then used as input into other CAs. The propagation of random variations in the design variables, parameters and states through a SA is also an area of current research interest. It should be noted that the overall distribution in the predicted system behavior as defined by the SA includes both bias errors and random variability. Discriminating between the sources and influence of each is an important design consideration. Thus the “total uncertainty” associated with the information provided by the system analysis, and required by the designer, is a combination of numerous sources and is not a single “number.”

As indicated above one might consider trying to develop additional insight into the random variability (mean and variation for system states) via repeated SAs using a Monte Carlo approach (or even analytic methods exist for the simplest of systems). This may be feasible if the computing time required to perform the SA is small and the number of design variables and

parameters are also relatively small. As the design matures and the level of abstraction changes, the SA becomes more complex and includes models or analyses not well suited to this approach. Quantifying the total uncertainty will require new approaches to provide the designer with the information on both the expected mean value and variability. Of particular interest will be approaches that do not require the extensive numbers of “samples” usually needed for conventional statistical methods of random data analysis.

In order to develop the information necessary to predict a system’s total uncertainty it appears as if the designer must at least:

1. Given the level of abstraction of the artifact at a given point in the design process, select analysis methods and appropriate fidelity for the models.
2. Define the design variables $\{x\}$, system parameters $\{p\}$ and system states $\{y\}$ necessary for the system analysis. This requires establishing the inner and outer environments for the system.
3. Estimate, based upon other sources, the bias errors associated with the individual CAs that makeup the SA. The variation in the bias errors across the design space must be considered. Those elements of the bias errors associated with lack of knowledge or incomplete information may be the most problematic.
4. Estimate statistical characteristics (i.e. variance, probability distribution, etc.) of the elements of the design variable vector $\{x\}$ and the system parameters $\{p\}$.

Once these steps have been performed, one can then consider how to estimate the bias errors and the variability associated with the system states. The total uncertainty in the predicted characteristics of the artifact (i.e. the information that will subsequently be used to make design decisions) will then be a combination of this information. It must be emphasized that, unlike the simple examples given above, the “real world” results, actual system realizations, are never known during the design process. Thus it is only the nominal value of the system states, as predicted by the SA, and the quantified estimate of the total uncertainty that the designer will have to estimate the expected range of behavior and the probability for that to occur. Since the total uncertainty information provides this range it will be an important factor in helping to quantify risk and influence the design.

ONGOING CHALLENGES FOR MDO

The final consideration in this discussion is related to how this total uncertainty information will eventually influence the design decision process. As with the preceding discussion, there is no attempt to offer solutions, but the goal is to highlight issues associated with applying these ideas to multidisciplinary design optimization. Consider the simple “design problem” illustrated graphically below in Figure 7. In this case there are only two elements of the design variable vector $\{x\}$ and no system parameters. There are three elements of the state vector $\{y\}$ that are represented on the plot. Two of the elements of the state vector represent performance constraints and they are the dashed lines. The third state represents the measure-of-merit for the design. Lines of iso-merit are plotted and the “optimum” design occurs near the upper-right-hand corner of the figure. These lines would represent “deterministic” representations of design requirements and objectives as provided by the SA. Assuming that in order to satisfy the design constraints, the values of the design variables must be “below” the two constraint lines. Visual inspection of this two-dimensional design space indicates that the “optimum” design occurs with a design vector $\{7,6\}$ and is labeled with an “o” on Figure 7.

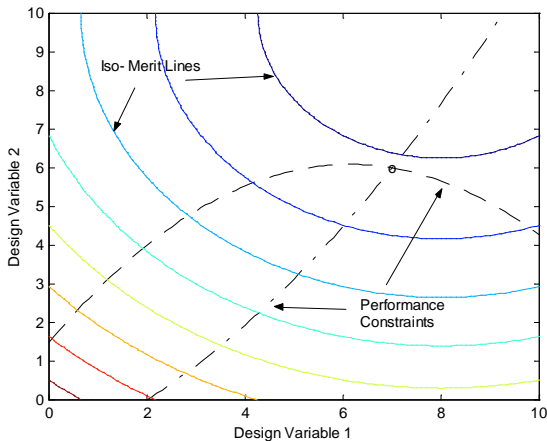


Figure 7. A simple design example

For this design both state constraints are satisfied (i.e. the design is feasible) and the design is as close to the optimum as any other feasible designs. But what happens if, when the designer stipulates a design with design variable attributes $\{7,6\}$, in reality she will only be able to achieve designs with attributes $\{7+\delta_1, 6+\delta_2\}$ where δ_1 and δ_2 are random variables representative of those introduced in manufacturing process, material variability or other sources of variability. Such a case is illustrated using the “+” symbols on the Figure 8.

For this set of 100 “designs” many of the resulting designs would not be feasible, that is they would violate the constraints (assuming for the moment that the states representing the constraints remain deterministic). If the designer were instead to select the design represented by the nominal values $\{7.5,5.2\}$ and encountered similar random variation in design variables nominal values (these designs are marked by “.” in Figure 9) a more robust situation would exist. Figure 9 illustrates that almost all of these design would be feasible, a more desirable condition.

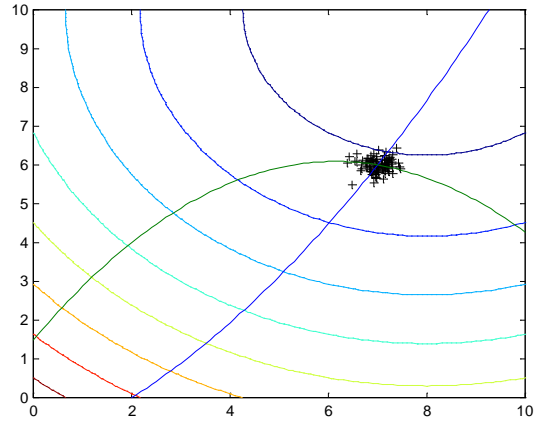


Figure 8. Effect of design variable “variability”

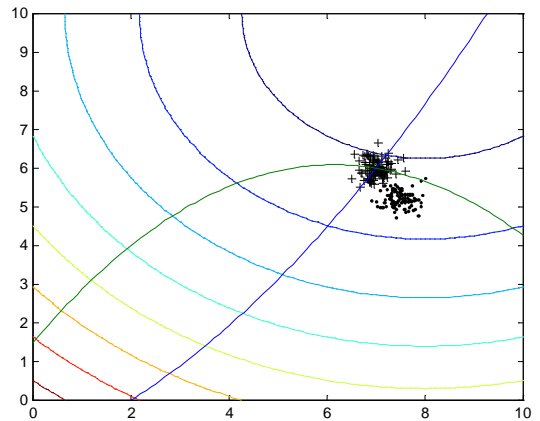


Figure 9. Robust design with variability

This simple example does not include the influence of variability, uncertainty and bias errors in the predicted states. The existence of these sources of total uncertainty would actually alter the location of the merit and constraint boundaries in the design space. The design space as shown represents those boundaries as predicted by the SA, which may not agree with “the real world” as a result of total uncertainty in the modeling and simulation provided by the SA. As

stressed above the system realizations cannot be known a priori and random numbers are just that, random!

This brief example is intended to illustrate just a few of the issues related to systems design optimization with uncertainty. The ongoing challenge is to deal efficiently and economically with the uncertainty in a manner that recognizes the influence that it will have in the design process.

FINAL THOUGHTS AND RECOMMENDATIONS

One of the key challenges in achieving widespread acceptance of the use of MDO techniques in engineering design practice will be the effective integration of issues related to technical risk and uncertainty in the model-based design of physical artifacts. The authors would like to stress three areas in which continued progress must be made in order to achieve this end. The first is the continued development of consistent terminology for discussing these issues. Developing basic concepts and associated terminology will allow for practitioners to effectively discuss and refine ideas. The second is fostering the recognition that modeling and simulation information used in the design process must have associated with it “best” estimates of uncertainty and these must be quantified. Modeling and simulation software that does not provide quantified estimates of uncertainty will be of limited value. It is often felt that “some number is better than nothing” but “some number” without some measure of the risk associated with basing a decision on the number may not be better than “nothing”. Lastly, techniques and design frameworks developed for MDO applications must integrate with the automated, rational design decision-making process the means to efficiently use quantified uncertainty information. The designer must eventually be able to express in a quantitative fashion the technical risk associated with a particular design, as the overall concept of “optimum” must include risk. Continued research emphasis and progress in each of these areas will be needed in order to achieve the goal of widespread use of MDO in engineering practice.

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