

Name: _____

Instructor: _____

Math 10-350, Calculus A
Fall Semester 2006
Exam 1
Tuesday, Sept 19: 8:00–9:15 a.m.

This Examination contains **16** problems, worth a total of **100** points, on **9** sheets of paper including the front cover. The first **12** problems (Section A) are multiple choice with no partial credit, and each is worth **5** points. Record your answers to these problems by placing an \times through one letter for each problem below:

- | | |
|-------------------------------|--------------------------------|
| 1. <input type="checkbox"/> a | 7. <input type="checkbox"/> • |
| 2. <input type="checkbox"/> a | 8. <input type="checkbox"/> a |
| 3. <input type="checkbox"/> • | 9. <input type="checkbox"/> a |
| 4. <input type="checkbox"/> a | 10. <input type="checkbox"/> a |
| 5. <input type="checkbox"/> a | 11. <input type="checkbox"/> • |
| 6. <input type="checkbox"/> a | 12. <input type="checkbox"/> a |

The last **4** problems (Section B) are partial credit problems worth **10** points each. For these problems, **show all your work** and **clearly mark your answers** on the page. Books and notes are not allowed. You may not use your calculator.

Sign the pledge: “On my honor, I have neither given nor received unauthorized aid on this Exam”:

GOOD LUCK!

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Part A: Multiple Choice Problems

1. (5 pts.) Find the limit

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}.$$

- a) 0 b) $\frac{1}{3}$ c) 3 d) 1 e) ∞

2. (5 pts.) Find the limit

$$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4}.$$

- a) -2 b) 0 c) $\frac{3}{4}$ d) ∞ e) $-\infty$

3. (5 pts.) Consider the function

$$f(x) = \frac{x^2 + 3x - 4}{x^2 + 2x - 3}.$$

Which one of the following statements is true?

- a) The graph of f has a vertical asymptote only at $x = -3$.
- b) The graph of f has a vertical asymptote only at $x = 1$.
- c) The graph of f has vertical asymptotes at $x = -3$ and $x = -4$.
- d) The graph of f has vertical asymptotes at $x = -3$ and $x = 1$.
- e) The graph of f has no vertical asymptotes.

4. (5 pts.) Find the limit

$$\lim_{x \rightarrow 3^+} \frac{x-1}{x-3}.$$

- a) -2 b) $-\infty$ c) 2 d) ∞ e) 1

5. (5 pts.) If $g(t) = \frac{t^2 - 2t - 4}{t + 3}$, find $g'(t)$.

- a) $\frac{2t-2}{t+3}$ b) $\frac{3t^2+2t-10}{(t+3)^2}$ c) $\frac{t^2+6t-2}{t+3}$
d) $2t-2$ e) $\frac{t^2+6t-2}{(t+3)^2}$

6. (5 pts.) Let $f(x) = x^2 \cos(x)$. Which one of the following is equal to the second derivative $f''(x)$ of f ?

- a) $-x^2 \cos(x) + 4x \sin(x) + 2 \cos(x)$ b) $x^2 \cos(x) + 4x \sin(x) + 2 \cos(x)$
c) $-x^2 \cos(x) - 2x \sin(x)$ d) $-x^2 \cos(x) - 4x \sin(x) + 2 \cos(x)$
e) $x^2 \cos(x) + 2 \cos(x)$

7. (5 pts.) Determine the point(s) on the graph of $f(x) = x^3 + 3$ at which the tangent line is parallel to the line $y = 3x - 5$.

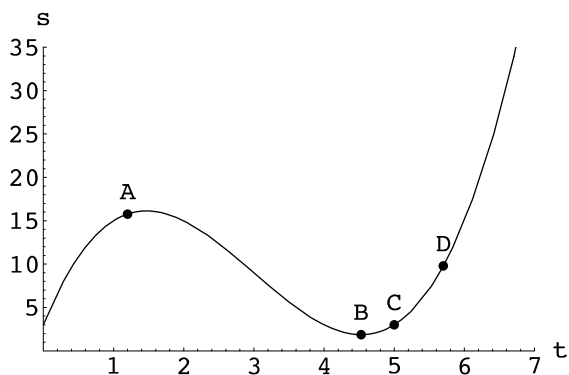
- a) $(1, 4)$ and $(-1, 2)$ b) $(1, 4)$ only c) $(-1, 2)$ only
d) $(1, -2)$ and $(-1, -8)$ e) $(1, -2)$ only

8. (5 pts.) Which of the following statements are true?

- I.** If $y = f(x) + g(x)$, then $y' = f'(x) + g'(x)$.
II. If $y = f(x)g(x)$, then $y' = f'(x)g'(x)$.
III. If $y = \pi$, then $y' = 0$.

- a) I only b) I and III only c) II and III only
d) I and II only e) II only

9. (5 pts.) The graph of the position function $s(t)$ of a moving object is given below.



At which of the labelled points is the object moving the fastest?

- a) It cannot be determined. b) B c) A d) C e) D

10. (5 pts.) Let

$$f(x) = \begin{cases} cx - 3, & \text{if } x \leq 1 \\ \frac{x^2 - 1}{x - 1}, & \text{if } x > 1 \end{cases}$$

where c is a constant. Find the value of c for which f is continuous on the entire real line.

- a) There is no such value of c . b) 4 c) 0 d) 1
e) 5

11. (5 pts.) A ball is thrown into the air. Its height (in feet) after t seconds is given by

$$h(t) = 32t - 16t^2.$$

What is the velocity of the ball when it hits the ground?

- a) -32 ft/sec b) -16 ft/sec c) 0 ft/sec
d) 16 ft/sec e) -24 ft/sec

12. (5 pts.) What is the value of the limit

$$\lim_{\Delta x \rightarrow 0} \frac{\sin(x + \Delta x) - \sin(x)}{\Delta x} ?$$

- a) $\sin(x)$ b) $\cos(x)$ c) $-\cos(x)$ d) $-\sin(x)$ e) 1

Part B: Partial Credit Problems

Remember to show all your work.

13. (10 pts.) Find the equation of the tangent line to the graph of $f(x) = \sin x$ at the point $(\pi/4, \sqrt{2}/2)$ on the graph.

Answer:

14. (10 pts.) Consider the function $f(x)$ defined by the formula

$$f(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ x^2, & \text{if } x > 0. \end{cases}$$

a) Compute the limit $\lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$.

b) Compute the limit $\lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0}$.

c) Is f differentiable at $x = 0$? If no, explain why not. If yes, explain why and give the value of $f'(0)$.

d) Is f continuous at $x = 0$? Explain.

15. (10 pts.) Let $f(x) = x^2 + x$.

a) Find the slope $m(h)$ of the secant line joining the points

$$(2, f(2)) \quad \text{and} \quad (2 + h, f(2 + h))$$

on the graph of $y = f(x)$.

Answer:

b) Using your answer to (a), compute the value of the limit

$$\lim_{h \rightarrow 0} m(h)$$

using only algebra and the limit laws.

Answer:

16. (10 pts.) The position function of a free-falling object is approximately given by the formula

$$s(t) = -5t^2 + v_0t + s_0$$

where s is measured in meters above ground level, t is the time in seconds, v_0 is the initial velocity and s_0 is the initial position.

A ball is thrown vertically into the air from a height of 2 meters, with an initial velocity of 20 meters per second upwards.

a) At what time will the ball be at a height of 2 meters again?

Answer:

b) Give a formula for the velocity of the ball at time t .

Answer:

c) What is the maximum height reached by the ball? (Hint: what will the velocity of the ball be at its maximum height?).

Answer: