Sample Test 3

1e, 2b, 3a, 4d, 5b, 6e, 7a, 8c, 9d,**10a**

Test 3, Problem 11

(a) State the divergence theorem.

Sol.

$$\iint_D \mathbf{V}\cdot\mathbf{n}\, d\sigma = \iiint_\Omega \text{Div } \mathbf{V} \;\; dxdydz$$

where D is a closed surface enclosing the volume Ω, $\mathbf{n}$ is the exterior normal.

(b) Compute $\iint_D \mathbf{V}\cdot\mathbf{n}\, d\sigma$, where D is the ellipsoid surface $\frac{x^2}{1}+\frac{y^2}{4}+\frac{z^2}{9}=1$, the vector $\mathbf{n}$ is the exterior normal vector, and $\mathbf{V}=y^2\mathbf{i}+y\mathbf{j}-z\mathbf{k}$.

Sol. Since Div $\mathbf{V}=0+1-1=0$,

$$\iint_D \mathbf{V}\cdot\mathbf{n}\, d\sigma = \iiint_\Omega \text{Div } \mathbf{V} \;\; dxdydz = 0$$

Problem 12

(a) If $f(x)$ is represented by a Fourier Sine Series

$$f(x)=\sum_{n=1}^{\infty} a_n \sin(nx), \quad -\pi < x < \pi$$

what is the formula for a_n?

Sol.

$$a_n = \frac{2}{\pi}\int_0^\pi f(x)\sin(nx)dx$$

(b) Compute the Fourier Sine series for the function

$$f(x)=\begin{cases} 2 & 0<x<\pi \\ -2 & -\pi<x<0 \end{cases}$$

Sol.

$$a_n = \frac{2}{\pi}\int_0^\pi f(x)\sin(nx)dx = \frac{4}{\pi}\int_0^\pi \sin(nx)dx = \frac{-4}{\pi}\frac{\cos(nx)}{n}\Big|_0^\pi = \frac{4(1-\cos(n\pi))}{n\pi}$$

Thus

$$f(x)=\frac{8}{\pi}\Big(\sin(x)+\frac{1}{3}\sin(3x)+\frac{1}{5}\sin(5x)+\frac{1}{7}\sin(7x)+\cdots\Big)$$

Problem 13
Using appropriate coordinate system, evaluate the integral

$$\iint_D e^{-x^2-y^2}\,dxdy,$$

where D is the unit solid disk. Show all your work.

Sol.

$$\iint_D e^{-x^2-y^2}\,dxdy = \int_0^{2\pi} d\theta \int_0^1 e^{-r^2} r dr = 2\pi(-\frac{1}{2}e^{-r^2})\Big|_0^1 = \pi(1-e^{-1})$$