ACMS 20550: Applied Math. Method I
Exam I, Version white
September 16, 2021

Name: ____________________________

Instructor: ________________________

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use a BASIC calculator. (Cell phone calculator -YES. Graphic calculators - NO).
- The exam lasts for 75 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

A formula sheet is included in the last page

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
Total	________

Multiple Choice

1.(6 pts.) The series $\sum_{n=1}^{\infty} \frac{2^n}{(n!)^2}$ is

(a) divergent by preliminary test

(b) convergent by ratio test

(c) divergent by ratio test

(d) convergent by preliminary test

(e) divergent by integral test

2.(6 pts.) The series $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)^{3/2}}$ is

(a) divergent by ratio test

(b) convergent by ratio test

(c) divergent by integral test

(d) convergent by integral test

(e) divergent by preliminary test

3.(6 pts.) The power series $\sum_{n=0}^{\infty} n(-5x)^n$ is

(a) convergent for all x

(b) convergent for $|x| > 1/5$ and divergent for $|x| < 1/5$

(c) convergent for all $x > 1/5$ and divergent for $x \leq -1/5$

(d) convergent for $|x| \leq 1/5$ and divergent for $|x| > 1/5$

(e) convergent for $|x| < 1/5$ and divergent for $|x| \geq 1/5$

4.(6 pts.) $\lim_{n\to\infty} \frac{5^n}{n!} =$

(a) e

(b) e^2

(c) 0

(d) ∞

(e) 1

5.(6 pts.) The sum of series $\sum_{n=1}^{\infty} 3e^{-n\ln 4}$ is: [*Hint: what is* $e^{-\ln 4}$*?*]

(a) $4/3$

(b) 3

(c) 1

(d) 4

(e) $3/4$

6.(6 pts.) $\sum_{n=1}^{\infty} \frac{2^n}{n!} \quad =$

(a) 1

(b) e^2

(c) e

(d) $e^2 + 1$

(e) $e^2 - 1$

7.(6 pts.) The series $\sum_{n=1}^{\infty} \dfrac{(-1)^n(1+\sqrt{n})}{1+n}$ is

(a) divergent by comparison test

(b) divergent by ratio test

(c) convergent but not absolutely convergent.

(d) convergent by the preliminary test.

(e) absolutely convergent

8.(6 pts.) The series $\sum_{n=1}^{\infty} \dfrac{(n-1)!}{(n+1)!}$ is

(a) divergent by the preliminary test

(b) convergent by the ratio test

(c) divergent by the comparison test

(d) convergent by the preliminary test

(e) convergent by the integral test

9.(6 pts.) By the ratio test, the series $\sum_{n=1}^{\infty} \frac{1}{n^2}\left(\frac{x}{2}\right)^n$ is convergent for $|x| < 2$. At the endpoints, the series

(a) is divergent for both $x = -2$ and $x = 2$

(b) is convergent for $x = -2$ and is divergent for $x = 2$

(c) is convergent for $x = 2$ and is divergent for $x = -2$

(d) is convergent absolutely for both $x = -2$ and $x = 2$

(e) is convergent for both $x = -2$ and $x = 2$, but not absolutely convergent for $x = 2$

10.(6 pts.) The series $\sum_{n=1}^{\infty}(-1)^n \frac{2n^2+1}{\sqrt{2n^2+2}}$ is

(a) divergent by the ratio test

(b) divergent by the preliminary test

(c) convergent but not absolutely convergent

(d) convergent because the first few terms are quite small

(e) absolutely convergent by the integral test

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(20 pts.) Find the Taylor (MacLaurin) series of the following functions. Write your answer in the format of a summation. For example, $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$.

(a) $\dfrac{1}{x} \ln\left(1 + \dfrac{x}{2}\right)$

(b) $\displaystyle\int_0^x \cos t^2 \, dt$.

Partial Credit

You must show your work on the partial credit problems to receive credit!

12.(20 pts.)

(a) Determine whether the series $\sum_{n=0}^{\infty} \frac{n^n}{3^n n!}$ is convergent. Indicate the test you use. Show all your work. You may need the formula $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \approx 2.7$.

(b) Use Taylor series to find $\lim_{x\to 0} \frac{e^{x^2}-1}{\sin^2(2x)} =$

Formula Sheet

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots, -1 < x < 1;$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \text{ all } x;$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \text{ all } x;$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots, \text{ all } x;$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, -1 < x \le 1;$$