This sample exam does not cover all materials of Chapter 1.

The format of exam will be the same as this one.

Name: ____________________

Instructor: Sample

Multiple Choice

1.(6 pts.)The series $\sum_{1}^{\infty} \frac{(-1)^n}{\sqrt{2n+7}}$ is

(a) divergent and there is no limit for the partial sum

(b) divergent and diverges to $+\infty$

(c) divergent and diverges to $-\infty$

(d) absolutely convergent

(e) convergent but not absolutely convergent

2.(6 pts.) The series $\sum_{1}^{\infty} \frac{n^8}{2^n}$ is

(a) divergent by ratio test

(b) convergent by ratio test

(c) cannot be decided by ratio test since the limit of the ratio is 1

(d) divergent because the numerator is too large after computing several terms

(e) convergent because this is a geometric series

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3.(6 pts.) The power series $\sum_{1}^{\infty} \frac{2n+1}{3^n} x^n$ is

(a) convergent for $|x| < 2$ and divergent for $|x| > 2$

(b) convergent for $|x| > 3$ and divergent for $|x| < 3$

(c) convergent for $|x| < 3$ and divergent for $|x| > 3$

(d) convergent for $|x| > 2$ and divergent for $|x| < 2$

(e) convergent for $|x| < 1$ and divergent for $|x| > 1$

4.(6 pts.) The series $\sum_{1}^{\infty} \frac{(-1)^n n^2}{100(n^2+2n+8)}$ is

(a) convergent because there are enough cancellations

(b) convergent by alternating series test

(c) convergent because first few terms are quite small

(d) divergent by divergent test (preliminary test)

(e) divergent by ratio test

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5.(6 pts.) $\lim\limits_{x\to 0} \dfrac{e^{x^2}-1}{\sin^2(2x)} \quad =$

(a) 1 (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{2}$

(d) 0 (e) $+\infty$

6.(6 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!} \quad =$

(a) e^2+1 (b) e (c) 1

(d) e^2 (e) e^2-1

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7.(6 pts.) $\lim_{n\to\infty} \frac{10^n}{n!} \quad =$

(a) 0

(b) 10

(c) does not exist and is not $+\infty$

(d) 1

(e) $+\infty$

8.(6 pts.) By ratio test, the series $\sum_{n=1}^{\infty} \frac{(2x)^n}{(n+1)3^n}$ is convergent for $|x| < \frac{3}{2}$. At the endpoints, the series

(a) is convergent for $x = \frac{3}{2}$ and is divergent for $x = -\frac{3}{2}$

(b) are divergent for both $x = -\frac{3}{2}$ and $x = \frac{3}{2}$

(c) are convergent absolutely for both $x = -\frac{3}{2}$ and $x = \frac{3}{2}$

(d) are convergent for both $x = -\frac{3}{2}$ and $x = \frac{3}{2}$, but not absolutely convergent for $x = -\frac{3}{2}$

(e) is convergent for $x = -\frac{3}{2}$ and is divergent for $x = \frac{3}{2}$

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9.(6 pts.) The series $\sum_{n=1}^{\infty} \frac{n^2+5}{\sqrt{n^7+3n^4+6}}$ is

(a) convergent by ratio test

(b) divergent by ratio test

(c) divergent by integral test

(d) convergent by special comparison test

(e) divergent by comparison test

10.(6 pts.) For $b > 0$, the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^b}$ is

(a) convergent for $b \geq 2$ and divergent for $0 < b < 2$, by integral test

(b) always convergent, by integral test

(c) convergent for $b > 1$ and divergent for $0 < b \leq 1$, by integral test

(d) convergent for $b \geq 1$ and divergent for $0 < b < 1$, by integral test

(e) convergent for $b > 2$ and divergent for $0 < b \leq 2$, by integral test

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(15 pts.) Find the Taylor series of the following functions. Write your answer in the format of a summation. For example, $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$.

(a) $\dfrac{1}{1+x^2}$.

(b) $\arctan x$ (use the result from (a))

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12.(15 pts.) The formula $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ is valid for all x.

(a) For $|x| \le 1$, write the formula **for the error estimates** covered in lecture using n-terms

$$\sum_{k=0}^{n} \frac{x^k}{k!}$$

as the approximation for e^x.

(b) Using (a), for $|x| \le 1$, find out the minimum number n needed in order to have an error less than 0.001. Justify your answer. For those without a calculator, we list the first few factorial numbers and the approximation of the number e here
($2! = 2,\ 3! = 6,\ 4! = 24,\ 5! = 120,\ 6! = 720,\ 7! = 5040,\ 8! = 40320$ and $e \approx 2.718$)

Answer: $n =$______

Justification:

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Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(10 pts.) Use Taylor series to find

$$\frac{d^8}{dx^8}\left(x^4[1-\cos(x^2)]\right) \text{ at } x = 0.$$

Show all your work.

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Formula Sheet

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots, -1 < x < 1;$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \text{ all } x;$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \text{ all } x;$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots, \text{ all } x;$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, -1 < x \leq 1;$$