ACMS 20550: Applied Math. Method I
Exam III, Version pink
November 11, 2021

Name: ______________________________

Instructor: __________________________

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use a BASIC calculator. (Cell phone calculator -YES. Graphic calculators - NO).
- The exam lasts for 75 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

It is a violation of the honor code to give or to receive unauthorized aid on this Exam.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
Total	________

Multiple Choice

1.(6 pts.)For the vectors $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, the work done by the force $\mathbf{B}$ on the displacement $\mathbf{A}$ is

(a) 0 (b) 3 (c) 4

(d) -2 (e) 10

2.(6 pts.) Find the speed (magnitude of the velocity) of a moving particle whose position is given by $\mathbf{r} = \mathbf{r}(t) = 2t^2\,\mathbf{i} - 3t\,\mathbf{j} + (t-1)^2\,\mathbf{k}$ at time $t = 1$.

(a) 3. (b) 2. (c) 0.

(d) 5. (e) 4.

3.(6 pts.) The volume between the planes $z = 2x + 3y + 5$ and $z = 2x + 7y + 7$ over the triangle with vertices $(0, 0)$, $(2, 0)$ and $(2, 1)$.

(a) $\frac{2}{3}$ (b) $\frac{9}{2}$ (c) $\frac{5}{2}$

(d) $\frac{10}{3}$ (e) $\frac{7}{3}$

4.(6 pts.) For which value of c is the vector field $\mathbf{F} = (cx^3y^4 + x)\mathbf{i} + (2x^4y^3 + y)\mathbf{j}$ conservative?

(a) 2 (b) -3 (c) -2

(d) -1 (e) 3

5.(6 pts.) Consider the surface area S formed by revolving the curve $y = 1 + \sin x$ around the x-axis from $x = 0$ to $x = 2\pi$. The surface area of S is given by the integral

(a) $2\pi \int_{x=0}^{2\pi} \sin x \sqrt{1 + \cos^2 x}\, \mathrm{d}x$

(b) $2\pi \int_{x=0}^{2\pi} \sqrt{1 + \cos^2 x}\, \mathrm{d}x$

(c) $2\pi \int_{x=0}^{2\pi} (1 + \sin x)^2 \sqrt{1 + \cos^2 x}\, \mathrm{d}x$

(d) $2\pi \int_{x=0}^{2\pi} \sin^2 x \sqrt{1 - \cos^2 x}\, \mathrm{d}x$

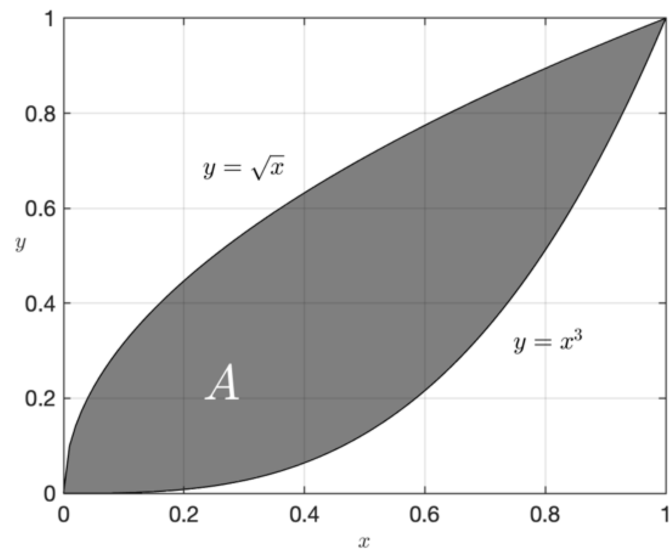
(e) $2\pi \int_{x=0}^{2\pi} (1 + \sin x) \sqrt{1 + \cos^2 x}\, \mathrm{d}x$

6.(6 pts.) Using Green's Theorem or otherwise, calculate the integral $\oint_C 2y\mathrm{d}x - 3x\mathrm{d}y$, where C is the square bounded by $x = 3$, $x = 5$, $y = 1$ and $y = 3$.

(a) 10. (b) 5. (c) -20.

(d) 4. (e) -5.

7.(6 pts.) For the shaded area A in the picture below, which of the following represents the integral $\iint_A f(x,y)dxdy$.



(a) $\int_{y=0}^{1}\left(\int_{x=y^2}^{y} f(x,y)dx\right)dy.$

(b) $\int_{x=0}^{1}\left(\int_{y=x^3}^{\sqrt{x}} f(x,y)dy\right)dx.$

(c) $\int_{x=0}^{1}\left(\int_{y=0}^{1} f(x,y)dy\right)dx.$

(d) $\int_{y=0}^{1}\left(\int_{x=0}^{1} f(x,y)dx\right)dy.$

(e) $\int_{y=0}^{1}\left(\int_{x=\sqrt{y}}^{y^3} f(x,y)dx\right)dy.$

8.(6 pts.) Find the directional derivative of $xy^2 + z^5$ at the point $(1,1,1)$ in the direction of $\mathbf{u} = (1,-1,1)$. Hint: remember to normalize the direction $\mathbf{u}$!

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{5}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}}$

(d) $\frac{4}{\sqrt{3}}$

(e) $\frac{8}{\sqrt{3}}$

9.(6 pts.) Find the area of the plane $2x - 3y - 4z = 9$ cut out by the cylinder $x^2 + y^2 = 4$.

(a) $\sqrt{29}\pi$ (b) 2π (c) 4π

(d) $\dfrac{\sqrt{29}}{3}\pi$ (e) $\dfrac{\sqrt{13}}{2}\pi$

10.(6 pts.) Consider a scalar field $\phi(x, y)$ and vector field $\mathbf{F} = F_1(x, y)\mathbf{i} + F_2(x, y)\mathbf{j}$. The quantities

(i) $\nabla \cdot (\nabla\phi)$

(ii) $\mathbf{F} \times \nabla\phi$

(iii) $\nabla \cdot (\phi\mathbf{F})$

are best described by which statement? (Recall: $\nabla = \dfrac{\partial}{\partial x}\mathbf{i} + \dfrac{\partial}{\partial y}\mathbf{j}$)

(a) (i) is a scalar and (ii), (iii) are vectors.

(b) (i),(iii) are scalars and (ii) is a vector.

(c) (i), (ii) and (iii) are scalars.

(d) (i), (ii) are scalars, (iii) is a vector.

(e) (i), (ii) and (iii) are vectors

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(20 pts.)

(a) [10pts] By changing to polar coordinates, evaluate the integral

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\sqrt{x^2+y^2}}\,dx dy.$$

(b) [10pts] Evaluate $\iint_A 3xy\,\mathrm{d}x\mathrm{d}y$ where A is the triangle with vertices $(0,0),(3,0),(1,2)$.

Partial Credit

You must show your work on the partial credit problems to receive credit!

12.(20 pts.)

(a) [10pts] By changing the order of integration, evaluate the double integral

$$\int_{y=0}^{1}\int_{x=y^2}^{1}\frac{e^x}{\sqrt{x}}dxdy.$$

(b) [10pts] Find the area of the surface $2z = x^2 + y^2$ over the unit disc $x^2 + y^2 < 1$.

Formula Sheet

1. Surface integral on D represented by $\phi(x,y,z)=0$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \frac{\sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2}}{\left|\frac{\partial\phi}{\partial z}\right|}.$$

Surface integral on D represented by $z=f(x,y)$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}.$$

2. Green's Theorem: For bounded region A and continuous $P(x,y)$, $Q(x,y)$

$$\iint_A \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right) dxdy = \oint_{\partial A} (Pdx+Qdy).$$

3. Volume generated by the vectors $\vec{A},\vec{B},\vec{C}$.

If $\vec{A}=(A_1,A_2,A_3)$, $\vec{B}=(B_1,B_2,b_3)$, $\vec{C}=(C_1,C_2,C_3)$, then

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

4. Polar coordinate system ($r\geq 0,\ 0\leq\theta\leq 2\pi$)

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}, \quad dxdy = rdrd\theta$$

5. Cylindrical coordinate system ($r\geq 0,\ 0\leq\theta\leq 2\pi$)

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}, \quad dxdydz = rdrd\theta dz$$

6. Spherical coordinate system ($r\geq 0,\ 0\leq\theta\leq\pi,\ 0\leq\phi\leq 2\pi$)

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}, \quad dxdydz = r^2\sin\theta drd\theta d\phi$$