Formula Sheet for Final exam, Dec 13, 2022

1. Taylor Series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots, -1 < x < 1;$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \text{ all } x;$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \text{ all } x;$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots, \text{ all } x;$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, -1 < x \le 1;$$

2. Tangent Plane and Normal Line:

If the normal vector direction is given by $\vec{n} = (n_1, n_2, n_3)$, then

Tangent plane at (x_0, y_0, z_0) is: $n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

Normal line at (x_0, y_0, z_0) is: $\dfrac{x - x_0}{n_1} = \dfrac{y - y_0}{n_2} = \dfrac{z - z_0}{n_3}$

Formula Sheet

3. Surface integral on D represented by $\phi(x, y, z) = 0$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \frac{\sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2}}{\left|\frac{\partial\phi}{\partial z}\right|}$$

Surface integral on D represented by $z = f(x, y)$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

4. Green's Theorem: For bounded region A and continuous $P(x, y)$, $Q(x, y)$

$$\iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \oint_{\partial A} (Pdx + Qdy).$$

5. Stokes' Theorem: For a surface σ in 3-dimensional space with its boundary $\partial\sigma$ oriented in consistency with the normal direction,

$$\oint_{\partial\sigma} \mathbf{V} \cdot d\mathbf{r} = \iint_\sigma \text{curl } \mathbf{V} \cdot \mathbf{n} \; d\,\sigma.$$

6. Volume generated by the vectors $\vec{A}, \vec{B}, \vec{C}$.

If $\vec{A} = (A_1, A_2, A_3)$, $\vec{B} = (B_1, B_2, b_3)$, $\vec{C} = (C_1, C_2, C_3)$, then

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

7. Polar coordinate system $(r \geq 0,\ 0 \leq \theta \leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}, \quad dxdy = rdrd\theta$$

8. Cylindrical coordinate system $(r \geq 0,\ 0 \leq \theta \leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}, \quad dxdydz = rdrd\theta dz$$

9. Spherical coordinate system $(r \geq 0,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi)$

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}, \quad dxdydz = r^2\sin\theta drd\theta d\phi$$