ACMS 20550: Applied Math. Method I
Final Exam, Version Sample
Date: Sample

Name: ______________________________
Instructor: ___________________________

- Be sure that you have all 15 pages of the test.
- You may use a BASIC calculator. (TI graphic calculators are not allowed).
- The exam lasts for 2 hours.
- Be sure that your name is on every page in case pages become detached.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

It is a violation of the honor code to give or to receive unauthorized aid on this Exam.

Two formula sheets are included at the end of this test

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e	9.	a	b	c	d	e
2.	a	b	c	d	e	10.	a	b	c	d	e
3.	a	b	c	d	e	11.	a	b	c	d	e
4.	a	b	c	d	e	12.	a	b	c	d	e
5.	a	b	c	d	e	13.	a	b	c	d	e
6.	a	b	c	d	e	14.	a	b	c	d	e
7.	a	b	c	d	e	15.	a	b	c	d	e
8.	a	b	c	d	e						

Please do NOT write below.

Multiple Choice	________
16.	________
17.	________
18.	________
Total	________

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Multiple Choice

1.(6 pts.)The series $\sum_{1}^{\infty} \frac{(-1)^n}{2n+7}$ is

(a) divergent and diverges to $+\infty$

(b) convergent but not absolutely convergent

(c) divergent and diverges to $-\infty$

(d) absolutely convergent

(e) divergent and there is no limit for the partial sum

2.(6 pts.)We use the following formula to approximate $\cos 1$

$$\cos 1 \approx 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!}$$

What is the best error estimates?

(a) $\frac{1}{8!}$ (b) $\frac{2^8}{8!}$ (c) $\frac{2^8}{5!}$

(d) $\frac{1}{7!}$ (e) $\frac{1}{5!}$

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3.(6 pts.) Suppose that $f(x) = \begin{cases} 2, & 0 < x < 1, \\ -1, & -1 < x < 0 \end{cases}$. What value will the Fourier series converges to at $x = 0$?

(a) -2 (b) -1 (c) 2

(d) 0 (e) $\frac{1}{2}$

4.(6 pts.) The disc of convergence of the series $\sum_{1}^{\infty} \frac{2^n + 1}{n^2 + 3} z^n$ is

(a) $|z| < 1$, by ratio test (b) $|z| < \frac{1}{2}$, by ratio test

(c) $|z| < \sqrt{2}$, by ratio test (d) $|z| < \frac{1}{\sqrt{2}}$, by ratio test

(e) $|z| < 2$, by ratio test

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5.(6 pts.) Find all roots of $(-1)^{1/4}$. Do not simplify.

(a) $e^{i(\pi/2)},\ e^{i(3\pi/2)},\ e^{i(5\pi/2)},\ e^{i(7\pi/2)}$

(b) $\sqrt{2}i,\ \sqrt{2},\ -\sqrt{2},\ -\sqrt{2}i$

(c) $e^{i(\pi/4)},\ e^{i(3\pi/4)},\ e^{i(5\pi/4)},\ e^{i(7\pi/4)}$

(d) $1, -1, i, -i$

(e) $e^{i(\pi/8)},\ e^{i(5\pi/8)},\ e^{i(9\pi/8)},\ e^{i(13\pi/8)}$

6.(6 pts.)The series $\frac{1}{3!} - \frac{1}{5!} + \frac{1}{7!} - \frac{1}{9!} + \frac{1}{11!} - \frac{1}{13!} + \cdots$ converges to

(a) 0

(b) $1 - \cos 1$

(c) $\cos 1$

(d) $\sin 1$

(e) $1 - \sin 1$

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7.(6 pts.) If $x^2 + y^2 = s + t$ and $2xy = s - 2t$, find $\partial x/\partial t$.

(a) $\dfrac{x-2y}{2(x+y)}$ (b) $\dfrac{x+2y}{2(x-y)}$ (c) $\dfrac{sx+2ty}{2(x-y)}$

(d) $\dfrac{x-2y}{2(x^2+y^2)}$ (e) $\dfrac{x+2y}{2(x^2-y^2)}$

8.(6 pts.)Find the derivative

$$\frac{d}{dt}\int_0^{\cos t} \frac{1}{1+x^4}dx$$

(a) $\dfrac{-\cos t}{1+(\cos t)^4}$ (b) $\dfrac{1}{1+(\cos t)^4}$

(c) $\dfrac{1}{1+(\sin t)^4}$ (d) $\dfrac{-\sin t}{1+(\cos t)^4}$

(e) $\dfrac{\cos t}{1+(\cos t)^4}$

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9.(6 pts.)If $\mathbf{F}$ is a conserved field, which one of the following statements is always true?

(a) curl $\mathbf{F} = \mathbf{0}$

(b) div $\mathbf{F} = 0$

(c) grad $\mathbf{F} = \mathbf{0}$

(d) $\nabla \cdot \mathbf{F} = 0$

(e) grad $|\mathbf{F}| = \mathbf{0}$

10.(6 pts.)Find the expression that represent the integral

$$\iint_D f(x, y)dxdy,$$

where D is the upper half disk of radius 2, that is, $D : x^2 + y^2 \le 4, \;\; y \ge 0.$

(a) $\displaystyle\int_{\theta=0}^{\pi} \Big(\int_{r=0}^{2} f(r\cos\theta, r\sin\theta)rdr \Big) d\theta$

(b) $\displaystyle\int_{\theta=0}^{\pi} \Big(\int_{r=0}^{2} f(r\cos\theta, r\sin\theta)dr \Big) d\theta$

(c) $\displaystyle\int_{\theta=0}^{2\pi} \Big(\int_{r=0}^{2} f(r\cos\theta, r\sin\theta)rdr \Big) d\theta$

(d) $\displaystyle\int_{\theta=0}^{2\pi} \Big(\int_{r=0}^{2} f(r\cos\theta, r\sin\theta)dr \Big) d\theta$

(e) $\displaystyle\int_{\theta=0}^{\pi/2} \Big(\int_{r=0}^{2} f(r\cos\theta, r\sin\theta)rdr \Big) d\theta$

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11.(6 pts.) If a sound wave is represented by $p(t) = \sum_{n=1}^{\infty} \frac{\cos(61nt)}{20(n-3)^2+1}$, what is the apparent frequency (the frequency you can hear)?

(a) $\frac{122}{2\pi}$ (b) $\frac{244}{2\pi}$ (c) $\frac{305}{2\pi}$

(d) $\frac{183}{2\pi}$ (e) $\frac{61}{2\pi}$

12.(6 pts.) Find the directional derivative of $x+3y^2+z^4$ at the point $(0,1,1)$ in the direction (a,b,c).

(a) $\frac{6b+4c}{\sqrt{a^2+b^2+c^2}}$ (b) $a+6b+4c$ (c) $\frac{6b+4c}{\sqrt{|a|+|b|+|c|}}$

(d) 0 (e) $\frac{a+6b+4c}{\sqrt{a^2+b^2+c^2}}$

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13.(6 pts.)Using Green's theorem, evaluate

$$\int_C (x-y)dy - (y+x)dx,$$

where C is a circle of radius 3 oriented counter-clockwise. This integral equals to

(a) 18π (b) 9π (c) 3π

(d) 0 (e) 6π

14.(6 pts.)Using differential, for large n, the expression

$$\frac{1}{n} - \frac{1}{n+2}$$

can be approximated by

(a) $\frac{2}{n}$ (b) $\frac{1}{(n+2)^2}$ (c) $\frac{2}{n^2}$

(d) $\frac{1}{n^2}$ (e) $\frac{1}{n}$

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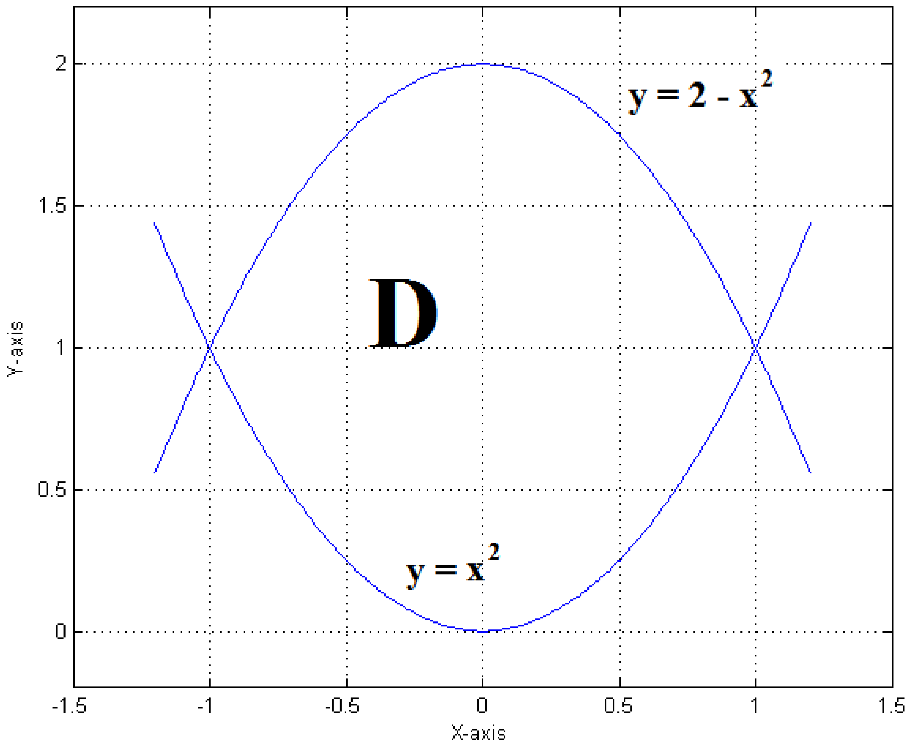
15.(6 pts.) For the area D in the picture below, which of the following represents the integral $\iint_D f(x,y)dxdy$?



(a) $\displaystyle\int_{y=0}^{2}\Big(\int_{x=-\sqrt{y}}^{\sqrt{y}} f(x,y)dx\Big)dy$

(b) $\displaystyle\int_{y=0}^{2}\Big(\int_{x=\sqrt{y}}^{\sqrt{2-y}} f(x,y)dx\Big)dy$

(c) $\displaystyle\int_{x=-1}^{1}\Big(\int_{y=x^2}^{2-x^2} f(x,y)dy\Big)dx$

(d) $\displaystyle\int_{y=0}^{2}\Big(\int_{x=-\sqrt{y}}^{\sqrt{2-y}} f(x,y)dx\Big)dy$

(e) $\displaystyle\int_{x=-1}^{1}\Big(\int_{y=0}^{2} f(x,y)dy\Big)dx$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

16.(20 pts.) (a) Find the Taylor series for $\sin(x^2)$ about $x = 0$.

(b) Find the Taylor series for the function $G(x) = \int_0^x \sin(t^2)dt$ about $x = 0$.

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17.(20 pts.) (a) Find the minimum distance from the origin to the plane $x + 2y + 3z = 14$.

(b) Find the equation of the normal line of the surface $x^2 + 2y^2 + 3z^2 = 4$ at the point $(1, 1, 1)$.

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18.(20 pts.) (a) What extension is needed for a function $f(x)$ defined on $(0, l)$ in order to have a Fourier Sine series?

(b) If $f(x) = 2$ for $0 < x < l$, find the Fourier Sine series.

Formula Sheet

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots, -1 < x < 1;$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \text{ all } x;$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \text{ all } x;$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots, \text{ all } x;$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, -1 < x \leq 1;$$

Formula Sheet

1. Surface integral on D represented by $\phi(x, y, z) = 0$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \frac{\sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2}}{\left|\frac{\partial\phi}{\partial z}\right|}$$

Surface integral on D represented by $z = f(x, y)$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

2. Green's Theorem: For bounded region A and continuous $P(x, y)$, $Q(x, y)$

$$\iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \oint_{\partial A} (Pdx + Qdy).$$

3. Stokes' Theorem: For a surface σ in 3-dimensional space with its boundary $\partial\sigma$ oriented in consistency with the normal direction,

$$\oint_{\partial\sigma} \mathbf{V} \cdot d\mathbf{r} = \iint_\sigma \text{curl } \mathbf{V} \cdot \mathbf{n} \, d\,\sigma.$$

4. Volume generated by the vectors $\vec{A}, \vec{B}, \vec{C}$.

If $\vec{A} = (A_1, A_2, A_3)$, $\vec{B} = (B_1, B_2, b_3)$, $\vec{C} = (C_1, C_2, C_3)$, then

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

5. Polar coordinate system $(r \geq 0,\ 0 \leq \theta \leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}, \quad dxdy = rdrd\theta$$

6. Cylindrical coordinate system $(r \geq 0,\ 0 \leq \theta \leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}, \quad dxdydz = rdrd\theta dz$$

7. Spherical coordinate system $(r \geq 0,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi)$

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}, \quad dxdydz = r^2\sin\theta drd\theta d\phi$$