

FOURIER TRANSFORM

$$\hat{f}(\alpha) = \int_{-\infty}^{\infty} f(t) e^{-i\alpha t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\alpha) e^{i\alpha t} d\alpha$$

Properties:

$$f(t) \quad \hat{f}(\alpha)$$

$$f(at) \quad \frac{1}{a} \hat{f}\left(\frac{\alpha}{a}\right)$$

$$f(t-a)$$

$$f(t) e^{-iat}$$

$$\hat{f}(\alpha) e^{-ia\alpha}$$

$$\hat{f}(\alpha + a)$$

$$\frac{d^m}{dt^m} f(t)$$

$$(-it)^m f(t)$$

$$\int_{-\infty}^{\infty} g(t-z)f(z) dz$$

$$(i\alpha)^m \hat{f}(\alpha)$$

$$\frac{d^m}{d\alpha^m} \hat{f}(\alpha)$$

$$\hat{g}(\alpha) \hat{f}(\alpha)$$

$$\int_{-\infty}^{+\infty} |f(t)|^2 dt$$

$$e^{-\frac{t^2}{2\sigma^2}}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |\hat{f}(\alpha)|^2 d\alpha$$

$$e^{-\frac{\alpha^2 \sigma^2}{2}}$$