UNIVERSITY OF NOTRE DAME
DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING

Professor H.M. Atassi
113 Hessert Center
Tel: 631-5736
Email:atassi@nd.edu

AME-60612
Mathematical Methods II

Homework 9

1. Thin Airfoil in Supersonic Flow

Consider an airfoil of chord length c placed in a uniform supersonic[1]stream, U_∞ at a small angle of attack α as shown in the Figure 1.[2] Let

$$y = f_u(x) \quad 0 \leq x \leq c, \tag{1}$$
$$y = f_\ell(x) \quad 0 \leq x \leq c, \tag{2}$$

represent the upper and lower surfaces of the airfoil. In linearized thin airfoil theory, the velocity is potential and can be represented by

$$\mathbf{V} = U_\infty \mathbf{e}_x + \nabla\phi, \tag{3}$$

where U_∞ is the upstream uniform velocity and ϕ is the perturbation potential due to the presence of the airfoil. $\nabla\phi = \{\partial\phi/\partial x, \partial\phi/\partial y\} = \{u, v\}$, the x and y components of the perturbation velocity. The velocity potential ϕ satisfies the following equation

$$\beta^2 \frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial y^2} = 0, \tag{4}$$

where $\beta^2 = M^2 - 1$. Along the airfoil surface we assume impermeability which, within the approximation of the thin airfoil theory, gives

$$(v)_{u,\ell} = U_\infty(\frac{df_{u,\ell}}{dx}). \tag{5}$$

At large distance, the velocity field is assumed to be finite.

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[1]A flow is supersonic if its velocity is higher than the speed of sound, i.e., the Mach number $M > 1$.
[2]Since $\alpha << 1$, $sin\alpha \approx \alpha$ and $cos\alpha \approx 1$.

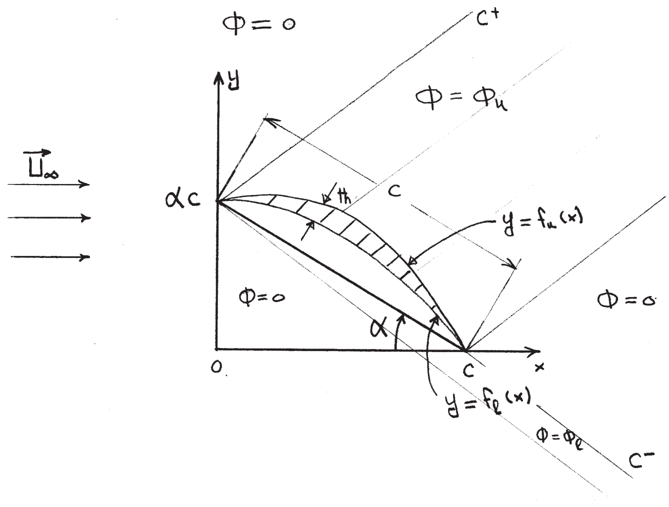

Figure 1: Airfoil in supersonic flow.

(a) Show that (4) is hyperbolic and write the equations for the characteristics $C^{\pm}$. Integrate theses equations and draw the characteristics straight lines with slopes $\pm 1/\beta$. In aerodynamics, the supersonic characteristic lines are known as the Mach lines and they make the angle $\pm sin^{-1} 1/M$ with x axis.

(b) Write the equations to be satisfied by $\{u, v\}$ and integrate these equations. Write the general solution for (4).

(c) Let D_+ and D_- be the upper and lower domains defined by the Mach lines passing by the leading and trailing edges shown in Figure 1. Show that since the flow is supersonic no perturbations can travel upstream and therefore,

$$\phi_u = \phi_u(x - \beta y) \text{ in } D_+, \tag{6}$$
$$\phi_\ell = \phi_\ell(x + \beta y) \text{ in } D_-. \tag{7}$$

and $\phi \equiv 0$ outside D_+ and D_-.

(d) Noting that

$$c_p = -\frac{2}{U_\infty}\frac{\partial \phi}{\partial x}, \tag{8}$$

show that the lift coefficient is given by

$$C_L = \frac{4\alpha}{\sqrt{M^2 - 1}}. \tag{9}$$

Compare with the subsonic lift formula:

$$C_L = \frac{2\pi}{\sqrt{1 - M^2}}(1 + 0.77\theta)(\alpha + 2m), \tag{10}$$

where θ is the thickness ratio and m, the camber.

2. Problems 2, 3, 4(a, b, c, d, e), and 5(a, b, c, d, e); page 597.
3. Problems 6, 13, and 14; page 598.