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Mathematical Methods II

Homework 6

1. Problem 3; page 241.
2. Problem 5; page 242.
3. Problem 8; page 243.
4. Problem 10, 11; page 245.
5. Problems 22: (b), (c), and (d); page 247.
6. Problems 23, 24; page 248.
7. We have solved in class the following boundary-value problem

$$\mathcal{L}y = y'' + k^2 y = k^2 x, \tag{1}$$
$$y(0) = 0, \tag{2}$$
$$y(1) = 0. \tag{3}$$

The solution depends on the parameter k.

(a) $k \neq n\pi$
The solution is unique and is given by

$$y = x - \frac{sinkx}{sink}. \tag{4}$$

(b) $k = n\pi$
The existence of the solution depends on whether the right-hand-side of (1) is orthogonal to $\mathcal{N}(\mathcal{L})$, which is one-dimensional and can be represented by the function $sinn\pi x$. It is easy to verify that this is not the case since,

$$\int_0^1 xsinn\pi x dx \neq 0.$$

Therefore we concluded there is no solution.

(c) It is interesting to examine what happens using the analogy between calculus and linear algebra. Equation(1) can be discretized using a second order scheme where we approximate y'' by

$$\frac{y_{n+1} + y_{n-1} - 2y_n}{\Delta^2}.$$

This leads to the algebraic system of equations

$$y_{n+1} + (-2 + k^2\Delta^2)y_n + y_{n-1} = k^2\Delta^2 x_n, \tag{5}$$

where $\Delta = x_{n+1} - x_n$.

Solve numerically this system for $k \neq n\pi$ and for $k = \pi$ and compare with the exact analytical solution. Examine the case where the right-hand-side of (1) is equal to $k^2(1 - 2x)$, which is orthogonal to $\mathcal{N}(\mathcal{L})$.