UNIVERSITY OF NOTRE DAME
DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING

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Mathematical Methods II

Homework 4

I. We have shown in class that if ρ is the car density per unit length and q is the number of cars crossing a point per unit time, ρ and q are connected by the conservation equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0. \tag{1}$$

As an example we consider the traffic through the Lincoln tunnel in New York. Data were gathered, and it was found that

$$\begin{aligned} q &= a\rho(1 - \frac{\rho}{\rho_j}) && (2)\\ a &= 17.2 \text{ mph} && (3)\\ \rho_j &= 250 \text{ vpm} && (4)\\ & && (5) \end{aligned}$$

We assume the following initial distribution for the cars.

$$\rho(x, 0) = \bar{\rho} e^{-2x^2}, \tag{6}$$

where $\bar{\rho} = 20$.

1. Calculate the car density at which the flux is maximum and plot the car flux, the wave speed $c(\rho)$, and $dc/d\rho$ versus the car density. What can you say about $dc/d\rho < 0$? Show that the wave will be steepen to the left and flatten to the right.
2. Calculate the breaking time and plot the distribution of the traffic density at different times starting with the initial distribution, including the distributions before, at and after breaking.

3. In order to avoid traffic jams, the tunnel authority opened a second lane. For simplicity, we assume the effect of the second lane is to create a sink with strength $S\rho$, where S is a constant characterizing the sink strength. Derive the conservation equation connecting ρ, q and S. Examine the traffic flow with a constant sink and derive the conditions for breaking to occur. Plot the distribution of the traffic density at different times when (i) breaking may occur and (ii) breaking may not occur.