UNIVERSITY OF NOTRE DAME
DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING

Professor H.M. Atassi
113 Hessert Center
Tel: 631-5736
Email:atassi.1@nd.edu

AME-562
Mathematical Methods II

Homework 12

1. In class we have shown that if $k > 0$ and $z = x + iy$

$$f(z) = \int_{-\infty}^{+\infty} \frac{e^{ik\tau}}{\tau - z} d\tau = \begin{cases} 2\pi i\, e^{ikz} & \text{if } \; y > 0, \\ \pi i\, e^{ikx} & \text{if } \; y = 0, \\ 0 & \text{if } \; y < 0. \end{cases} \tag{1}$$

The integral for $y = 0$ must be understood as a Cauchy principal value and was obtained by indenting the contour. The function $f(z)$ is a remarkable example of a discontinuous function along the x axis. This results may appear startling and can be extended to Cauchy integrals along open or closed contours.

   (a) Calculate numerically $f(z)$ and compare the results with the analytically derived expressions. This makes you feel more comfortable with discontinuities.

   (b) If $k < 0$, show that

$$f(z) = \int_{-\infty}^{+\infty} \frac{e^{ik\tau}}{\tau - z} d\tau = \begin{cases} 0 & \text{if } \; y > 0, \\ -\pi i\, e^{ikx} & \text{if } \; y = 0, \\ -2\pi i\, e^{ikz} & \text{if } \; y < 0. \end{cases} \tag{2}$$

   (c) For $z = 0$, verify that (1) and (2) give

$$\int_{-\infty}^{+\infty} \frac{sink\tau}{\tau} d\tau = sgn(k)\pi. \tag{3}$$

2. Problem 90, (b) and (c); page 611.

3. Problem 91, (b) and (c); page 611.

4. Problem 92, (b) and (c); page 611.

5. Problem 104; page 614.